746

PROCEEDINGS

OF THE

PHYSICAL SOCIETY OF LONDON

From December 1920 *to August* 1921.

VOL. XXXIII.

LONDON:
FLEETWAY PRESS, LTD.,
3–9, DANE STREET, HIGH HOLBORN, W.C.1.
MDCCCCXXI

10629

LONDON:
PRINTED BY FLEETWAY PRESS, LTD.,
3-9, DANE STREET, HIGH HOLBORN, W.C.1.

CONTENTS.

VOL. XXXIII.

PAGE

X-ray Spectra, Discussion on 1

Ionisation and Excitation of Radiation by Electron Impact in Helium. By Frederick S. Goucher, Ph.D. 13

The Location of Interference Fringes. By J. Guild, A.R.C.Sc., D.I.C., F.R.A.S. (Optics Department, National Physical Laboratory) 32

Fringe Systems in Uncompensated Interferometers. By J. Guild, A.R.C.Sc., D.I.C., F.R.A.S. (Optics Department, National Physical Laboratory) 40

A New Relay for Moderately Heavy Currents. By Guy Barr, B.A., D.Sc. (From the National Physical Laboratory) 53

Some Slide Rule Improvements. By J. St. Vincent Pletts 57

The Current Density in the Crater of the Carbon Arc. By N. A. Allen, B.Sc. 62

A Sodium Vapour Electric Discharge Tube. By F. H. Newman, M.Sc., A.R.C.Sc., University College, Exeter 70

Absorption of Gases in the Electric Discharge Tube. By F. H. Newman, M.Sc., A.R.C.Sc., University College, Exeter 73

Magnetic Separation of Neon Lines and Runge's Rule. By H. Nagaoka, Professor of Physics, Imperial University of Tokyo 83

A Method of Demonstrating the Retroactive Property of a Triode Oscillator. By E. V. Appleton, M.A., M.Sc..... 100

The Quickness of Response of Current to Voltage in a Thermionic Tube. By D. Owen, B.A., D.Sc., and R. M. Archer, B.Sc., M.I.E.E., Birkbeck College, London...... 104

On the Mechanical Equilibrium of a Sphere of Gravitating Fluid. By Charles H. Lees, D.Sc., F.R.S. (Part of the Presidential Address to the Physical Society, Feb. 13, 1920). *(Continued from Vol. XXXII, p. 270.)*.......... 116

PAGE

Discussion on "The Absolute Measurement of Electrical Resistance, and Instruments Based on the Temperature-Variation of Resistance" 125

On Tracing Rays through an Optical System. (Fourth Paper.) By T. Smith, M.A., F.Inst.P. (From the National Physical Laboratory) 174

On the Refractometry of Prisms. By J. Guild, A.R.C.Sc., F.Inst.P., F.R.A.S. (From the National Physical Laboratory) 179

A Note on the Hot Wire Inclinometer. By R. H. Humphry, M.Sc. (Birm.) (The Sir John Cass Technical Institute) 190

On the Magnetic Susceptibility of Certain Natural and Artificial Oxides. By Prof. E. F. Herroun, F.I.C., and Prof. E. Wilson, M.I.C.E., M.I.E.E. 196

A New Registering Microphotometer. By Dr. W. J. H. Moll, University of Utrecht 207

A Balance Method of Using the Qaudrant Electrometer for the Measurement of Power. By H. Parry 217

Application of the Ionisation Spectrometer to the Determination of the Structure of Minute Crystals. By Prof. Sir W. H. Bragg, F.R.S. 222

The Effect of Viscosity on Orifice Flows. By W. N. Bond, B.Sc. (Eng.), M.Sc. (Lond.), A.R.C.S., A.Inst.P. 225

Viscosity of Water at Low Rates of Shear. By A. Griffiths, D.Sc., and C. H. Griffiths, M.Sc. 231

A Method of Measuring Frequencies. By B. S. Smith and G. F. Partridge, B.Sc. 243

Note on Vacuum Tubes Used as Detectors of Electrical Oscillations. By L. Hartshorn, D.I.C., A.R.C.S., B.Sc., and E. S. Keeping, D.I.C., A.R.C.S., B.Sc. 249

On the Coefficient of Diffusion of Certain Saturated Solutions. By B. W. Clack, M.Sc. (Lond.), Lecturer in Physics at Birkbeck College 259

Experiments on Thermal Transpiration Currents. By G. D. West, D.Sc. (Lond.) 266

PAGE

The Sixth Guthrie Lecture.—Some Recent Applications of Interference Methods. By Prof. A. A. MICHELSON, University of Chicago 275

A Method for the Micro-Analysis of Gases. (From the Research Laboratory of the GENERAL ELECTRIC Co.) 287

On the Reflection of the X-ray Spectrum of Palladium from Fluorspar. By H. PEALING, M.Sc. 297

The Intensity of X-ray Reflection by Diamond. By Prof. Sir W. H. BRAGG, K.B.E., M.A., F.R.S. 304

Capacity and Eddy Current Effects in Inductometers. By S. BUTTERWORTH, M.Sc. 312

New Specific Heat Apparatus. By EZER GRIFFITHS, D.Sc... 355

On Encounters between Non-Spherical Gas Molecules. By Prof. A. O. RANKINE, D.Sc. 362

An Electro-Culture Problem. By C. CHREE, Sc.D., LL.D., F.R.S. 377

Demonstration of Acoustical and Optical Experiments. By Prof. C. V. RAMAN 388

The Stability of Atoms. Lecture by Sir ERNEST RUTHERFORD, F.R.S., Cavendish Professor of Physics, Cambridge, and Professor of Natural Philosophy at the Royal Institution 389

Addendum.—Fig. 7 of Paper on A New Registering Microphotometer. By Dr. W. J. H. MOLL, University of Utrecht. (*See Vol.* XXXIII., *Part* IV., 1921, *p.* 207).. 395

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1920–1921.

October 25, 1920.

Meeting held at the Institution of Mechanical Engineers, jointly with the Faraday Society.

Sir **Robert Hadfield**, Bart., F.R.S., in the Chair.

A Joint Discussion was held on "The Physical and Chemical Properties of Colloids, and their Bearing on Industrial Problems."

Prof. Sir W. H. **Bragg**, K.B.E., M.A., F.R.S., presided over the Discussion.

November 12, 1920.

Meeting held at the Imperial College of Science.

Prof. Sir W. H. **Bragg**, F.R.S., President, in the Chair.

The following Papers were read :—

1. "Ionisation and Excitation of Radiation by Electron Impact in Helium." By F. S. **Goucher**, Ph.D.

2. "Fringe Systems in Uncompensated Interferometers." By J. **Guild**, D.I.C. (From the National Physical Laboratory.)

3. "The Location of Interference Fringes," By J. Guild, D.I.C. (From the National Physical Laboratory.)

4. "A New Relay for Moderately Heavy Currents." By Guy Barr, D.Sc. (From the National Physical Laboratory.)
The relay of Paper 4 was exhibited in action.

November 26, 1920.

Meeting held at the Imperial College of Science, jointly with the Optical Society.

Chairmen : R. S. Whipple, Esq., and W. R. Cooper, Esq.

A Joint Discussion was held on "The Making of Reflecting Surfaces."

December 10, 1920.

Special General Meeting, held at the Imperial College of Science.

Prof. W. Eccles, D.Sc., Vice-President, in the Chair.

On a motion from the Chair the following resolutions were passed :—

1. That Article 31 of the Articles of Association, which now reads :—

"The persons whose names are set forth in the second Schedule hereto, other than those whose names are marked ||, shall pay an annual subscription of £2 2s. each,"

be amended by the following addition :—

"Except that the Council shall have the power to authorise a reduction in the subscription of any Fellow who is a member of the Institute of Physics and also a member of one or more other societies which may be co-operating with the Physical Society of London for any common purpose approved by the Council, according to the scale agreed to by all the co-operating societies ; and that a new

Fellow who at the date of his election is already a member of the Institute of Physics and of one of these co-operating societies shall not be required to pay an entrance fee."

2. That Article 33, which now reads :—

"Unless he be transferred from the class of Student, in which case no entrance fee or transfer fee shall be paid, every Fellow hereafter to be elected shall pay an entrance fee of £1 1s., and an annual subscription of £2 2s.,"

be amended to read as follows :—

"Unless he be transferred from the class of Student, in which case no entrance fee or transfer fee shall be paid, every Fellow who is not a Member of the Institute of Physics and also a Member of one of the Societies participating with that Institute, shall pay an entrance fee of £1 1s."

December 10, 1920.

Meeting held at the Imperial College of Science.

Prof. W. ECCLES, D.Sc., F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. "Some Slide Rule Improvements." By J. ST. VINCENT PLETTS.

2. "The Current Density in the Crater of the Carbon Arc." By N. A. ALLEN, B.Sc.

3. "A Sodium Vapour Electric Discharge Tube." By F. H. NEWMAN, M.Sc., A.R.C.Sc., University College, Exeter.

January 5, 1921.

Special General Meeting held at the Imperial College of Science.

Prof. Sir W. H. BRAGG, K.B.E., F.R.S., President, in the Chair.

On the motion of the President, seconded by Prof. A. W. Porter, the Resolutions passed at the Special General Meeting of December 10, 1920, were unanimously confirmed.

January 5 and 6, 1921.

The Eighth Annual Exhibition of Scientific Apparatus was held in conjunction with the Optical Society.

Discourses were given by—

Sir W. H. **Bragg**, K.B.E., F.R.S. (January 5), on "Sounds in Nature."

Emeritus Professor **Archibald Barr**, D.Sc., LL.D. (January 5 and 6), on "The Optophone—an Instrument that Enables the Totally Blind to Read Ordinary Print," with demonstration by a blind person.

Prof. C. R. **Darling**, F.I.C. (January 6), on "Some Unusual Surface Tension Phenomena."

January 28, 1921.

Meeting held at the Imperial College of Science.

Prof. Sir W. H. **Bragg**, K.B.E., F.R.S., in the Chair.

The following Papers were read :—

1. "Magnetic Separation of the Neon Lines and Runge's Rule." By Prof. H. **Nagaoka**, University of Tokyo.

2. "A Method of Demonstrating the Retro-active Property of a Triode Oscillator." By E. V. **Appleton**, M.A., B.Sc., R.E.

3. "The Quickness of Response of Current to Voltage in a Thermionic Tube." By D. **Owen**, B.A., D.Sc., and R. M. **Archer**, B.Sc., M.I.E.E.

February 11, 1921.

Annual General Meeting.

Held at the Imperial College of Science.

Prof. Sir W. H. **Bragg**, K.B.E., F.R.S., President, in the Chair.

A discussion was held on "Absolute Measurements of Electrical Resistance, and Instruments based on the Temperature-variation of Resistance."

GENERAL BUSINESS.

The Report of the Council was read by the Secretary, Mr. F. E. Smith, O.B.E., F.R.S., and the Report of the Treasurer was read by the Treasurer, Mr. W. R. Cooper, M.A., B.Sc. Both Reports were unanimously adopted.

REPORT OF THE COUNCIL.

In the year 1920, 14 ordinary Science Meetings and one Special General Meeting of the Society were held. Of these the Special General Meeting and 12 ordinary meetings were held at the Imperial College of Science; one ordinary meeting, by invitation of Professor Eccles, was held at the City and Guilds Technical College, Finsbury; and a joint meeting with the Faraday Society was held at the Institution of Mechanical Engineers.

Four of the ordinary meetings were devoted to discussions. One on Relativity was opened by Professor Eddington, F.R.S.; the second on X-Ray Spectra was opened by the President, Professor Sir W. H. Bragg; the third was in co-operation with the Faraday Society and dealt with Colloids, and the fourth was a joint meeting with the Optical Society, the subject being "The Making of Reflecting Surfaces." Each of these meetings was attended by over 100 Fellows and visitors, the average attendance being about 160.

The Guthrie Lecture was delivered on April 23rd, by Dr. Guillaume, who chose as his subject "The Anomaly of the Nickel Iron Alloys: its Causes and its Application." The lecture was attended by 114 Fellows and visitors. Subsequently the Council had the pleasure of congratulating Dr. Guillaume on the occasion of the award to him of the Nobel Prize for Physical Research.

Excluding the Discussions and the Guthrie Lecture the average attendance at the ordinary meetings was 63.

In co-operation with the Optical Society the Annual Exhibition of Apparatus was held on January 4th and 5th. About 40 firms exhibited and over 2,000 Fellows and visitors attended. A number of captured German instruments were exhibited by the Air Ministry and the Admiralty Compass Department. Professor F. J. Cheshire gave two lectures on "Some Polarisation Experiments," and Professor Rankine gave two lectures on "The use of Light in the Transmission and Reproduction of Speech." The lectures were attended by large audiences.

A sub-committee appointed to consider the award for distinguished research of a gold medal, to be known as the Duddell Memorial medal, reported in favour of such a project. A special fund, known as the Duddell Memorial Fund, has been raised by voluntary contributions, and it is hoped to proceed shortly with the preparation of the medal and to circulate particulars of the scheme.

A Report on Atomic Structure by Professor Bohr is in preparation.

At a Special General Meeting held on December 10th, 1920, the following resolutions were submitted and carried *nem con.*

1. That Article 31 of the Articles of Association which now reads :

> "The persons whose names are set forth in the second Schedule hereto, other than those whose names are marked shall pay an annual subscription of £2 2s. each,"

be amended by the following addition :

> "Except that the Council shall have the power to authorise a reduction in the subscription of any Fellow, who is a member of the Institute of Physics and also a member of one or more other societies which may be co-operating with the Physical Society of London for any common purpose approved by the Council, according to the scale agreed to by all the co-operating societies ; and that a new Fellow who at the date of his election is already a member of the Institute of Physics and one of these co-operating societies shall not be required to pay an entrance fee."

2. That Article 33, which now reads :

> "Unless he be transferred from the class of Student, in which case no entrance fee or transfer fee shall be paid, every Fellow hereafter to be elected shall pay an entrance fee of £1 1s., and an annual subscription of £2 2s.,"

be amended to read as follows :

> Unless he be transferred from the class of Student, in which case no entrance fee or transfer fee shall be paid, every Fellow who is not a Member of the Institute of Physics and also a Member of one of the Societies participating with that Institute, shall pay an entrance fee of £1 1s.

The number of Honorary Fellows on the Roll on December 31st, 1920, was nine, and the number of Ordinary Fellows and Students was 507.

Thirty-three new Fellows and one Student were elected during the year.

The Society has to record with regret the deaths of Professor Righi, Sir Norman Lockyer, Sir William Abney and Professor Perry. Professor Righi was an Honorary Fellow of the Society from 1903, and Sir William Abney and Professor Perry were past Presidents of the Society.

REPORT OF THE TREASURER.

From the financial point of view, the past year has been somewhat critical owing to the increasingly high cost of printing. Fortunately there has been a substantial increase in the revenue, the total income being £1,513 2s. 8d., as against £1,261 2s. 7d. for the previous year. This is partly due to the larger membership, but mainly to the much increased sale of publications, amounting to £436 15s. 4d., as compared with £280 16s. 10d. for the previous year.

INCOME AND EXPENDITURE ACCOUNT.

FROM JANUARY 1ST TO DECEMBER 31ST, 1920.

INCOME AND EXPENDITURE ACCOUNT.

From January 1st to December 31st, 1920.

Income.	£	s.	d.	£	s.	d.
Entrance Fees	30	9	0			
Subscriptions by Fellows	598	16	0			
" Voluntary	15	15	0			
" by Students	6	6	0			
" Arrears paid	94	2	0			
" Paid in Advance	17	15	0			
" for "Science Abstracts" and Advance Proofs	24	7	6			
				787	10	6
Composition Fees				63	0	0
Dividends :—						
Furness Debenture Stock	11	3	10			
Midland Railway	28	0	0			
Metropolitan Board of Works	7	0	0			
Lancaster Corporation Stock	8	8	0			
New South Wales Stock	8	17	10			
London, Brighton & South Coast Railway	18	7	6			
Great Eastern Railway	14	0	0			
India 3½% Stock	17	10	0			
Funding Loan	18	4	0			
				131	11	2
Income Tax refunded				34	5	6
Interest on deposit account				20	3	8
Sales of Publications (Fleetway Press, Ltd.)				436	15	4
Optical Society (for Exhibition)				39	16	6
				£1,513	2	8

Expenditure.	£	s.	d.	£	s.	d.
"Science Abstracts"	287	12	0			
" " Extra copies	5	11	6			
				293	3	6
Fleetway Press, Ltd. :—						
"Proceedings"	580	15	9			
Bulletin	68	14	0			
Distribution (Postage)	58	7	3			
General	135	18	3			
				843	15	3
Periodicals	2	14	0			
International Catalogue	11	12	4			
Reporting	56	5	6			
Refreshments and Attendance	92	0	11			
Petty Cash—						
Secretaries' Expenses	20	16	8			
Treasurer's Expenses	16	1	7			
Guthrie Lecture (Honorarium)	10	0	0			
Insurance	3	2	6			
F. S. Spiers (registered address)	2	2	0			
Bank Charges		7	4			
Cheque Book		10	0			
				215	12	10
Donations—						
Conjoint Board of Scientific Societies	10	0	0			
Watt Memorial	10	10	0			
				20	10	0
Total Expenditure for the Year				1,373	1	7
Balance, being excess of income over expenditure				140	1	1
				£1,513	2	8

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,
G. W. O. HOWE, } *Honorary Auditors.*
ROBERT S. WHIPPLE, }

February 8th, 1921.

BALANCE SHEET AT DECEMBER 31ST, 1920.

ASSETS.	£	s.	d.	£	s.	d
Subscriptions in arrears	80	6	6			
Less reserve for subscriptions probably unrealisable	30	0	0			
				50	6	6
Investments (valued at Dec. 31):—						
£533 Furness 3 per cent. Debenture Stock	250	0	0			
£1,600 Midland Railway 2½ per cent. Perpetual Preference Stock	612	0	0			
£200 Metropolitan Board of Works 3½ per cent. Consolidated Stock	160	0	0			
£400 Lancaster Corporation 3 per cent. Redeemable Stock	196	0	0			
£254 2s. 9d. New South Wales 3½ per cent. Ordinary Stock	219	0	0			
£500 London, Brighton & South Coast Railway Ordinary Stock...	250	0	0			
£500 Great Eastern Railway 4 per cent. Debenture Stock	310	0	0			
£500 India 3½ per cent. Stock......	270	0	0			
£650 4% Funding Loan, 1960-90	435	0	0			
				2,702	0	0
Stock of Publications (Treasurer's valuation)				400	0	0
Cash at Bank on Deposit				300	0	0
Cash at Bank, Current Account at Dec. 31	90	17	10			
Adjustment for outstanding cheques	16	15	5			
				107	13	3
Cash in hand (Treasurer's Petty Cash)				1	3	10
				£3,561	3	7

LIABILITIES.	£	s	d.
Life Compositions	1,938	10	0
Balance, General Fund	1,622	13	7
	£3,561	3	7

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,

G. W. O. HOWE,
ROBERT S. WHIPPLE, } *Honorary Auditors.*

February 8th, 1921.

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1920.

		£	s.	d.
140	Fellows paid £10	1,400	0	0
3	Fellows paid £15	45	0	0
10	Fellows paid £21	210	0	0
9	Fellows paid £31 10s.	283	10	0
162		£1,938	10	0

Audited and found correct,

W. R. COOPER, *Honorary Treasurer*.

February 8th, 1921.

G. W. O. HOWE,
ROBERT S. WHIPPLE, } *Honorary Auditors.*

During the past year the Annual Exhibition was resumed, jointly with the Optical Society. This involved a considerable expense, both in refreshments and printing, which was shared by the two Societies. The main increase in expenditure, however, was due to the ever rising cost of printing the publications. The expenditure for the year has thus risen to £1,373 1s. 7d., as compared with £987 6s. 8d .for the previous year, but fortunately the increase in revenue has resulted in a credit balance of £140 12s. 1d. for the year. If the normal activities of the Society are to be maintained there is no doubt that the expenditure must rise still further, but it is hoped that the revenue from the publications (the price of which has now been raised by 50 per cent.) will also increase sufficiently to meet the needs of the case.

The Balance Sheet shows that the position in regard to arrears of subscriptions is much more satisfactory than it was last year. The figure included for the stock of publications has been increased from £250 to £400 as their value is certainly materially greater than it was a few years ago, and their price has now been raised.

The investments, as in previous years, have been valued at market prices through the courtesy of the London County, Westminster & Parr's Bank. They show a further considerable depreciation in value. No further investment has been made, although some life compositions have been received, as it seemed preferable at the moment to retain these fees in the deposit account at the Bank. The cash thus on deposit at December 31st has increased by £50.

On the whole it may be said that the financial position of the Society is much more satisfactory than might have been expected, considering the difficult times through which most of the scientific societies are at present passing.

ELECTION OF OFFICERS AND COUNCIL.

The election of Officers and Council for the ensuing year resulted as follows :—

President.—Prof. Sir W. H. BRAGG, K.B.E., M.A., F.R.S.

Vice-Presidents, who have filled the office of President.—C. CHREE, Sc.D., LL.D., F.R.S.; Prof. H. L. CALLENDAR, M.A., LL.D., F.R.S.; Prof. R. B. CLIFTON, M.A., F.R.S.; Sir RICHARD GLAZEBROOK, K.C.B., D.Sc., F.R.S.; Sir OLIVER J. LODGE, D.Sc., F.R.S.; Prof. C. H. LEES, D.Sc., F.R.S.; Prof. A. W. REINOLD, M.A., F.R.S.; Sir ARTHUR SCHUSTER, Ph.D., Sc.D., F.R.S.; Sir J. J. THOMSON, O.M., D.Sc., F.R.S.; Prof. C. VERNON BOYS, F.R.S.

Vice-Presidents.—Prof. W. ECCLES, D.Sc.; Prof. A. S. EDDINGTON, M.A., M.Sc., F.R.S.; The Rt. Hon. Lord RAYLEIGH, F.R.S.; Prof. Sir ERNEST RUTHERFORD, D.Sc., F.R.S.

Secretaries.—F. E. Smith, O.B.E., F.R.S., National Physical Laboratory, Teddington; D. Owen, B.A., D.Sc., 62, Wellington Road, Enfield, Middlesex.

Foreign Secretary.—Sir Arthur Schuster, Ph.D., Sc.D., F.R.S.

Treasurer.—W. R. Cooper, M.A., B.Sc., 82, Victoria Street, S.W.1.

Librarian.—Prof. A. O. Rankine, D.Sc., Imperial College of Science and Technology.

Other Members of Council.—G. B. Bryan, D.Sc.; C. R. Darling, F.I.C.; Prof. C. L. Fortescue, O.B.E.; E. Griffiths, D.Sc.; F. L. Hopwood, D.Sc.; E. H. Rayner, M.A., D.Sc.; A. Russell, M.A., D.Sc.; T. Smith, B.A.; J. H. Vincent., D.Sc., M.A.; Prof. W. B. Morton, M.A.

On the motions of Dr. Chree, Mr. Clack and Dr. Thomas, respectively, seconded by Mr. Lister, Dr. Borns, and Mr. Paul, respectively, votes of thanks to the Honorary Auditors, to the Officers and Council, and to the Governing Body of the Imperial College of Science, were unanimously passed.

February 25, 1921.

Meeting held at the Imperial College of Science.

Prof. Sir W. H. Bragg, K.B.E., F.R.S., President, in the Chair

The following Papers were read:—

1. "On Tracing Rays through an Optical System." By T. Smith, B.A.

2. "On the Refractometry of Prisms." By J. Guild, A.R.C.Sc.

3. "A Note on the Hot Wire Inclinometer." By R. H. Humphry, M.Sc.

4. "The Magnetic Susceptibility of Certain Natural and Artificial Oxides." By Prof. E. F. Herroun, F.I.C., and Prof. E. Wilson, M.I.C.E., M.I.E.E.

March 11, 1921.

Meeting held at the Imperial College of Science.

Prof. Sir W. H. **Bragg**, K.B.E., F.R.S., in the Chair.

The Sixth Guthrie Lecture was delivered by Prof. A. A. **Michelson**, University of Chicago ; *subject*—

"Some Recent Applications of Interference Methods."

April 8, 1921.

Meeting held at the Imperial College of Science.

Mr. W. R. **Cooper**, M.A., B.Sc., Treasurer, in the Chair.

The following Papers were read :—

1. "A New Registering Microphotometer." By Dr. W. J. H. **Moll**, University of Utrecht.

2. "Application of the Ionisation Spectrometer to the Determination of the Structure of Minute Crystals." By Prof. Sir W. H. **Bragg**, K.B.E., F.R.S.

3. "A Balance Method of Using the Electrometer for the Measurement of Power." By Mr. **Hubert Parry**.

April 22, 1921.

Meeting held at the Imperial College of Science.

Prof. W. **Eccles**, D.Sc., F.R.S., Vice-President, in the Chair.

The following Papers were read :—

1. "The Effect of Viscosity on Orifice Flows." By Mr. W. N. **Bond**, M.Sc.

2. "The Viscosity of Water at Low Rates of Shear." By A. **Griffiths**, D.Sc., and **Constance** H. **Griffiths**, M.Sc.

3. "A Method of Measuring Frequencies." By B. S. **Smith**, and G. F. **Partridge**, B.Sc.

May 13, 1921.

Meeting held at the Imperial College of Science.

Prof. Sir W. H. Bragg, K.B.E., F.R.S., President, in the Chair.

The following Papers were read :—

1. "Notes on Vacuum Tubes used as Detectors of Electrical Oscillations." By L. Hartshorn, A.R.C.Sc., D.I.C., and E. S. Keeping, A.R.C.Sc., D.I.C.

2. "On the Coefficient of Diffusion of Certain Saturated Solutions." By B. W. Clack, M.Sc.

3. "Experiments on Thermal Transpiration Currents." By G. D. West, D.Sc.

May 27, 1921.

Meeting held at University College, London.

Prof. Sir W. H. Bragg, K.B.E., F.R.S., President, in the Chair.

The following Papers were read :—

1. "A Method for the Micro-Analysis of Gases." From the Research Laboratory of the General Electric Co.

2. "On the Reflection of the X-ray Spectrum of Palladium from Fluorspar." By H. Pealing, M.Sc.

3. "The Intensity of the X-ray Reflection by Diamond." By Sir W. H. Bragg, F.R.S.

Sir William Bragg exhibited and explained photographs by M. le Duc de Broglie of the "Magnetic Spectra" obtained if the β-rays produced when X-rays strike copper be deflected on to a photographic plate by a magnetic field.

June 10, 1921.

Meeting held at the Imperial College of Science.

Dr. C. Chree, F.R.S., Vice-President, in the Chair.

A Lecture on "The Stability of Atoms" was delivered by Prof. Sir **Ernest Rutherford**, F.R.S.

June 24, 1921.

Meeting held at the Imperial College of Science.

Prof. Sir W. H. Bragg, K.B.E., F.R.S., President, in the Chair.

The following Papers were read :—

1. "Capacity and Eddy Current Effects in Inductometers." By S. **Butterworth**, M.Sc.

2. "New Specific Heat Apparatus." By Dr. **Ezer Griffiths**.

3. "On Encounters between Non-spherical Gas Molecules." By Prof. A. O. **Rankine**.

4. "An Electro-Culture Problem." By Dr. C. **Chree**, F.R.S.

A demonstration of experiments in Acoustics and Optics was given by Prof. C. V. **Raman**, of Calcutta.

X-RAY SPECTRA.

A Discussion on this subject was held on May 28th, 1920.

Sir W. H. **Bragg**, F.R.S., President, who opened the discussion, said: The X-ray spectrometer, although but a few years old, is an instrument of very considerable precision. I propose to discuss briefly the nature of the measurements that can be made with it, the means of ensuring accuracy, certain results that have been obtained, and a few notable features of those results.

The outbreak of war practically put a stop to the work with the spectroscope, which had been commenced in England, and we have fallen behind other countries which have been able to push on with it. I think it is not generally known how far the subject has advanced abroad, and I hope that the brief discussion which has been arranged for this afternoon may serve to bring its fascinations before members who have not yet had the opportunity of examining it.

In the first place, let us recall that the spectrometer gives the angle at which X-rays of a given wave-length are reflected by a given set of planes belonging to a crystal. The quantities are connected by the relation $n\lambda=2d \sin \theta$, where λ is the wave-length, d the spacing of the planes, θ the reflecting angle, and n an integer representing the order of the spectrum. The spectrometer gives θ, and hence the ratio of λ to d can be calculated at once; but the spectrometer does not find λ or d separately; it gives angles, not lengths. It yields, however, so much information about the relative spacings of the different sets of planes in a crystal that we can, taking also into account the information given by a study of the crystal form, determine the architecture of the crystal, at any rate in the case of such simple crystals as rock-salt, diamond, calcite and so on.

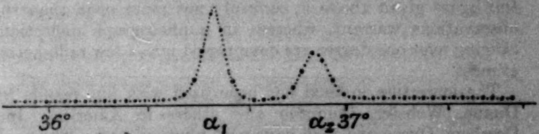
When we know the structure, and if we assume values for the actual weights of the different atoms of which the crystal is composed, and if we also know the specific gravity of the crystal, we can find the spacing of any given set of planes. It is, by the way, convenient to express spacings in terms of the Angstrom unit (10^{-8}cm.). Into these determinations the accuracy of the spectrometer does not enter at all. No more than a rough observation with the spectrometer is required to give the structure, and the remainder of the calculation is concerned entirely with quantities with which the spectrometer has nothing to do—viz., the specific gravity and the weights of the atoms in grams.

Let us remember that the weights of the atoms are found by measuring first the ratio of the charge of an electron to the mass of the ion in electrolysis, and secondly the charge on the electron itself. The best determination of the latter is probably that by Millikan, who used the oil-drop method—viz., $(4{\cdot}774 \pm 0{\cdot}005) \times 10^{-10}$ in electrostatic units. The derived value of the weight of the H atom is $(1{\cdot}662 \pm 0{\cdot}002) \times 10^{-24}$.

The accuracy with which the spectrometer angle can be measured is probably much greater than the accuracy with which the weight of the atom can be found. Fortunately, a knowledge of the angle alone is sufficient for most purposes. Every observer who uses the same kind of crystal starts on the same basis, for the spacing of a crystal depends on the properties of the atoms of which it is composed, and must be the same from whatever source the crystal comes.

Observers do not generally leave their results in terms of spectrometer angles, but assume a spacing for the crystal planes and then calculate wave-length or frequency. This is quite convenient for comparison so long as the same value of the spacing is always used. The crystals in most frequent use are rock salt and Iceland spar. It would be convenient if all workers assumed that the spacing of Na.Cl. (100) was 2·814 A.U. and of $CaCO_3$ (100) was 3·029 A.U. They are no doubt slightly in error—more so absolutely than relatively to each other; but, for most purposes, only relative values of wave-lengths are wanted, and it is more important that every one should have the same standard of reference than that it should be highly accurate.

The spectrometer may be fitted with an ionisation chamber or with a photographic plate. Each method has its advantages. The former gives the angle of reflection directly in terms of the angular graduations of the spectrometer, and involves no determinations of length. A fine pencil of X-rays falls on the crystal, which is revolved about an axis contained by the planes under investigation, until the reflection flashes out; observations of the intensity of the action in the ionisation chamber are made at short intervals, and the maximum point is found. This can be done to a few seconds of arc. The figure shows observations of the lines $K\alpha_1$, $K\alpha_2$ in the rhodium spectrum of the fourth order, diamond being the crystal employed. The angles of reflection lie between 36° and 37°. Any zero of scale error is eliminated by making a corresponding set of observations on the other side. The ionisation chamber is not moved during either set; but, placed in its proper position, is left at rest with a wide open slit. The photographic method gives the positions of lines and absorption edges on a plate; and it is necessary to know the distance between the point of reflection in the crystal and the plate before the angle of reflection can be found. There is an uncertainty in the position of the point of reflection; it is, in fact, no point at all, because the rays are not reflected exactly at the surface of the crystal, but penetrate some distance within. In order to get over this difficulty, Dershem ("Physical Review," June, 1918, p. 461) and Overn ("Physical Review," August, 1919, p. 137) have used extremely thin crystals. Siegbahn ("Phil. Mag.," November 19, p. 639) has used a fairly thin crystal of rock salt placed at right angles to the ray, so that the reflecting planes are in the body of the crystal and are of small area. Uhler and Cooksey ("Physical Review," December, 1917, p. 645) have observed the right and left images for two positions of the plate, first when the plate is a short distance, say 10 cm., from the crystal, and secondly when it has been withdrawn 70 cm. The amount of withdrawal must be exactly known, and can be conveniently regulated by a distance piece, which can afterwards be measured by the same apparatus that is used to measure the separation of the images on the plates. It is easy to calculate the angle of reflection from these data.



$K\alpha_1$ & $K\alpha_2$ of Rhodium in diamond: 4th order.

The photographic method has the advantage of leaving a permanent record, and it is cumulative, since it can be left in operation for hours, during which no observations are required. An observable result is obtained much more quickly with the ionisation chamber than with the photographic plate. I am not aware of any careful comparison, but, from rough measurement of my own, the former is of the order of a thousand times the sensitivity of the latter. But the ionisation method requires a number of successive measurements over separate points in the spectrum in order to make a survey of the region in question, whereas one photograph is alone required. The former method requires greater care in avoiding errors due to fluctuation of the X-ray bulb and to imperfect insulation of the chamber or the electroscope; the latter requires more power. The ionisation method is far more reliable for the purpose of comparing the intensities of different lines, and is also, I would suppose, more accurate in determining their position, since the scale is so much more open. The scale of the figure given above is certainly not more open than the observations warrant, whereas in a photograph deflections ranging over one degree are compressed into a few millimetres at most.

The ionisation method has been used with fine results by Duane, Webster, Compton and others in America; but Dershem, Overn and Hull have used photography. The work of these observers is to be found for the most part in the "Physical Review." Photography has also been used by many other observers doing excellent work, as, for instance, by de Broglie, whose papers are to be found in the "Comptes Rendus"; by Wagner, by Debye; also by Siegbahn and his collaborators at the University of Lund, who contribute to the "Philosophical Magazine." The refined apparatus of the latter is described in the "Philosophical Magazine" of June and November, 1919.

An X-ray spectrum shows:—

1. Fine lines due to the material of the anti-cathode. It is usually intended that the anti-cathode shall be of pure material, impurities may show as faint lines imposed on the true spectrum. There may also be faint lines due to monochromatic rays scattered by the anti-cathode, but not originally produced there.

2. Absorption effects due to the substances traversed by the rays. These are of two kinds:—

(*a*) Any substance traversed by the rays on their way to the plate, such as absorbing screens intentionally placed there, or gases, or the wall of the bulb, or the crystal, or even the substance of the anti-cathode itself will absorb the rays on the short side of certain critical wave-lengths more than the rays on the long side; the critical wave-lengths are characteristic of the atoms passed through. Consequently, the plate shows a darkening (intenser action) on the *long* wave-length side; the dividing line may be referred to as an absorption edge.

(*b*) Any substance in the photographic film itself or sufficiently near to it will, owing to this same absorption effect, cause more action on the film on the *short* side of a critical wave-length than on the *long*, because high absorption means excessive production of cathode rays and it is these that affect the plate.

There are no absorption lines, or bands properly so called; unless the term can be applied to certain narrow fluctuations of intensity close to an absorption edge. These have been announced as observable in certain cases; but the edge is usually so sharp as to deserve the name of a discontinuity.

The lines due to a given substance used as anti-cathode are, as is well known, divided into series, *K*, *L* and *M*, the first-named consisting of the shortest wave-lengths. These constitute the characteristic radiations discovered by Barkla. From certain indirect evidence the existence of a *J* series, shorter than the *K*, has been presumed; but this is not confirmed by a direct examination (Duane and Shimizu, "Physical Review," November, 1919, p. 389).

The subjoined table is compiled from the results of a few of the workers in this subject, and is arranged to show generally the wave-lengths and intensities of the lines in the *K* and *L* series, so far as they have been observed, and incidentally the extent of the concordance between different observers. They all refer to one substance, tungsten.

There are certain definite numerical relations between the frequencies of lines and absorption edges. These are apparently in all cases of the form :—

Difference between the frequencies of two absorption edges =frequency of a line.

Kossel drew attention some time ago to the probable existence of relations between the frequencies, but supposed that they involved line frequencies only. Duane and his collaborators find that their accurate expression brings in the

SPECTRUM LINES.

K Series.

	Siegbahn $CaCo_3=3·029$	Duane $CaCO_3=3·028$	Dershem $NaCl=2·814$	Ledoux-Labard & Dauvillier.
$K\alpha_2$	0·21352	0·21341	0·2124	0·2128
$K\alpha_1$	0·20885	0·20860	0·2076	0·2053
$K\beta$	0·18436	0·18420	0·1834	0·1826
$K\gamma(\beta_2)$	0·17940	0·12901	0·1784	0·1768

L Series.

Intensity (Siegbahn)	Siegbahn $NaCl=2·81400$ $CaCO_3=3·02904$	Overn $NaCl=2·814$	Dershem $NaCl=2·814$	Compton $CaCO_3=3·0279$
3 λ......	1·67505	...	...	...
3 α_2 ...	1·48452	1·4839	1·4828	1·4846
10 α_1 ...	1·47348	1·4731	1·4722	1·4736
η ...	1·417	1·4177	1·4163	...
2 β_4 ...	1·29874	1·2984	1·2977	1·2987
...	1·2871	1·2872	1·2868	...
8 β ...	1·27917	1·2793	1·2784	1·2792
3 β_3 ...	1·26000	1·2598	1·2586	1·2602
6 β_2 ...	1·24191	1·2434	1·2416	1·2421
...	1·2395	1·2355	...	...
...	1·2205	1·2212	1·2202	1·2187
...	1·2118	1·2132	...	...
...	...	1·2097	1·2098	...
β_5 ...	1·2031	1·2021	1·1773	...
...	1·1284	1·1302	1·1292	...
3 γ_1 ...	1·09553	1·0967	1·0953	1·0965
...	...	1·0794	...	...
...	...	1·0724	1·0705	...
1 γ_2 ...	1·06584	1·0659	1·0648	1·0653
1 γ_3 ...	1·05965	1·0596	1·0587	1·0584
...	...	1·0446	1·0427	...
γ_4 ...	1·02647	1·0263	1·0253	1·0251

absorption edges. For example, in a recent Paper ("Physical Review," July, 1919, p. 67, April, 1920, p. 329), he gives the following figures :—

TUNGSTEN.

K Spectrum.

	Wave-length.	Reciprocal of wave-length.
$K\alpha_2$	0·21341	4·6858
$K\alpha_1$	0·20860	4·7938
$K\beta$	0·18420	5·4290
$K\gamma$	0·17901	5·5864
Absorption edge *Ka*	0·17809	5·6151

There are at least two absorption edges in the L series, given by Duane as $La_1=0{\cdot}8240$ and $La_2=0{\cdot}9323$.

$Ka-La_1$ (the difference between the reciprocals of two absorption edges)

$$=5{\cdot}6151-0{\cdot}8240$$
$$=4{\cdot}7911$$
$$=K\alpha_1 \text{ within errors of experiment,}$$
$$Ka-La_2=5{\cdot}6151-0{\cdot}9323$$
$$=4{\cdot}6828$$
$$=K\alpha_2 \text{ within errors of experiment.}$$

Also, Stenström has observed three absorption edges in the M series of uranium and thorium, from which the reciprocal for tungsten can be found by extrapolation; these are:—

$$Ma_1=0{\cdot}152,\ Ma_2=0{\cdot}157,\ Ma_3=0{\cdot}182.$$

The difference $Ka-K\beta=5{\cdot}6151-5{\cdot}4290$

$$=0{\cdot}1861,$$

and again the agreement is within the experimental errors.

The K series of lines peculiar to any substance can be excited either by X-rays, or by electrons. In the former case the incident X-rays must have a frequency greater than that of the absorption edge; in other words, the quantum energy of the exciting X-ray must be greater than the quantum energy of the edge. In the latter case, the energy of the electron must exceed the same limit. The whole series is excited at once, and the intensities of the various numbers appear to be always in the same proportion. If in the X-ray bulb the speed of the whole electron stream is increased, the intensity of the series increases with it. Papers dealing with this branch of the subject are by Hull ("Physical Review," January, 1916), Webster ("Physical Review," June, 1916), Webster and Clark ("Proceedings" Natural Academy of Science, 1917, p. 181), and Blake and Duane ("Physical Review," December, 1917).

It is reasonable to suppose that any one electron cannot excite more than one member of a series since its energy is not greater than the quantum energies of any two lines. In that case the relative strength of the lines must represent the relative chances of those lines being excited by any one electron. This is in agreement with the fact pointed out by

Duane that the well-marked absorption edge La_1 goes with the strong line $K\alpha_1$, and the weaker La_2 with the weaker line $K\alpha_2$.

An electron can excite X-rays of all frequencies for which the quantum energy is less than its own energy; so that a stream of cathode rays excites a continuous spectrum limited at the high-frequency end. But it does not excite any of the series of lines unless, as explained above, its energy exceeds the quantum energy of the edge belonging to those lines.

To each edge in the L series belongs its own set of lines. There are certain differences of character between set and set. The frequencies of the lines α_2, α_1, β_2 and β_5 which belong to La_1 increase linearly with the atomic number of the atom from which they come; while the frequencies of the lines $\eta\beta_4\beta_1\gamma_1\gamma_2$ which belong to La_2 increase rather more rapidly. (*See* Webster, "Physical Review," March, 1920, p. 238.)

The frequency of the absorption edge, perhaps the most singular of all the frequencies associated with an atom, increases steadily with the atomic number, following Moseley's principle. Its importance has led to its being carefully measured by de Broglie, Duane, Siegbahn, Wagner and others; and it is clearly shown that the ratio of frequency to atomic number though constant for the smaller numbers rises somewhat sharply for the larger. Duane puts forward an interesting view that the *velocity* of the electron associated with the absorption edge rises linearly with atomic number, and that the uneven rise of the *energy* (energy of electron or quantum energy of exciting X-ray) is due to the increase of the mass of the electron with its speed.

The relation between the frequencies of lines and edges in the X-ray spectra have been closely examined because of the bearing which they have on the theories of Sommerfeld.

For although Sommerfeld's main discussion refers to the origin of the fine structure of the ordinary spectrum lines in the cases of helium and hydrogen, yet he has also considered the case of the very high frequencies of the X-rays. The frequencies of X-rays are of the order of a thousand times as great as those of visible rays, the electron speeds which excite and are excited by them are so large that the movements of the electrons are readily detected and measured; and, generally, the whole phenomena depending on the connection between wave motion and electron movement are displayed on a far more open scale than in the case of light. At the same time, they are much less complicated. Moreover, and again as a consequence of high electron speed, the effects of the varying mass of the electron are much more obvious. Thus the study of X-ray spectra, now that measurements can be made with so much precision, may be expected to make a contribution of special value to the radiation question.

Mr. Charles Darwin gave the following account of the theory underlying the production of X-ray spectra: We are to consider the application of Bohr's theory to X-rays, and especially the work of Sommerfeld in this country. To recapitulate the theory, the parts of an atom move according to the laws of dynamics, but not all dynamical motions are permissible. There is a series of permissible states in which the atom can exist without radiating, the elements of the orbits being determined by quantum principles. Radiation occurs by the system passing with loss of energy from one permissible state to another, according to the well-known law $W_2 - W_1 = h\nu$, to determine ν. Without any more definite idea of the structure we can get out a good deal about X-rays. First, Bohr's idea leads at once to the Ritz principle of combination, by which the sum or difference of frequencies is also an observed frequency. This is, in fact, observed in X-ray spectra, and hence the study of them can be brought down from that of the lines themselves to the energies of the initial and final states of each. It is convenient to have a word for these energies, and they are called levels. By the general study of the X-ray spectra we find that there is one and only one lowest level in the K region—*i.e.*, W_κ has a much larger *—ve* value than for any other permissible state. In the L region there are two fairly near together, W_{L} and W_{L}' and a third and fourth (hitherto unexplained) called W_{A} and W_{A}'. The next group of levels is called M, and so on. They increase rapidly in complication. Then we can obtain radiations of frequencies given by $\nu(K_\alpha) = \frac{W_{\mathrm{L}} - W_\kappa}{h}$; $\nu(K_\alpha') = \frac{W_{\mathrm{L}}' - W_\kappa}{h}$ (a close doublet); $\nu(K_\beta) = \frac{W_{\mathrm{M}} - W_\kappa}{h}$; &c. Again, $\nu(L_\alpha) = \frac{W_{\mathrm{M}} - W_{\mathrm{L}}}{h}$; $\nu(L_\beta) = \frac{W_{\mathrm{M}} - W_{\mathrm{L}}'}{h}$, and so on. All this follows without any more precise assumption about the natures of the permissible orbits. A further support to our explanation is given by the intensities. Suppose the level W_L is more likely to arise than W_L', and that $W_L > W_L'$. Then $\nu(K_\alpha)$ is stronger than $\nu(K_\alpha')$, and it is of higher frequency. But as W_L occurs with the $-$ sign in the *L* series, we conclude that in this series the line of lower frequency will be the stronger. This is borne out by observation. So much for the general developments. To get any further, more definite assumptions must be made about the structure of the atom. We therefore imagine that it is made up of rings of electrons round a nucleus. We call the innermost ring *K*, the next *L*, and so on. Perhaps, *K* has 3 and *L* 9 electrons for the permissible position which has least energy. Suppose that a shock is given, say, by an incident X-ray. Nothing happens unless the frequency is such that $h\nu$ is energy sufficient to change the arrangement to another permissible position. If it is great enough absorption occurs. Now suppose we enquire what conditions are required to lift out a *K* electron. The *L*, *M*, &c., rings are full up, so it is no use trying to put it there. Therefore, the minimum frequency which can be absorbed corresponds to W_κ, the energy necessary to lift a *K* electron right out of the atom. When this has occurred the *K* ring is short by one electron, and it proceeds to replace it. This it may do by borrowing one from one of the other rings. Say it comes from the *L* ring. Then the subsequent energy will be W_L instead of W_κ, and so the emission will be K_α. If from the *M* ring it will be K_β. Thus the absorption takes place in one step, but the subsequent emission may be in several. This explains the fact that the absorption frequency is higher than the emission frequencies of the lines.

Dr. E. A. Owen referred to the four kinds of frequencies of vibrations associated with X-ray spectra. In the case of the *K*-series spectra there are (1) several emission frequencies, (2) one frequency such that when multiplied by Planck's constant the product is equal to the minimum of energy the electron in the X-ray tube must have to produce that emission series, (3) one critical absorption frequency, and (4) one critical ionisation frequency. Moseley found from the examination of the emission spectra of the elements that the square root of the frequencies was approximately a linear function of the atomic numbers of the elements emitting them, the heaviest elements showing a slight divergence from the linear relation. Duane, examining the critical absorption frequencies of the elements from manganese to lead, found a similar relation with a similar irregularity in the case of the heavier elements.

Calculating the velocity of the electron in the X-ray tube necessary to produce a given radiation from the quantum relation

$$\frac{\frac{1}{2}m_0\nu^2}{\sqrt{1-\nu^2/c^2}}=h\nu,$$

he observed that the relation $v=\nu_0(N-3/2)$, where $\nu_0=0{\cdot}00678c$ was a more accurate representation of the experimental results ; the greatest variation in the value of v given by this equation from that deduced from experimental observations amounted to a little over 1 per cent. The critical absorption frequency is found to exceed the frequency of the γ-line in the emission series by 0·2 or 0·3 per cent., and the critical ionisation frequency to be equal to the critical absorption frequency to within less than 0·1 per cent. In the L-series spectra three critical absorption frequencies are observed, and Duane finds that each α-line frequency is equal within experimental error, to the difference between the K-critical absorption frequency and one of the L-critical absorption frequencies. Assuming the same law to hold for all emission spectra it ought to be possible to deduce the critical absorption frequencies of other series.

Some experiments on the measurement of the relative intensities of the α and $\beta\gamma$ peaks reflected by different crystals were described. For crystals containing atoms which do not selectively absorb radiation incident upon them, the relative intensities of radiations of two nearly equal frequencies should remain constant. If, however, one of the radiations is selectively absorbed whilst the other is not, the relative intensities of the reflected beams will be greatly altered ; in fact, the measurement of the relative intensity of two radiations of nearly equal frequencies would serve as a method of determining the critical absorption frequencies of atoms comprising the crystal, by comparison with a suitable standard of reference.

The method of measuring the intensity of a peak was that employed by Bragg, by turning the crystal at a definite rate from one side of the maximum to the other. This agreed well with another method of measurement in which the ordinates of the peak were experimentally determined and plotted, and the area measured directly. The K-series lines of Palladium and Rhodium were employed and reflections measured from the five crystals $CaCO_3$, CaF_2, SiO_2, S, CSi. No regular change in the ratio of the intensities of the peaks with specific gravity of crystal, was observed, but in both crystals containing calcium the ratio was higher than in the other cases. On the assumption that the crystal selectively absorbs the radiation, the high ratio for the crystals containing calcium could be explained if calcium emitted a radiation of wave-length somewhere in the neighbourhood of $0{\cdot}52 \times 10^{-8}$ cm. This would have to be a radiation of the J-series. At present there is no definite proof that this series exists, so that possibly the phenomenon might be due to another cause.

Dr. E. H. **Rayner** asked the order of magnitude of the quantities W_{κ}, &c.

The **President** replied that they were of the order 10^{-11} c.g.s. units.

Prof. **Fortescue** proposed a vote of thanks to the President and the other speakers for their able survey of the work that had been done in X-ray spectroscopy.

Prof. **Rankine** seconded, pointing out the value to teachers of these co-ordinated accounts of a particular branch of physics at which they might not be working themselves.

I. *Ionisation and Excitation of Radiation by Electron Impact in Helium.* *By* Frederick S. Goucher, *Ph. D.*

Note.—This work has been carried out with the aid of a grant from the Council of Scientific and Industrial Research during the months February to July, 1919, at the Carey-Foster Physical Laboratory, University College, London University.

Introduction.

This investigation was undertaken as a continuation of the experiments of Davis and Goucher (1) on the ionisation and excitation of radiation by electron impact in various gases.

The method there employed was a modification of that first used by Lenard for the direct determination of the ionisation potentials of gases; the modification being introduced for the purpose of determining whether or not the effect hitherto interpreted as resulting from ionisation was not due at least in part to a photo electric action on the metal parts of the apparatus, produced by radiation from the bombarded gas atoms. It had been applied to the case of mercury vapour, hydrogen and nitrogen showing that for these gases at least two critical potentials existed, the lowest one of which in each case corresponded to the production of radiation by the electron impacts on the gas atoms, while ionisation did not take place until a higher potential was reached.

Intermediate critical potentials were found which, along with these others, bore definite relations to the spectra of these gases as calculated by the quantum theory of radiation. Analogous effects have been shown for the case of a number of the metallic vapours, by other experimenters using a different method (2).

The application of the method to the case of helium seemed specially desirable both from the theoretical point of view and because indirect experimental evidence pointed to a higher ionisation potential for this gas than had been found by direct measurement hitherto.

It was with this end in view that the investigation was undertaken. Before its completion, however, Horton and Davies (3) published an account of experiments on helium in which this method with extensions was applied. These showed that two critical potentials did exist for this gas, the lower of which—20·4 volts—corresponded to the production of radiation only, while the higher—25·6 volts—corresponded to the ionisation potential. For a full discussion of the theoretical significance of these results and a comparison with previous experimental evidence, reference should be made to their Paper. It was considered advisable to continue with the investigation, however, because it was proposed to introduce certain modifications into the method which would render it more sensitive, with special regard to the separation of the ionisation and radiation effects.

The results obtained were in good agreement with those of Horton and Davies with the exception that some ionisation was found to accompany the production of radiation at the lower critical potential. The effect of impurity was suspected, but this could not be established before the investigation was discontinued. A recent Paper by Horton and Bailey (4) gives evidence for this view and offers an explanation for the marked effect of minute traces of impurity. That it is a result of impurity is probably correct, though the exact part this impurity plays is not so clear. Other experimenters (5) using the valve method for the measurement of ionisation independent of radiation effects, find ionisation at the lower critical potential only in cases where the gas must be quite pure and the apparatus comparatively free from electrode gas, though Rentschler (6) reports experiments in which the method was used indicating that ionisation did not take place until the bombarding electrons passed through a field of 26 volts.

It seems desirable, therefore, to describe in detail the results obtained in the course of this investigation and to discuss them in the light of these recent experiments.

Description of the Apparatus.

The apparatus used in this experiment was in general similar to that used in the experiments on mercury vapour, hydrogen and nitrogen, and need not be described here in detail. Certain modifications were, however, introduced in the construction of the measuring vessel, and improvements made in the method of gas manipulation, which seem worthy of note.

The platinum thimble used as an equipotential surface electron source in the former experiments was done away with, both on account of the difficulty of freeing it of occluded gas, and also because the method of estimating the critical values of potential was to be a purely relative one in which corrections for distribution of velocities due to temperature and to the potential drop in the filament were no longer necessary. The source of electrons employed was simply a loop of tungsten wire requiring currents of the order of 2 amperes to heat it, and of such a length that the drop of potential as measured between the filament leads outside the vessel did not exceed one volt. Also, the amount of metal was reduced to a minimum in order to facilitate the pumping out of the apparatus and render the evolution of gas during the course of measurement as small as possible. Instead, therefore, of using solid metal electrodes, so called "gas free," nickel wire gauze—mesh 20 to the inch—such as that used in the construction of wireless valves, was employed throughout. All supports were made of glass in so far as possible.

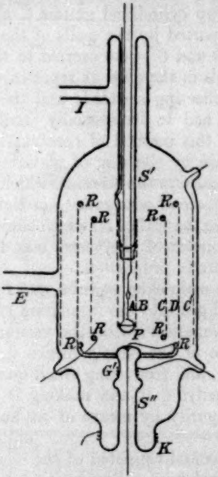


FIG 1

The cylindrical form of vessel was employed as before, with the filament along the axis.

Fig. 1 is a sectional view of the measuring vessel showing the arrangement of the essential parts. *A* represents the tungsten loop used as the source of electrons, heated and supported by means of the leads sealed into the glass part S′S′ as shown. S' also served as a support for the gauze B and contained the seal of the lead to B as shown. The gauze B was bound to S' at its upper end by a fine metal wire, and to a glass cap P at its lower end, which served to insulate it from the filament support wire, which was so bent as to keep A concentric with B.

The glass part S'', to the enlarged parts of which glass cane frames were attached—the *R*'s in the diagram—served as the support for the two cylindrical gauzes C and D. A fourth gauze C' was supported by the walls of the vessel as shown. The leads to C, D and C' were carried to the outside of the vessel through seals in the glass as represented.

D, being the collecting electrode and the one attached to the electrometer, had to be specially insulated, which was rendered easy by this method of construction. It was protected on the inside by the ring of platinum wire G', which was partially embedded in the glass and which could be earthed. On the outside, a piece of wire gauze was tightly bound about the glass part above the seal at K, which could also be earthed.

The outside diameter of the vessel was 4 inches, and the other dimensions are in proportion.

E represents the main exhaust tube, and I an inlet tube for the introduction of gas into the measuring vessel.

Fig. 2 is a diagramatic view of the vacuum system, and is of interest because of the simple means it affords of introducing, circulating, and recovering small quantities of gas, at the same time purifying it and making it possible to conveniently test for purity by means of an auxiliary discharge tube.

The system in general consisted of the measuring vessel V, and connections, including a liquid air trap T_1; a side tube leading to the main pumping system containing a mercury cut-off C; a side tube to McLeod gauge G; a small water cooled mercury condensation pump P_2 connected to the reservoir of a small Töpler pump R_1; connections to the gas reservoir R through the stopcock S_1; to a small Plücker tube D, and to the gas inlet tube I of the measuring vessel through the stop-cocks S_2 and S_3 and the charcoal tube T_2, as shown.

C_1, C_2 and C_3 represent mercury columns controlling the mercury cut off C, the Töpler pumps R_1 and for filling the mercury reservoir R_2 used for operating the condensation pump P_2.

Having thoroughly exhausted the system, liquid air being kept on the traps T_1 and T_2, the cut off C could be closed and the required amount of gas admitted through S_1. By means of the condensation pump it was possible to circulate this gas through V, out through E, and the trap T_1, and in through the charcoal tube T_2 and the inlet I. Also by closing S_2 it was possible to readily remove the gas which could be quickly stored in R_1 at the low pressures used in these experiments. Or by closing S_3 the gas could be compressed into D, for purposes of spectroscopic examination.

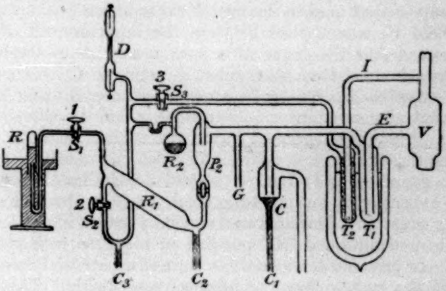


FIG. 2.

The main pumping system consisted of a large mercury condensation pump of the Langmuir type backed by a double Fleüss oil pump electrically driven.

The McLeod gauge was sensitive to ·00001 mm. of mercury.

The measuring vessel was furnished with an asbestos oven heated by means of a bunsen burner in which it could be baked out for many hours at a temperature over 300°C. During this heating the main pumping system could operate continuously and the filament be maintained at a high temperature throughout. This process could be kept up until the evolution of residual gases from the metal parts of the apparatus and the walls of the glass parts was reduced sufficiently to carry on the measurements.

The helium used in these experiments was very carefully purified by Professor Collie. The writer takes this opportunity to express his thanks to Professor Collie for this and other gases furnished for these experiments and for his kindly interest taken in the work.

The electrical apparatus used needs no special comment. Small storage cells were used as sources of potential. The photo-electric and ionisation currents were measured by means of an electrometer of the Dölezalek type. The electron currents were measured by means of a galvanometer.

Method of Measurement.

The method of making the measurements was in general the same as that used in the former experiments.

A field V_1 was applied between the negative end of the filament *A* and the gauze *B* in such a way as to accelerate electrons from *A* through *B* into the region *B C*, where they were retarded by a field V_2 applied between *B* and *C*. V_2 was kept at a constant value—usually about 4 volts greater than V_1—in order to prevent any of the original electrons from passing through *C*.

The gauzes *C* and *C′* were connected together throughout these experiments serving to control the field about the collecting gauze *D*. This field was varied by means of a potential V_3 which maintained *CC′* positive or negative with respect to *D*. In practice *D* was always kept at a potential zero, the rest of the system being controlled accordingly. This was done because of the electrometer used in measuring the current to *D*.

Curves could then be obtained showing the variation of current to *D* with variation of the accelerating field V_1 under different conditions of V_3. These curves furnish a means of estimating the critical potentials of the gas and of differentiating between the current effects due to actual ionisation in the gas and those due to a photo-electric action of the radiation from the bombarded gas atoms.

With V_3 small as compared with V_1 and V_2 positive ions when formed would reach *D* whether V_3 aided or opposed their motion, so that when V_1 exceeds the ionising potential, the current to *D* will be increasingly positive with increase of V_1 due to this cause alone, the tendency to increase occurring at the values of V_1 corresponding to the critical potential at which ionisation takes place. If, on the other hand, no positive ions were formed, and if all or part of the energy of the impacting electron—when exceeding a definite critical value—were transformed into radiation emitted from the bombarded atom, this radiation would fall on C, D, and C' and a photoelectric current would take place either from C and C' to D, or from D to C and C', depending on the direction of V_3. In other words D would charge with either an increasing negative, or an increasing positive current with values of V_1 progressively greater than the critical value at which the effect begins.

Both of these causes may, of course, be operative at the same time, but the interpretation of the curves rests entirely on these assumptions, discontinuities corresponding to critical potentials and the slope of the curves beyond these indicating the predominating effect.

Corrections, of course, had to be applied in the estimation of the absolute values of the critical potentials for which special measurements had to be made. This follows because it was not possible for all the electrons which were accelerated by the field V_1 to have an energy corresponding to this field, both on account of their initial velocity due to temperature, and on account of the drop of potential along the filament. Also, contact differences of potential are likely to occur between the filament and accelerating gauze.

In these experiments on helium it was proposed to measure only the relative values of critical potential and compare them with the ionising potential of mercury as measured in the same apparatus and under similar conditions, thereby eliminating the necessity for making these auxiliary measurements.

The method consisted in making a set of curves first in mercury vapour, then in helium under as nearly similar conditions as possible.

The mercury vapour was admitted to the measuring vessel by the removal of the liquid air from the traps and thoroughly pumping out by means of the main pumping system until the residual-gas evolution was reduced to an amount too small to register on the McLeod gauge. The pump was kept running during these measurements.

The mercury vapour could be removed sufficiently from the measuring vessel, so that it at least contributed no appreciable amount to the ionisation currents. This was done by means of circulating pure helium through the system with liquid air on the traps for a short time, usually less than an hour being required.

For the purpose of making the measurements under similar conditions of electron emission in the two gases it was found necessary to insert a galvanometer in the battery circuit supplying the potential between *A* and *B*. For with equal filament current the emission was found to be quite different in the two cases, due to the effect of the gas, even when the potential V_1 was below the ionising potential of either gas. The filament current was therefore adjusted so as to give equal electron currents in the two cases at a given value of V_1 below the ionising potential of mercury vapour.

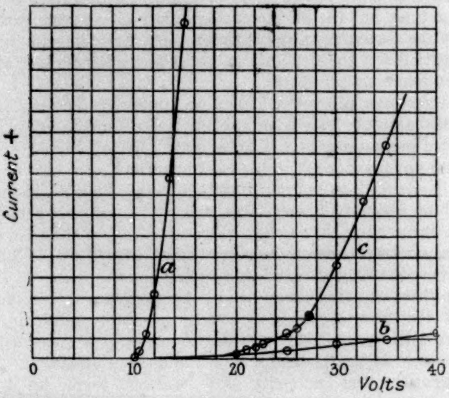


FIG. 3

Experimental Results.

Fig. 3 shows a typical set of curves obtained in mercury vapour and helium for the case where V_3 is zero. This is the case where the effect of radiation on the collecting electrode should be zero provided the state of field about *D* is actually zero.

The presence of the large field V_2 would effect the state of field between *C* and *D* to come extent, and this might be sufficient to cause a slight tendency for *D* to charge positively in the presence of radiation even though no ionisation were present.

Curve (*a*) was obtained with mercury vapour present in the measuring vessel at a pressure corresponding to the temperature of the vessel, which in this case was slightly above room temperature. This should give a pressure of the order of 0·001 mm. The electron emission was adjusted so as to give a convenient reading on the electrometer over a range of a few volts above the ionising potential.

This curve shows a critical potential for mercury vapour at a value between 10 and 10·5 volts, which is in very good agreement with experimental values already obtained and with the value calculated by means of the quantum theory from the wave-length of the line 1188 Angstrom units, which is the head of one of the combination series for mercury vapour. This calculated value is about 10·4 volts, and probably represents the true ionising potential of mercury vapour.

We may then fairly assume that the great majority of the bombarding electrons in this apparatus, working under these conditions, have velocities, the energy of which corresponds to the values of field applied and measured as V_1.

Curve (*b*) was obtained after the removal of the mercury vapour from the measuring vessel. This was done by washing for some time with purified helium, liquid air being kept on both traps and the auxiliary condensation pump being kept running. This helium was then pumped out and the vessel exhausted by the main pumping system for several hours. Then, with this pump running, the measurements were made which are represented in the curve (*b*). That this positive current was not due to the ionisation of residual gas was considered likely, because further pumping did not cause it to diminish, and the pressure was too low to be recorded on the McLeod gauge. It was considered as probably due to the emission of positive ions from the gauze *B* under the bombardment by electrons from *A*, as has been pointed out by Horton and Davies, who obtained a similar "no gas" curve.

Curve (*c*) was obtained with purified helium in the measuring vessel at a pressure less than 0·01 mm. It is typical of a large number obtained at these pressures, and shows definitely two critical potentials, one about 20 volts where it breaks away from the "no gas" curve and one at about 26 volts.

This would indicate two critical ionising potentials, since in this case the field V_3 was adjusted so as to render the collecting electrode comparatively insensitive to the photo-electric effect. This condition may not be exactly fulfilled, as has been pointed out.

Fig. 4 shows curves obtained with variation of the field V_3. from $+1{\cdot}3$ volts to $-1{\cdot}3$ volts. (*a*) is the "no gas" curve, which does not change with the reversal of V_3, (*b*) the curve obtained when V_3 is such as to cause *D* to charge negatively under the action of radiation alone, viz., $V_3-1{\cdot}3$ volts; (*c*) the curve obtained when V_3 is such as to cause *D* to charge positively due to radiation alone, viz., $V_3+1{\cdot}3$ volts.

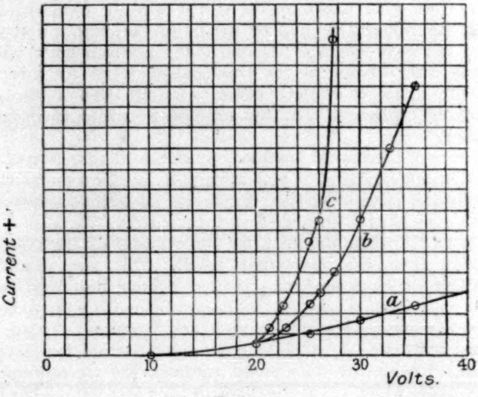


FIG. 4.

Since (*b*) as well as (*c*) have a positive tendency above the lower as well as the higher critical voltage, it would appear that under these conditions ionisation did occur at both the critical potentials, and that if radiation did take place the ionisation effect predominated.

Fig. 5 shows curves obtained with a higher pressure of helium—viz., 0·06 mm. V_3 was given the same value as before and reversed to test for radiation. Curve (*a*) was obtained with $V_3+1{\cdot}3$ volts, curve (*b*) with $V_3-1{\cdot}3$ volts. In this case the radiation effect predominated and showed that strong radiation occurred at the lower critical potential, viz., about 20 volts. The slight tendency to a more positive slope of the curve above 26 volts indicates the increase of ionisation at that point.

The Effect of Radiation Alone on the Metal Parts.

In order to test the assumption that the positive slope of the curve (*b*), Fig. 4, above 20 volts, proves the presence of ionisation, and would have been negative if radiation alone were responsible for the charging up of *D*, the following modification was made in the measuring vessel.

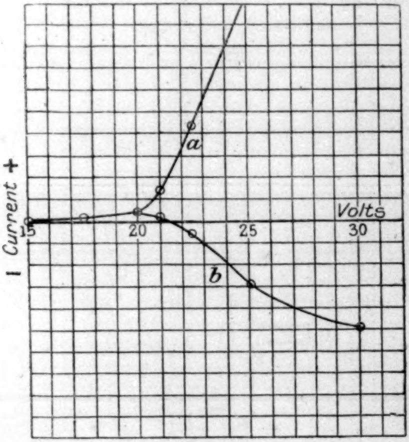


FIG. 5.

The vessel in its modified form is shown in Fig. 6. Instead of the filament source of electrons, a source of ultra-violet radiation was substituted. This was done by means of a silica tube, *Q*, closed at its lower end as shown, containing a pool of mercury, *M*, above which was supported an oxide coated platinum filament. The electrons from the filament could be utilised to produce a luminous discharge in mercury vapour, using the mercury pool as an anode. The silica tube was of the same diameter as the cylindrical gauze *B*, Fig. 1, and served as its support in the modified form, so that the geometrical arrangements of the gauzes remained the same.

The silica tube was sealed into the measuring vessel by means of the glass parts and wax joints, as shown, and was connected with a separate exhaust tube, *I′*.

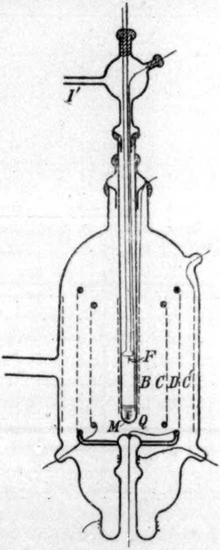


FIG. 6.

The exhaust tube *I′* was not connected to *I*, Fig. 2, as in the first part of the experiment; but directly to the large pump system beyond the mercury cut-off (*c*). *I* was sealed off, otherwise the vacuum system was the same.

The volumes both inside and outside *Q* with the cut-off open could be thoroughly exhausted. The cut-off could then be closed, and gas to any desired pressure could be then introduced into the volume outside of Q. The pressure inside Q could be maintained at a constant low value as regards gases other than mercury vapour by keeping the pump running so that when temperature equilibrium was attained after heating the filament the pressure of mercury vapour remained quite constant.

The arc could easily be struck by applying a high potential between F and M and then maintained by a 25-volt field under the conditions here obtaining. The intensity of the discharge, and hence of the radiation emitted, could be controlled by the electron emission from the cathode.

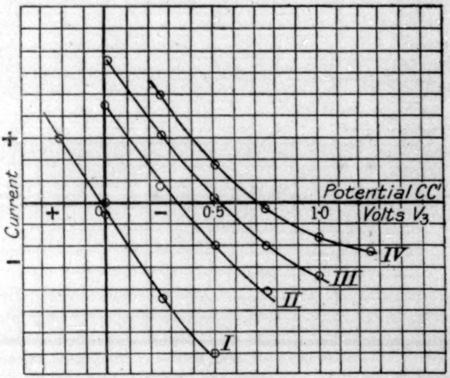


FIG. 7.

B was kept at the same potential as M, and was used to control the field V_2. So that it was possible to duplicate all the field conditions obtaining in the measurements made in the first part of the experiment, with the excption of the field V_1, which in this case was fixed at 25 volts; but this is of the same order of magnitude as the value of V_1 obtaining at the portion of the curves 20-26 volts with which we are specially concerned. In any case, the effect of V_1 on the field about D would be so small that this difference in the field conditions can well be neglected, as compared with the effect of V_2.

The following measurements were made: The arc was struck and maintained by a 25-volt field. Then, with V_2 fixed at a series of values ranging from zero to the highest values obtaining in the measurements made in helium in the first part of the experiment, a curve was obtained showing the variation of current to *D* with a variation of V_3 for each value of V_2. Fig. 7 shows such a set of curves obtained when the gas pressure in the measuring vessel was so low that there could be no appreciable ionisation. The cut-off *C* was kept open, so that this low pressure could easily be maintained. In these curves, *I.*, *II.*, *III.* and *IV.*, V_2 had the values 0, 6, 12 and 24 volts respectively, while V_3 was varied over the ranges shown, sufficient in each case to give a curve cutting the V_3 axis. The electrometer sensitivity was the same as that used in measuring the ionisation and photo-electric currents in the first part of the experiment, and the intensity of the mercury vapour discharge was adjusted to give photo-electric currents of the same order of magnitude.

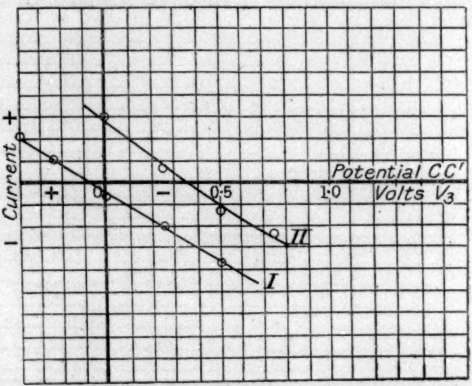


Fig. 8.

These curves show that the assumption that V_3 controlled the field about *D* does not hold entirely, otherwise all of the curves should cut the axis at the point $V_3=0$. The shift, which is in the direction expected as a result of the field V_2, is never more than 0·7 volt when V_2 has its maximum value. The slight shift of 0·1 volt to the left when $V_2=0$ shows that V_1 is here effective to this extent; but this would not be effective in the original measurements, as V_1 is there variable, and would be approximately zero when V_2 was zero.

In order to show the effect of helium on these radiation effects, a set of curves was obtained, Fig. 8, with helium present in the measuring vessel at a pressure corresponding to that used when the curves, Fig. 4, were obtained. Here *I* was obtained when $V_2=0$ volts, *II*. when $V_2=30$ volts. It is seen that this amount of helium diminished rather than increased the effect of the field V_2 on the photo-electric currents.

These curves, therefore, prove that with the value of V_3 used in making the original measurements in helium, if *D* charges positively it must be attributed to a predominance of ionisation currents, and could in no case be attributed to radiation, for with this value—1·3 volts—*D* would certainly charge negatively under the action of radiation alone.

For the purpose of further testing the apparatus as used in these experiments, curves were obtained both in mercury vapour and hydrogen. The results obtained agreed will with those obtained in the experiments of Davis and Goucher, except that in the case of hydrogen no ionisation was obtained below 16 volts.

Discussion of Experimental Results.

The conclusions arrived at in these experiments agree well with those obtained by Horton and Davies with the exception of the ionisation shown to accompany the production of radiation in the helium at 20 volts.

That this is due to the presence of a trace of gas having a lower ionisation potential than helium as pointed out by Horton and Bailey is probably correct. For even though this gas contributed no measurable ionisation current below the 20-volt critical potential due to direct electron impacts, there might still be enough to contribute a measurable current above this value due to radiation from the radiating helium atoms. This would be true both because the volume of gas from which positive ions could be drawn to the collecting electrode *D* would be larger when depending on radiation as the means of ionisation than it would when depending on electron impact, and also because a higher percentage of the gas ions could be ionised by the radiation than by electron impact at the low pressures obtaining.

That the impurity causing this effect is mercury vapour which has not been removed from the measuring vessel seems very unlikely, both because of the washing out with pure gas passing through liquid air, and also because in the observations made on pure hydrogen no ionisation was observed below 16 volts. If there were sufficient mercury vapour present to contribute a measurable ionisation current it would have done so below 16 volts in the presence of the radiation which is known to take place at about 11 volts in hydrogen. The impurity is more likely to be electrode gas or gas coming off of the glass walls of the apparatus than any impurity present in the helium, because this was very carefully purified.

The gases coming off of the glass and metal parts of the apparatus are known to be mainly hydrogen carbon monoxide and water vapour. As charcoal immersed in liquid air was used in the gas circulating system, the hydrogen would be the most likely gas to accumulate in the circulating gas and cause the effect measured. If this is the case, part of the positive current in the "no gas" curve (*b*), Fig. 3, may be due to a steady evolution of hydrogen sufficient to maintain a slight pressure, even though the main pumping system was in operation. In this case the slight increase of positive current observed above the 20-volt critical potential could easily be accounted for by the additional positive ions produced by the radiation.

If the production of positive ions at the lower critical potential in helium could be shown not to be due to the presence of impurity—which was found to be impossible in this investigation without the construction of new apparatus —it would indicate the likelihood that some of the normal helium atoms were absorbing sufficient radiant energy, so that they could be ionised with a 20-volt electron impact.

Summary.

Measurements have been made of the critical potentials for helium by the method used in the experiments of Davis and Goucher and compared with the ionising potential of mercury vapour taken as a standard.

Assuming the ionising potential of mercury to be 10·4 volts, two critical potentials occur in helium. One at about 20 volts and the other at about 26 volts. These critical values agree well with those obtained by Horton and Davies.

The effect of radiation alone on the metal parts of the apparatus was studied under conditions which would yield evidence of use in the interpretation of the results obtained when the production of both ionisation and radiation was taking place simultaneously.

The conclusion was reached that the lower critical potential was a radiation potential, though some ionisation was produced also at this potential. This, however, was attributed to the presence of impurity, probably hydrogen. The higher critical potential was that at which ionisation took place.

The writer wishes to thank Sir William Bragg for the apparatus and facilities used in the carrying out of this investigation and for his kindly interest in the experiments.

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(6) H. C. Rentschler. "Phys. Rev.," Vol. XIV., p. 503, 1919.

DISCUSSION.

Mr. B. S. Gossling, speaking on behalf of himself and Mr. J. W. Ryde, said: Dr. Goucher has given a most illuminating critical discussion of his results. Material furthering this discussion may perhaps be found in some observations made by us last year in the laboratories of the G. E. C. bearing on the question of the existence and effect of impurities.

The object of the experiment was actually to trace the variation with time of the effects of a small quantity of impurity known to be present at the beginning. The method of observation was that described in a recent Paper by G. Stead and B. S. Gossling, ("Phil. Mag.," 1920, v. 40, p. 413), which, in the light of these later results requires qualification in respect of the statement that the ionisation potential of helium is 21 volts.

The invariable final result shown in the cases I have to bring forward, and in the very similar case described by Horton and Bailey ("Phil. Mag.," 1920,

v. 40, p. 440), is that no positive ions appear until voltages considerably higher than 20 or 21 volts are reached. This indicates that the lower ionisation point is not a characteristic of pure helium, but is due in some way to an admixture removable by the discharge.

The observations were made on about a dozen valves, all of which behaved in a generally similar manner. These were subjected to a more severe exhaust with heating by electronic bombardment lasting an hour or more on a better pumping system than that used by Stead in the earlier case, but after filling with helium to 0.7 mm. pressure were generally sealed off instead of being left over charcoal and liquid air.

Taking the current-voltage curves for a typical case and plotting the logarithms of current and voltage so that the "no gas" curve is very nearly straight, and the curvature at the various voltage values where ionisation sets in correspondingly more distinct, we find a pronounced breaking away from the "no-gas" curve at about 15.5 volts, indicating the presence of a very considerable amount of impurity, presumably hydrogen or carbon monoxide, or both. After half an hour's running, however, with an anode potential of 150 volts there is a very marked change, the curve obtained being like Stead's with the critical point rather above 21 volts. But later curves taken after seven, nine and twelve hours show a further, but much slower, progressive change, and finally no positive ions are found to appear before 25 volts at the least. In some of the sealed valves the final critical voltage was well above this value, and there were two cases where the valve was left over liquid air where the first definite curvature is at 27 volts. In a few sealed valves curves were obtained showing, like Dr. Goucher's, two critical points, at 20-21 volts and 24-25 volts.

What was happening seems to be that with the exhaust treatment given the valve was left in such a state that the impurities disappeared under the action of the discharge just as they would have done had the helium not been there.

Pumping with the electrodes cold will not remove the last traces of impurity, nor will charcoal, as Dr. Goucher and Mr. Stead agree. Very possibly the impurity is "condensed" on the surfaces of the electrodes, and is ionised in situ. The use of gauze electrodes of large total surface would favour this. A given quantity so condensed would give many more ions than the same amount spread throughout the tube.

On the other hand, prolonged passage of the discharge does in the first put a stop to the formation of positive ions between 15 and 20 volts, apparently by removing the impurity, even in a sealed tube, to a place where it is no longer ionised by impact. Above 20 volts the impurity may still be accessible to helium radiation for a time, but ultimately its effects disappear and we are left with an ionisation at between 25 and 27 volts of which there is a better probability that it is really due to helium.

The considerations here put forward seem to apply to the recent work of Compton ("Phil. Mag.," 1920, v. 40, p. 553) equally with that of Dr. Goucher.

The **President** asked if argon behaved in a similar way to helium as regards the effect of impurities.

Mr. **Gossling** mentioned that Stead's values for argon were also low, but whether this was due to impurities or not he could not say.

The **President** asked if these critical potentials corresponded to the production of lines in the spectrum of the gas.

Dr. **Goucher** said the radiation potential did correspond to a line. In mercury, for instance, the radiation potential was 4.9 volts, which, as McLennan had shown, corresponded to the production of a single line in the mercury spectrum. The many-lined spectrum was produced when the potential was at or above the ionisation potential. The results obtained by Messrs. Gossling and Ryde portray in a most striking manner the effect of impurity on the discharge in helium above its radiation potential. They give no evidence of any secondary ionisation of the helium produced by the absorption of radiation combined with electron impact such as Compton appears to have found. When the present investigation was being carried out this secondary ionisation was considered as a possible explanation for the positive currents observed to accompany the production of radiation between 20 and 26 volts, and an attempt was made to test for this by studying the current voltage curves for different conditions of pressure and electron current density. It was expected that with increasing pressure and electron current density the 26 volt discontinuity would tend to disappear, showing an increase in the relative amount of secondary to primary ionisation. This effect was observed, but was not considered conclusive evidence, as the effect of impurity would also be to obscure the 26 volt discontinuity.

II. *The Location of Interference Fringes.* *By* J. GUILD, *A.R.C.Sc.,* *D.I.C.,* *F.R.A.S.* (*Optics Department, National Physical Laboratory.*)

IT is frequently of importance in making certain determinations by the aid of interference fringes to know precisely where the fringes are situated. The problem has been worked out by various writers,* but their methods of treatment always seem somewhat arbitrary, inasmuch as the physical conditions involved in the problem are not clearly emphasised, and the dependence of the results on the circumstances under which the fringes are observed is never pointed out. Unnecessary assumptions are usually made which confuse the reader, and tend to invest the problem with a certain degree of mystery. For instance in Michelson's treatment, which is probably the best known to English readers, the source is assumed to coincide with one of the surfaces of the film. The assumption is unnecessary, as the position of the source does not affect the phenomena under the conditions which Michelson investigates. The following method of treatment may therefore be worth recording.

Let 1, 1′ and 2, 2′ (Fig. 1) be the section of a thick plate, the surfaces of which are very nearly but not quite parallel. Let *SNA* be a ray which undergoes reflection from the front surface. Let *SN′B* be a ray emanating from the same point of the source (the same electron actually) which after reflection at the back surface intersects *NA* in some point such as *P*. Since all rays emanating from the same electron have a constant phase relationship interference or reinforcement will take place at *P*. But other rays from the same point in the source will intersect *NA* at different distances from *N*, consequently interference phenomena exist at all distances from the plate. What then determines the place where they will be most distinctly visible ?

Suppose that the observer's eye is at *E*. The pupil subtends an appreciable angle at the point *P* ; so it receives from *P* not only the rays *PA* and *PB* (which in reality are so close together that if drawn to scale they would coincide), but a conical pencil made up of similar pairs of rays which come to *P* from adjacent parts of the source via adjacent parts of the plate.

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* *E.g.*, Michelson, " Phil. Mag." (5), 13, 236 ; Feussner, Winkelmann's Handbuch der Physik, VI., p. 958.

Since the thickness of the plate is not uniform, the phase retardation for the component pairs of the pencil will vary, and the illumination at *P*, which is due to all the light which reaches the eye from that point, can neither be of maximum nor zero brightness. If the phase variation comprised within the pencil is a few half wavelengths, the illumination will be the same as if there were no interference whatever. The same applies to other points in the field of view, such as *P′* and *P″*, so that the illumination is uniform and no fringes are visible. It so happens, however, that the rate at which the phase at *P* varies with the inclination of the rays which pass through it is not the same at all distances from the plate, and at a certain distance it has a minimum value. In this neighbourhood fringes will be seen if the cone of rays utilised from each point of the field of view is not too wide.

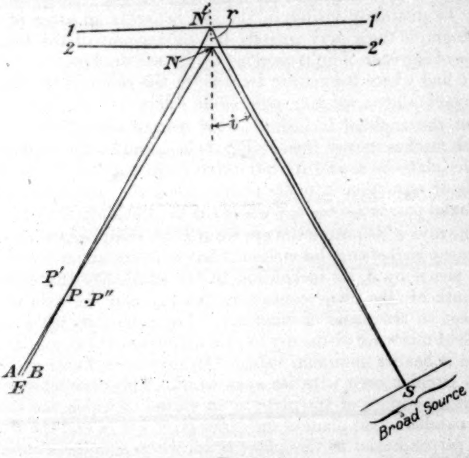

FIG. 1.

It is important to bear this last point in mind. Interference fringes are not entities like wires or focussed images, which have a definite position in space. They have no existence, except in relation to the observing instrument. In general the disturbance in the ether is just as violent, and comprises as many phase components, at one point in space as another.* Visible fringes only indicate that the phase variation is very slight *within the particular bundles of rays which are utilised.* In some circumstances the slightest movement of the eye may cause a complete change in the character of the fringe system by slightly changing the position or direction of the bundles of rays employed for their observation. Also fringes may be distinctly visible to the eye, whereas an attempt to photograph them may entirely fail on account of the larger angle of the cone of light received by the camera lens.

To find where fringes are located, in the sense of the word indicated above, we may proceed as follows :—

Let the angle of incidence of the central ray of the cone, which reaches the eye from P (Fig. 1) be i_0 and let the thickness of the plate be t, and its refractive index μ. Let r_0 be the angle of refraction into the plate. Then the retardation for the axial pair is $2\mu t \cos r_0$,† where t is the thickness at N'.

The rays which reach the eye from P are comprised within a narrow pencil of circular section. Let us define any ray within this pencil by δ, its inclination to the axial ray, and φ, the azimuth of the plane containing the ray and the axis with respect to the plane of incidence. Let φ be zero when the angle of incidence of the ray has its maximum value, and 180° when it has its minimum value. All rays of inclination δ lie on a circular cone with its apex at P. This cone intersects the front surface of the plate in an ellipse of which the semi axis parallel to the plane of incidence is $x\delta/\cos i_0$, and the semi axis perpendicular to this plane is $x\delta$, where x is the distance PN. The distance from the centre to the point corresponding to azimuth φ is $\dfrac{x\delta}{\cos i_0} \cdot \dfrac{\cos \varphi}{\cos \psi}$, where $\tan \psi = \tan \varphi \cos i_0$. ψ is the inclination to the plane of incidence of the line joining the point of incidence of the axial ray to the point of incidence of the ray whose azimuth in the incident pencil was φ.

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* Except when light from a point source is used, as described later.

† This relation is only strictly true when the two interfering rays are parallel both before and after incidence. Within the range of departure from this condition that may arise in practice, it is correct to a first approximation. Thus the retardation at P depends only on the thickness of the plate at the point where the ray is incident on the back surface, and on the angle of incidence on the back surface. It is independent of the distance of P or S from the plate.

Thus the point of incidence of the ray δ, φ, is separated from the point of incidence of the axial ray by

$$\frac{x\delta}{\cos i_0} \cdot \frac{\cos\varphi}{\cos\psi}$$

in a direction making an angle ψ with the plane of incidence. If the angle between the surfaces of the plate is θ, and the line of greatest slope makes an angle α with the plane of incidence, the difference in thickness at the two points is clearly

$$\frac{x\delta}{\cos i_0} \cdot \frac{\cos\varphi}{\cos\psi} \cdot \theta\cos(\psi-\alpha),$$

θ being assumed positive when the azimuth of the direction of *increasing* thickness lies between 90° and −90°. Thus

$$\frac{dt}{d\delta}=\frac{x}{\cos i_0}\ \frac{\cos\varphi}{\cos\psi} \cdot \theta\cos(\psi-\alpha). \quad . \quad . \quad . \quad (1)$$

The angle of incidence of the ray δ, φ, is given by

$$\cos i=\cos i_0 \cos\delta-\sin i_0 \sin\delta\cos\varphi$$

(from the properties of spherical triangles). Where δ is very small, as in the case we are considering

$$\cos i=\cos i_0-\delta\sin i_0\cos\varphi.$$

The phase retardation between this ray and its partner from the back surface is, to a first approximation,

$$\begin{aligned}\rho&=2\mu t\cos r=2t\sqrt{\mu^2-1+\cos^2 i}\\ &=2t\sqrt{\mu^2-1+\cos^2 i_0-2\delta\cos i_0\sin i_0\cos\varphi},\\ &=2\mu t\cos r_0-2t\,\frac{\delta\cos i_0\sin i_0\cos\varphi}{\mu\cos r_0}. \quad . \quad . \quad . \quad . \quad . \quad (2)\end{aligned}$$

From (1) and (2)—

$$\begin{aligned}\frac{d\rho}{d\delta}=&-2t\,\frac{\cos i_0\sin i_0\cos\varphi}{\mu\cos r_0}\\ &+2\left(\mu\cos r_0-\frac{\delta\cos i_0\sin i_0\cos\varphi}{\mu\cos r_0}\right)\frac{x}{\cos i_0}\ \frac{\cos\varphi}{\cos\psi}\,\theta\cos(\psi-\alpha).\end{aligned}$$

In the immediate neighbourhood of the axial ray (*i.e.*, $\delta=0$), this will be zero, and the fringes will therefore be most distinct, when

$$x=\frac{t\sin r_0}{\mu\theta} \cdot \frac{\cos^2 i_0}{\cos^2 r_0} \cdot \frac{\cos\psi}{\cos(\psi-\alpha)}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From equation (3) it follows that x can only be independent of ψ, the condition that P should be the point of minimum confusion for rays in all diametral planes of the pencil entering the eye, if $a=0$—*i.e.*, if the plane of incidence is parallel to the direction of slope of the surfaces. In this case the fringes are perpendicular to the plane of incidence and the terms in ψ cancel out. As the direction of the fringes departs from this, the visibility falls off. Thus, with thick films at oblique incidence fringes can only be seen distinctly when their direction is perpendicular to the plane of incidence.

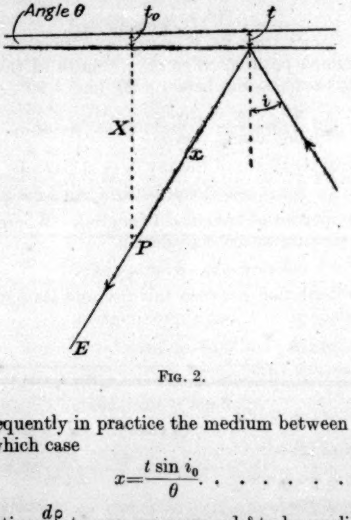

FIG. 2.

Most frequently in practice the medium between the surface is air, in which case

$$x=\frac{t \sin i_0}{\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3\text{A})$$

In equating $\frac{d\rho}{d\delta}$ to zero we assumed δ to be negligibly small. The retardation is therefore only constant for a very small cone, and may vary quite appreciably towards the boundary of the pencil which enters the eye. It is therefore usually possible to sharpen up the fringes considerably by placing a small diaphragm in front of the pupil to restrict its angular aperture.

Equation (3A) is equivalent to that obtained by Michelson; but he expresses it somewhat differently. Michelson's formula is $X=\frac{t_0 \tan i}{\theta}$; where X is the *perpendicular* distance from the plate to the fringes and t_0 is the thickness of the plate *at the point where the perpendicular cuts it.* The equivalence will be clear from Fig. 2.

$$x=\frac{X}{\cos i}\text{; also } t=t_0+X \tan i \,.\, \theta.$$

$$X=\frac{t_0 \tan i}{\theta}=\frac{t \tan i}{\theta}-X \tan^2 i,$$

or,

$$\frac{X}{\cos^2 i}=\frac{t \tan i}{\theta}.$$

$$\therefore \quad x=\frac{X}{\cos i}=\frac{t \sin i}{\theta}.$$

It is most convenient, in the author's estimation, to express the result in terms of the distance along the ray and the thickness at the point of incidence. These quantities always exist, and are measurable; but the quantities X and t_0 in Michelson's formula can usually only be obtained indirectly by calculation from x and t. We see from equations (3) or (3A) that in general the fringes lie in front of the plate when θ is positive on the convention adopted—*i.e.*, when the observer's eye is on the same side of the point of incidence as the line of intersection of the surfaces. If θ is negative they lie behind the plate. If θ is zero they are at infinity, and if either i or t is zero they are located at the plate. In the usual cases which arise in practice the value of θ is constant for all parts of the field of view. In viewing Newton's rings between a curved surface and a flat plate θ varies; but t is very small, so that the fringes are always at or close to the surface.

In cases where the interference is not produced at an actual film, but by an arrangement which is optically equivalent to a film, it is possible for such an equivalent film to have different angles for rays incident on it in different directions, and the location surface of the fringe system observable by the naked eye in such cases is affected accordingly.

Fringes in Parallel Light.—Fringes may be formed by a point source of light at infinity. In this case the rays meet every part of the plate at the same angle of incidence. In order that the eye may receive light from more than one point of the plate at a time, the rays after reflection are received by a lens, in the focal plane of which they produce an image of the point source. The eye is placed at this point, and sees an area of the plate, equal to that of the lens, illuminated. Any fringes observed can only be due to variations in the optical thickness of the plate, since the angle of incidence is constant. They are, therefore, contours of film thickness. It is evident that the pairs of interfering rays from one point of the surface never intersect these from any other point; consequently the considerations which we applied in the case of a broad source do not apply at all in this case. From any point *P*, Fig. 3, only the rays from the points *N* and *N′* ultimately reach the eye at *E*.

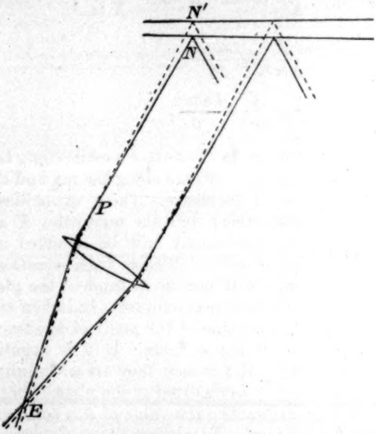

FIG. 3.

Similarly, for any other point along the ray *NP*, the illumination is due simply to that ray, and to the ray from the rear surface which meets it in the point considered. The actual phase difference varies at different distances along *NP*, except when the film is parallel, but there will be no components of varying phase at any one point. Fringes are therefore equally distinct at all distances from the plate, and will appear to be located either at the plate or at any aperture which bounds the field of view. Thus, in using fringes of this type for testing optical parts such as prisms and lenses by placing them in one beam of a Michelson interferometer, the fringes always appear coincident with the aperture of the system under test.

ABSTRACT.

The Paper treats of the conditions under which interference fringes, produced by reflection of light from the two surfaces of a "thick plate," are visible to an observer. The treatment lays stress on the physical significance of the term "location" as applied to interference fringes, and the dependence of the observed phenomena on the conditions of observation. For a broad source of light a formula is obtained which is equivalent to that derived by Michelson. For a point source of light at infinity, it is shown that the fringes obtainable are equally visible at all distances from the plate.

III. *Fringe Systems in Uncompensated Interferometers.*

By J. Guild, *A.R.C.Sc., D.I.C., F.R.A.S.* (*Optics Dept., National Physical Laboratory.*)

In the investigations of the fringe systems obtained with interferometers, such as Michelson's for instance, in which the instrument is equivalent to an air film between two parallel reflecting surfaces, the instrument is always assumed to be accurately compensated, in the sense that for all possible rays the thickness of glass to be traversed in either of the two beams is identical. This is accomplished by the well-known means of providing a compensating plate of the same thickness and refractive index as the semi-silvered plate, placing it parallel to the latter in the beam which only passes once through it. The glass paths of the two beams are then exactly equivalent for all possible incident rays, and the interferometer is in effect reduced to a simple air-film, of thickness depending on the positions of the mirrors, the character of the fringes being readily deduced from the theory of thick plates of refractive index unity.

But in the recently developed method of testing optical parts by placing them in one of the beams of a large Michelson interferometer *, it is impossible to secure exact equality of glass paths, except in the simple case when parallel-sided slabs are under test. Even then it would require a separate compensator for every plate tested, and in practice the instrument is usually employed without any compensator at all.

As the interference phenomena in an uncompensated interferometer do not appear to have been described, an investigation of them may be of interest.

We shall assume, to commence with, that the mirrors have been adjusted to be at an equal distance from the semi-silvered surface, and that the surfaces of the equivalent film are perfectly parallel. The arrangement is really equivalent to a thin film, but the rays reflected from one of the surfaces pass through an oblique slab of glass before and after reflection, whereas the rays reflected from the other surface do not pass through the slab.

Let *A*, Fig. I, be a ray which does not pass through the slab and B a ray which does. The latter, in passing from

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* F. Twyman, "Phil. Mag.", Jan. 1918, p. 49, and "Phot. Journ.", Nov., 1918, p. 239.

q to m, traverses a path the air-equivalent of which is $qn+\mu.nm$. The other ray, for which the slab is non-existent, traverses the air path pm. On account of its passage through the slab, therefore, the ray B suffers a retardation with respect to the ray A of $(qn+\mu\,.\,nm)-pm=\mu\,.\,mn-mo$

$$=\mu\,.\,mn-mn\cos\widehat{omn}.$$

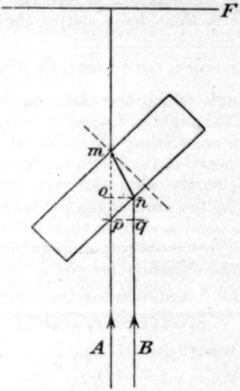


FIG. 1.

Thus if we denote the retardation by ρ, the thickness of the slab by t, and the angles of incidence and refraction by i and r respectively,

$$\rho=\mu\frac{t}{\cos r}-\frac{t}{\cos r}\cos(i-r)$$

$$=\mu\frac{t}{\cos r}-t\cos i-\frac{\mu t}{\cos r}\sin^2 r$$

$$=t(\mu\cos r-\cos i).$$

If these rays strike the surfaces of the film normally, they retrace their respective paths exactly and arrive back to the observer with a retardation 2ρ.

This retardation is clearly the same for all corresponding pairs of rays which meet the film normally; but it varies for pairs of rays which are inclined to the normal. It is only necessary to consider the case of rays whose incidence on the film F is nearly normal; for, as we shall see, the fringes quickly become too close to be visible as the angle of incidence increases.

Consider then a pair of rays, not necessarily in the plane of the paper, inclined at a small angle δ to the pair AB. If i_0 is the angle of incidence on the oblique slab of the ray B, and i is the angle of incidence of the ray which is inclined to it by an angle δ, then, by a simple theorem in spherical trigonometry,

$$\cos i = \cos i_0 \cos \delta + \sin i_0 \sin \delta \cos \varphi,$$

where φ is the angle which the plane of the angle δ makes with the plane of the angle i_0, *i.e.*, with the plane of the paper in the figure. By considering all values of φ we consider all parts of a conical pencil coaxial with the rays A and B.

After reflection at the film, the rays will return inclined to the central ray by the same angle, but with its sign reversed, and will meet the oblique slab on their return journey with a new angle of incidence corresponding to the substitution of $-\delta$ for δ in the equation for $\cos i$.

Let ρ_1 be the retardation for the journey towards F.

Then $\rho_1 = t(\mu \cos r_1 - \cos i_1)$

whence $2 \cos i_1 = (\mu^2-1)\, t/\rho_1 - \rho_1/t \quad . \quad . \quad . \quad . \quad . \quad (i)$

and $2\mu \cos r_1 = (\mu^2-1)\, t/\rho_1 + \rho_1/t. \quad . \quad . \quad . \quad . \quad (ii)$

Similarly for ρ_2 and i_2 the retardation and incidence angle on the return journey.

Also $\cos i_1 = \cos i_0 \cos \delta + \sin i_0 \sin \delta \cos \varphi,$

and $\cos i_2 = \cos i_0 \cos \delta - \sin i_0 \sin \delta \cos \varphi.$

From (i) and the sum and difference of $\cos i_2$ and $\cos i_1$ we get

$$\left[(\mu^2-1)\frac{t}{\rho_1\rho_2} + 1/t\right](\rho_2-\rho_1) = 4 \sin i_0 \sin \delta \cos \varphi \quad . \quad (iii)$$

and

$$\left[(\mu^2-1)\frac{t}{\rho_1\rho_2} - 1/t\right](\rho_2+\rho_1) = 4 \cos i_0 \cos \delta. \quad . \quad . \quad (iv)$$

We may expand ρ_1 and ρ_2 in a series of powers of $\sin \delta$, say

$$\rho_1 = \rho_0 - \alpha \sin \delta + \beta \sin^2 \delta - \quad . \quad . \quad . \quad . \quad . \quad . \quad \&c.$$

$$\rho_2 = \rho_0 + \alpha \sin \delta + \beta \sin^2 \delta + \quad . \quad . \quad . \quad \&c.$$

Substituting in (*iii*) and taking, as a first approximation,

$$\rho_1\rho_2=\rho_0^2,$$

$$2\sin i_0\cos\varphi=a\left[(\mu^2-1)\frac{t}{\rho_0^2}+1/t\right]=\frac{2a}{\rho_0}\,.\,\mu\cos r_0,$$

whence $$a=\frac{\sin i_0\cos\varphi}{\mu\cos r_0}\,\rho_0.$$

Similarly, by putting $\cos\delta=1-\frac{1}{2}\sin^2\delta$ and taking $\rho_1\rho_2$ to a second approximation, viz.,

$$\rho_1\rho_2=\rho_0^2-a^2\sin^2\delta+2\beta\rho_0\sin^2\delta,$$

we obtain from (*iv*)

$$\cos i_0=\beta\left[\frac{(\mu^2-1)t}{\rho_0^2}+\frac{1}{t}\right]-\frac{a^2(\mu^2-1)t}{\rho_0^3}.$$

From which

$$\beta=\frac{t}{2}\left[\cos i_0-\frac{\cos^2 i_0}{\mu\cos r_0}+\frac{(\mu^2-1)\sin^2 i_0\cos^2\varphi}{\mu_3\cos^3 r_0}\right]$$

$$+\text{terms in }\sin^4\delta,\ \&\text{c.}$$

Thus the difference between the retardation of the ray δ, φ, and the retardation of the normal ray ($\delta=0$) is

$$R=\rho_1+\rho_2-2\rho_0=2\beta\sin^2\delta$$

$$=t\sin^2\delta\left[\cos i_0-\frac{\cos^2 i_0}{\mu\cos r_0}+\cos^2\varphi\,\frac{\sin^2 i_0(\mu^2-1)}{(\mu\cos r_0)^3}\right]. \quad . \quad (1)$$

Hence, if the circumstances of the observation are such that different parts of the field of view correspond to rays of different inclinations, as, for instance, if the fringes are due to a broad source and are observed at infinity or in the focal plane of a lens (*i.e.*, the plane conjugate to infinity) a fringe system will be seen. The centre of the system will be the point in the field corresponding to normal incidence on the film ; and on moving outwards from this point, a fringe will be encountered every time $\rho_1+\rho_2-2\rho_0$ increases by one wave-length. For any point distant δ (in angular measure) from the centre of the system, the excess of phase retardation over that at the centre, is $$R=\rho_1+\rho_2-2\rho_0=t\sin^2\delta(A+B\cos^2\varphi)$$

in which A and B are essentially positive constants. Each fringe is the locus of points for which R is constant, so that δ, the (angular) radius vector of the fringe, is given by

$$\sin^2\delta,\text{ or simply }\delta^2,\ =R/t(A+B\cos^2\varphi). \quad . \quad (1a)$$

The fringes are, therefore, ellipses of which the principal axes are respectively parallel and perpendicular to the plane of incidence of the central ray of the system on the oblique slab. The minor axis is parallel to this plane and is $\{R/t(A+B)\}^{\frac{1}{2}}$. The major axis is $\{R/tA\}^{\frac{1}{2}}$.

We may regard these results indifferently as applying to a slab placed in one of the paths of an otherwise compensated interferometer, or to the half-silvered slab of an uncompensated interferometer. With an uncompensated interferometer, therefore, adjusted so that the mirrors are equally distant from the semi-reflecting surface, the fringes observed at infinity are ellipses distributed and shaped in accordance with 1(a). The system is clearly identical with the Newton's rings obtained between an astigmatic lens and a flat surface.

In the case of the half-silvered slab, i_0 is usually 45 deg., in which case the values of A and B are 0·334 and 0·266 respectively, assuming $\mu=1{\cdot}52$. In this case, therefore, the distances of points where the retardation differs by half a wavelength from that at the centre, *i.e.*, the distances of the black fringes if the centre happens to be bright, or vice-versa, are given by

$$\delta_h^2=\frac{(2n-1)\lambda/2}{0{\cdot}600t}$$

in the horizontal direction, and by

$$\delta_v^2=\frac{(2n-1)\lambda/2}{0{\cdot}334t}$$

in the vertical direction. The ellipticity of the fringes, as measured by the ratio of their axes, is $\sqrt{0{\cdot}600/0{\cdot}334}=1{\cdot}34$.

The horizontal distance to the first fringe $=\sqrt{\dfrac{\lambda/2}{0{\cdot}600t}}$,

which, for the green line of mercury, $=0{\cdot}0067/\sqrt{t}$ radians, $=23'/\sqrt{t}$. If the fringes are observed in the focal plane of a lens of focal length f, the linear distance is $0{\cdot}0067f/\sqrt{t}$.

Taking f as 30 cm. and $t=1{\cdot}8$ cm., which are common values in Hilger interferometers, the horizontal radius of the first fringe is 1·5 mm.

Effect of Additional Air Film.—The preceding treatment assumes that the distances of the two mirrors from the half-silvered surface are equal. In practice this is not necessarily so. In fact, it is not usually so. If the distances differ by x, one of the rays has, in effect, to travel twice through a slab of air which the other ray has not. The central ray of the system passes through it normally, and the retardation due to it is $2x$. For rays inclined at an angle δ to the central ray, the retardation is $2x \cos \delta=2x(1-\frac{1}{2}\delta^2)=2x-x\delta^2$. If the extra air path is in the same side as the glass slab, the total retardation is the algebraic sum of that due to the slab and the extra air path :—

$$R=\delta^2 t(A+B\cos^2\varphi)-x\delta^2$$

$$=\delta^2\{(A-x/t)+\mathrm{B}\cos^2\varphi\} \quad . \quad . \quad . \quad . \quad (2)$$

If the extra air path is in the other side, the total retardation is the difference of that due to the slab and that due to the extra air path, and

$$R=\delta^2 t\{(A+x/t)+B\cos^2\varphi\} \quad . \quad . \quad . \quad . \quad 2(\mathrm{a})$$

It is evident that the fringes will change in form as x varies. If there is a large excess of air path in the same side as the glass, the quantity $A-x/t$ in equation 2 will be large and negative. The fringes will be ellipses with their major axes horizontal. As the air path is diminished, the numerical value of $A-x/t$ decreases, and the fringes become of greater ellipticity until, when $A-x/t=-B$, they are horizontal straight lines, similar to the Newton's "rings" obtainable with a cylindrical lens and a flat plate. As x is diminished still further the fringes are hyperbolas, whose asymptotes rapidly open out from the horizontal axis and, after passing through the rectangular position when $A-x/t=-B/2$, close in on the vertical axis. When $A=x/t$, the asymptotes coincide with this axis and the fringes are vertical straight lines. A further reduction of x renders $A-x/t$ positive but small. The fringes are now elongated ellipses with major axes vertical. As $A-x/t$ increases, they crowd closer together and the ellipticity diminishes until, when $x=0$, the condition is arrived at which we treated in the first instance.

If we continue to advance the mirror, which is equivalent to introducing an air film into the other beam, the effect is a continuous enlargement of $A+x/t$, so that the fringes crowd more closely together, and become less elliptical on account of the greater preponderance of the term which is independent of φ. A particular case of practical interest is when x is approximately equal to ρ, the retardation of the oblique slab. When this is so the total optical paths in the two sides of the interferometer are equal, and the fringes are of maximum visibility. With most practical sources of monochromatic light the visibility of the fringes falls off fairly rapidly when the path difference is great, so that the adjustment of the mirrors is usually made to secure the most distinct fringes. For this case $x=t(\mu \cos r-\cos i)=t\times 0{\cdot}638$, and equation 2($a$) becomes

$$R=\delta^2 t(0{\cdot}334+0{\cdot}638+0{\cdot}266 \cos^2\varphi).$$

The ellipticity of the fringes is $\delta_v/\delta_h=\sqrt{1{\cdot}238/0{\cdot}972}=1{\cdot}13$. The horizontal radius of the first fringe (when centre is bright) is $\sqrt{\dfrac{\lambda/2}{1{\cdot}238\,t}}$ radians or $0{\cdot}0046 f/\sqrt{t}$ measured in the focal plane of a lens. For the values of f and t assumed earlier, this is just about 1 mm.

Effect of a Second Glass Slab.—Reverting to the case in which the mirrors are equally distant from the reflecting surface, if we insert another slab in one of the beams, the resulting retardation is the sum or difference of that due to each. If the second slab is of thickness t', and the angle of incidence on it is i', and if it is placed in the beam which does not traverse the glass of the semi-reflecting plate, the resulting retardation for an inclination δ to the central ray is

$$\begin{aligned} R &= t\delta^2(A_{45}+B_{45}\cos^2\varphi)-t'\delta^2(A_{i'}+B_{i'}\cos^2\varphi) \\ &= \delta^2\{A_{45}t-A_{i'}t'+(B_{45}\,t-B_{i'}t')\cos^2\varphi\}=\delta^2(a+b\cos^2\varphi). \end{aligned}$$

where $A_{i'}$ and $B_{i'}$ are calculated from the refractive index of the second slab.

The fringes may be either elliptic or hyperbolic, including as special cases straight lines or circles, depending on the signs and relative values of a and b.

For instance, in the case of a second slab of thickness 1·25 cm. and refractive index nearly equal to that of the interferometer plate (thickness 1·8 cm.) the fringes, when the plate is normal to the rays, are ellipses of which the vertical axis is approximately twice the horizontal. When the plate is turned from the normal position the ellipticity diminishes, and when i' is in the neighbourhood of 52° the fringes are circles. On further increasing the obliquity they again become elliptic, but with the long axis horizontal. At about 65° the ellipses have flattened out into horizontal straight lines, and at still higher angles they become hyperbolic.

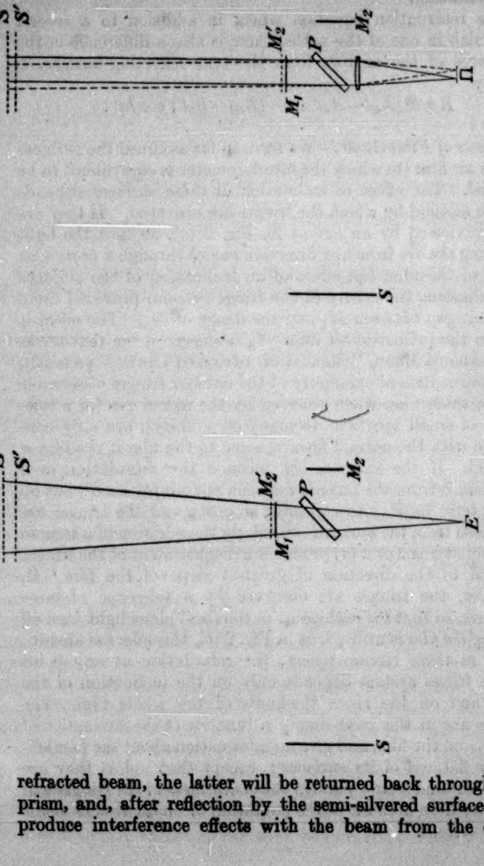
It is only if the plates are of the same material and thickness, and are equally inclined to the incident light, that a and b can be zero simultaneously, which is the condition for exact compensation.

The retardation equation when, in addition to a second glass slab in one of the paths, there is also a difference in the distances of the mirrors from the semi-reflecting surface, is clearly

$$R=\delta^2\{A_{45}t-A_i t'\pm x+(B_{45}t-B_i t')\cos^2\varphi\}.$$

Effects of Film Angle.—We have so far assumed the surfaces of the air film (to which the interferometer is equivalent) to be parallel. The effect of inclination of these surfaces depends on the method by which the fringes are observed. If they are simply viewed by an eye at E, Fig. 2 (a), so that the light reaching the eye from any direction passes through a particular point of the film, the effect of an inclination of the surfaces is to displace the centre of the fringe system, provided there is an air gap between M_1 and the image of M_2. The effect is due to the prismatic air film. It is shown in the text-books (for instance Mann, "Manual of Advanced Optics," page 53), that the centres of symmetry of the circular fringes observable with a thick film, when received by the naked eye (or a telescope of small aperture focussed on infinity), are only concentric with the normal from the eye to the film if the film is parallel. If the surfaces are inclined the retardation in a direction δ from the normal depends not simply on δ^2, but on δ^2+a term in δ^3. As a result the centres of the fringes are displaced from the normal. Similarly the addition of a term in δ^3 in equations 2 or 2 (a) produces a displacement of the fringes parallel to the direction of greatest slope of the film. If, however, the fringes are observed by a telescope of large aperture, so that for each point in the focal plane light *from all parts of the film* is utilised, as in Fig. 2 (b), this effect is absent; since, in these circumstances, the retardation at any point in the fringe system depends only on the inclination of the rays, and on the *mean* thickness of the whole film. The fringes are in this case simply a function of the inclination of the rays on the film, and give no information about the paralellism or flatness of its surfaces; except that unless they are nearly flat and nearly parallel the *distinctness* of the fringes will be affected on account of the confusion of phase at each point in the system.

Effects with Prisms and Lenses.—If we insert a refracting prism in one of the paths of the interferometer, and adjust the mirror so as to be perpendicular to the mean direction of the refracted beam, the latter will be returned back through the prism, and, after reflection by the semi-silvered surface, will produce interference effects with the beam from the other



(a) (b)

FIG. 2.

S Broad Source.
P Semi-silvered surface.
M_1, M_2 Interferometer Mirrors.
M'_2 Image of M_2 in P, forming with M_1 the equivalent film.
S', S'' Images of S in M_1 and M'_2.

mirror. The excess of retardation of one of the beams is made up of a certain amount of extra glass path and a certain amount of extra air path, the former progressively decreasing and the latter increasing from the base to the apex of the prism. The resulting fringe system due to rays inclined to the mean ray is complicated by this fact, and also by the fact that the deviation varies with the angle of incidence and with the inclination of the ray to the principal plane of the prism. These latter peculiarities give rise to the result that an incident ray, inclined at an angle δ to the ray which ultimately meets the mirror normally, is not inclined to it at an angle $-\delta$ after returning through the prism, and consequently is inclined to the corresponding ray in the other beam after recombination at the semi-silvered plate. This is in effect equivalent to the interference film having a different angle between its surfaces for rays incident in different directions. As a result of these various factors, a general expression for the resulting fringe system would be of a somewhat complicated form.

It is easy to deduce, however, what the phenomena observed in the focal plane of a telescope must be. For each point in the field of view, light is received from all parts of the prism; and, conversely, each element of the prism supplies light to every part of the field. The resultant fringe system observed through the eyepiece is, therefore, the resultant of the systems corresponding to each element of the prism, which may be approximately regarded as elements of slabs of the appropriate thicknesses. The systems will vary in character from the base to the apex, in accordance with the variation in the relative glass and air paths. The only point in common is the centre of the systems ($\delta=0$) and its immediate neighbourhood. Thus, we should expect to see one or possibly two fringes at the centre of the field, the outer parts being uniformally illuminated owing to the confusion of the outer parts of the various component systems. This is, in fact, what may be observed in practice. Usually only the central fringe is visible; it is approximately circular, elliptical or hyperbolic, depending on which form predominates among the component systems.

When a lens is placed in one of the beams in conjunction with a coaxial convex mirror, situated with its centre of curvature at the focus of the lens, the beam after returning through the latter will produce interference with the other beam. The resulting effects are clearly similar in character to those deduced in the case of a prism.

Contour Fringes.—The fringes which we have discussed so far are due to the varying obliquity with which the rays corresponding to different parts of the field of view meet the various surfaces of the interferometer. In the case of naked eye vision there is, as we have seen, a slight displacement of the system if the equivalent film is prismatic, and there would be a slight distortion of their form if the surfaces were irregular; but if a telescope of large aperture is used, so that for any part of the field of view rays from the whole of the illuminated area of the film are utilised, the fringes are unaffected, except in distinctness, by either the inclination or flatness of the surfaces of the film. Such fringes are, of course, useless for testing the surfaces of the mirrors or the uniformity of optical path through a piece of optical apparatus placed in one of the beams.

For such purposes it is necessary to utilise the fringes due to a *point source* situated at infinity. Rays from such a source, which may be a very small pinhole at the focus of a telescope object glass, reach all parts of the equivalent film at the same inclination. If, when the beams have passed through the interferometer and have been recombined, they are collected by a second telescope objective, two images of the pinhole will be formed in its focal plane. If the film is nearly parallel these will coincide, and an eye placed at this point will see the whole of the film illuminated. Each point is seen by a separate ray; and, if the optical thickness of the film is uniform, it will appear uniformly bright or dark. If, however, the film thickness varies, the retardation for rays reaching the eye from different parts of it will vary, and fringes will be seen which are true contours of the film thickness; for, since δ is the same for all parts of the field, the retardation for any ray can only depend on $\Sigma\mu t$, the total optical path which the ray traverses, and is independent of what proportion of the path is in glass or air. Thus the contour fringes are not affected in form by the inequality of glass path in the two sides of the interferometer; but the conditions under which they will be distinctly visible are affected. There is obviously no such thing as a point source in practice: the pinhole must have an appreciable area in order that the field may be reasonably bright. We may, therefore, regard it as a small portion of a large source, and its image in the focal plane of the telescope objective as a small area of the fringe system which a broad source would produce there. If the light meets the mirrors normally, the image will be in the centre of this system, where the retardation is varying slowly, and there will be no appreciable change in phase between one part of the pinhole and another. In these circumstances, the contour fringes seen when the eye is placed at the image will be most distinct. If, however, the light is not normal to the film, the image of the pinhole is located in an outer portion of the infinity fringe system, where the phase variation is rapid. The retardation, therefore, varies appreciably from one side of the pinhole to the other, and the eye receives light of a range of retardations from each point in the contour fringe system. As a result, the distinctness of the fringes is much reduced.

With an interferometer exactly compensated, and with the mirrors adjusted for equality of path, the fringes would be visible for all angles of incidence. Thus, one consequence of the lack of compensation is to render it necessary to adjust carefully for normal incidence in order to obtain clear contour fringes. The accuracy required varies as the diameter of the central fringe produced in the focal plane of the telescope with a broad source. When testing certain lenses, for instance, the normality has to be very exact or the contour fringes are scarcely visible. In other cases there is more latitude.

Another point to which attention has to be directed in using an uncompensated interferometer for contour testing is that the point source must be exactly at the focus of the collimating lens. If it is not so, the rays from the outer parts of the lens are inclined to the axial ray, and the retardation will vary from the centre of the field outwards, quite apart from contour variations. Excessive spherical aberration in the collimating lens will have the same effect.

ABSTRACT.

The Paper mainly consists of an investigation of the form of the fringe system observable at infinity, or in the focal plane of a telescope, when a broad source is employed with a Michelson interferometer, in which the glass paths of the two interfering beams are not equal. The fringes may be elliptical or hyperbolic, with circles and straight lines as special cases. In the recently developed method of using the instrument for optical testing, the fringes due to a point source at infinity are employed. It is shown that the form of the fringes in this case is unaffected by lack of compensation, but that the visibility of the fringes is conditioned by the nature of the fringe system due to a broad source.

DISCUSSION.

Mr. F. TWYMAN communicated the following remarks :—

As I shall not be present on the occasion of Mr. Guild's Papers on "Fringe Systems in Uncompensated Interferometers" and on "The Location of

Interference Fringes," I am writing to say how greatly I welcome the papers. The class of interferometers dealt with is becoming more and more used in the workshops with which I am connected, and it is of very great advantage to us to have the conditions for satisfactory distinctness of the fringes combined with good illumination clearly set forth.

So far as optical elements with flat surfaces are concerned, Mr. Guild's papers seem to me to provide everything that is needed in this respect.

I hope that at some future time Mr. Guild will find time to deal in a similar way with the fringes of the lens interferometers.* We are, of course, acquainted in a vague and general way with the desirable conditions for securing plenty of light simultaneously with good definition of the fringes, but the precise formulae are as yet not available.

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* F. Twyman, "Phil. Mag.," Jan., 1918, p. 49, and "Phot. Journ.," Nov., 1918, p. 239.

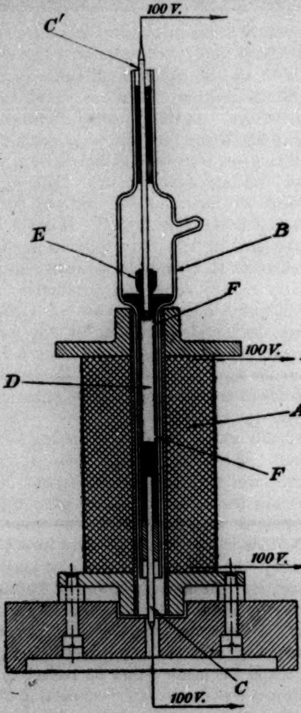
IV. *A New Relay for Moderately Heavy Currents.* By GUY BARR, *B.A., D.Sc.* *(From the National Physical Laboratory.)*

(COMMUNICATED BY F. E. SMITH, O.B.E., F.R.S.)

THE need frequently arises in physical laboratories for a relay which will switch on and off currents of a higher order than can be dealt with by the simple Post Office or similar type in which an electro-magnet is made to bring two platinum terminals into contact. At the National Physical Laboratory a solenoid device has been in use for some years which makes a circuit by lifting an iron core attached to a lever which lowers a copper fork into mercury cups. This form has been in fairly constant use for cutting in and out currents up to 4 or 5 amperes on a 100-volt supply. If there is much inductance in the circuit, the arc produced on break, even with smaller currents than this, is, however, so pronounced that the mercury rapidly fouls and "splutters" out of the cups. The difficulties due to the use of higher currents could, of course, be largely overcome by the use of a step relay with a more powerful magnet, &c., but it has recently been found possible to construct a relay in which a solenoid operating with about 0·03 ampere at 100 volts is capable of breaking currents up to 20 amperes.

Its action depends on the fact that no arc can be maintained between mercury electrodes in hydrogen at atmospheric pressure, at any rate, with ordinary voltages. A first form of switch was made consisting of a short glass tube of about 1 cm. bore, provided with small-bore side-tubes near each closed end; the side-tubes were then bent into U-tubes and filled with mercury, the wider tube being also about one-third full of this liquid and the remainder full of hydrogen. Into the open arms of the U-tubes were led iron or nichrome wires from the circuit to be operated, and the tubes were then closed with sealing-wax. By tilting the apparatus the mercury in the wide tube could be made either to collect in a pool at one end or to form a continuous layer connecting the U-tubes. It was found that this gave very clean breaks with quite large currents (20 amperes), but trouble was experienced when it was attached to the lever of the usual solenoid relay. A much more satisfactory form was eventually designed, which will be most readily understood from a diagram.

The lower end of the glass tube *B* of the shape shown fits into the axis of the solenoid *A*; *B* is provided with iron leads *C*, *C′*, of about 0·15 cm. diameter, which are enlarged by collars fitting loosely into the ends of *B*. These iron collars are cemented gas-tight in position with sealing-wax; the gas-tightness of the upper collar is further improved by a seal of mercury above the wax, and another layer of wax serves to contain the mercury. Moving in the part of *B*



which lies within the solenoid is an iron core *D*, carrying at its upper end a glass or silica cup *E*, a short tube projecting from which is inserted into a hole in the core. The lower end of the upper lead *C′* fits with a small clearance into the bore of the tube of *E*. Mercury fills *B* up to a suitable level, such that the rim of *E* is some 1 cm. clear of the mercury level when the core floats, and about 0·5 cm. below the mercury when the core is sucked down by the solenoid. The core does not fit the tube of *B*, but is provided at the bottom and at two-thirds of its length with three centering projections *F*; by this means a free passage for mercury is ensured and the friction against the sides is very small. A central position is further ensured when the core is floating by the fit of *C′* into the "tail" of *E*; the fit here is intended to absorb some of the shock occurring when the glass cup is arrested in its quick upward movement by the end of the iron rod *C′*. The space above the mercury in *B* is filled with hydrogen introduced before the sealing of the small side tube shown. In use the cup *E* becomes filled with mercury and the break of the current occurs on the glass rim of the cup, *i.e.*, between mercury electrodes; this is important since other metals are likely to produce fouling of the contact. The edge of *E* should be thin, to prevent cracking by the spark, or preferably it would be made of silica.

In the examples which have so far been made the core has been of mild steel; this is quite satisfactory with a core 6 cm. by 0·4 cm., using a solenoid 8 cm. long containing ½-lb. of copper wire of resistance 3,000 ohms. If electrolytic iron, stalloy or similar more permeable metal were used the dimensions of the core could be smaller or the solenoid weaker. The use of mild steel is further disadvantageous, since contact between the core and *C* must then be prevented, *e.g.*, by a small wooden plug, lest the remanent magnetism should cause sticking when the current through the solenoid is turned off. This precaution could probably be omitted if suitable soft iron were used.

One of these relays has been run for several days, switching a current of 17·5 amperes on and off at intervals of about a minute and the mercury has remained quite clean. The resistance of the system betweem *C* and *C′*, with core sucked down, is about 0·01 ohms when a current of 10 amperes or more is running; half of this is accounted for by the iron leads. When the instrument has been out of action overnight, the contact resistance between the iron wire and the mercury rises, in one model which has been examined, to 0·2 ohms for a current of 1 ampere, but this resistance rapidly falls to the normal low value when the current is increased. With a larger model the resistance was initially about 1 ohm for a current of 0·1 ampere, and fell to 0·18 ohms at 1 ampere and to ·01 ohms at 10 amperes; on then passing the smallest current the resistance remained 0·01 ohms.

The rise in resistance is presumably due to the formation by diffusion of a thin film of gas between the iron and mercury; when a current flows the thickness of this film would be rapidly reduced, owing to the heat developed at the junction, and to the rotation of the mercury in the magnetic field, which would assist the escape of bubbles.

The author would like to take this opportunity of thanking Mr. G. H. Glaysher for the interest he has taken in the development of this relay and for certain improvements suggested in details of mechanical construction of the final form exhibited.

ABSTRACT

The action of the relay depends on the fact that no arc can be maintained between mercury electrodes in hydrogen. One lead is brought to mercury contained in a vertical tube within a solenoid. An iron rod, at the upper end of which is a glass cup, floats in the mercury. The cup also contains mercury, and the other lead is connected to an iron rod which dips into this. When no current flows in the solenoid, the rim of the cup is about 1 cm. above the level of the main body of mercury. When the relay current (about 0·03 ampere) is running the iron rod is sucked down until the rim of the cup is submerged by about 0·5 cm. The space above the mercury contains hydrogen. The relay can be used to break quite large currents (20 amperes) without much spark.

DISCUSSION.

Mr. C. C. Paterson asked on what voltage the apparatus would work, as the energy in the spark would depend on this.

Dr. Barr said he had not tried it above 100 volts.

Mr. F. E. Smith suggested as a simplification that a U tube might be employed, in one limb of which was the iron float. When this was sucked down the rising mercury in the other limb of the U could make connection between two iron electrodes.

Dr. Barr said it was important that the break should take place between mercury surfaces, otherwise the mercury got contaminated.

Mr. Guild suggested that if the mercury in the second limb of the U were made to flow over and make connection with some stationary mercury in a side tube the difficulty of Mr. Smith's arrangement would be overcome.

Mr. Paterson asked how hot the apparatus would get if the current were broken as frequently as, say, 10 or 20 times per minute.

Dr. Barr replied that he had not had occasion to try it as rapidly as that.

Dr. Hopwood asked if the break was "clean," or if the oscillations set up in the mercury were sufficient to make and break the current after the first break had occurred.

Dr. Barr said there was no evidence of this. There was very little disturbance of the mercury in actual practice.

V. *Some Slide Rule Improvements. By* J. ST. VINCENT PLETTS.

RECEIVED SEPTEMBER 23, 1920.

ABSTRACT.

The relation existing between the log-log scale and the log scale in slide rules at present on the market is dealt with, and it is shown that if this relation is made that of the common logarithm the properties of characteristics and mantissæ enable the range of the non-recurring log-log scale to be indefinitely extended. In addition an equally divided scale is arranged to give ae^x and $\log_e ax$, where a is a number on the log scale, and scales for all the circular and hyperbolic functions are arranged on the back of the slide so as to read on the log scale. All such functions as $e^n \sin x$ and $\log n \cosh x$ are, therefore, obtained with a single setting of the slide and cursor. Other minor modifications are made, and a cursor for magnifying and subdividing the divisions is described.

THERE are now several slide rules on the market with log-log scales, by means of which any number can be raised to any power, or the logarithm of any number to any base can be obtained. As, however, the log log scale is non-recurring it is usually made two or three times the length of the rule, with the result that confusion sometimes arises as to which part of the scale is being used. The endeavour to get a suitable range of numbers on this scale has also led to curious relations between it and the log scale with which it corresponds. The numbers on the latter are, of course, the logarithms of the numbers opposite them on the former; but, taking three well-known examples, the base of these logarithms is 2 in the Jackson-Davis double slide rule, 2·5 in the Perry rule, and 3·5 in the Yokota slide rule. It is interesting to note that in the original Roget slide rule, as published in the "Philosophical Transactions" for 1815 (Part I., pages 9 to 28), the base was 10; but the scale was in two parts, running from 1·0025 to 1·26, and from 1·26 to 10^{10} respectively.

It occurred to me some time ago that the range of the log-log scale might be indefinitely extended, without making the scale longer than the rule, by taking advantage of the properties of characteristics and mantissæ of common logarithms. To this end I made a slide rule (Fig. 1), in which the relation between these scales was that of the common logarithm and

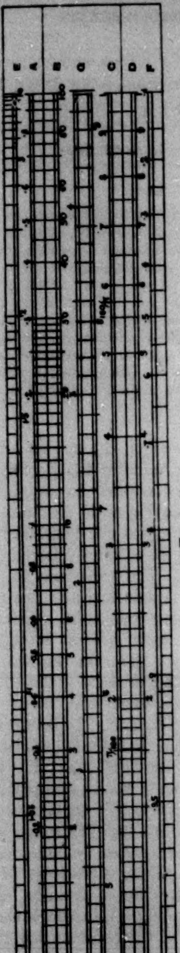

E
A
B
G
C
D
F

Fig 1.

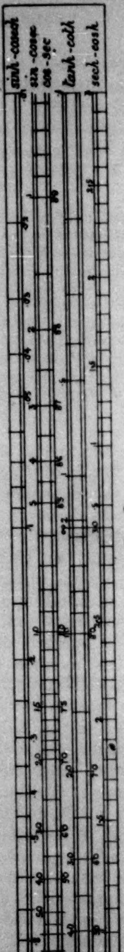

sinh-cosech
sin-cosec
cos-sec
tanh-coth
sech-cosh
sinh-cosech
sin-cosec
cos-sec
tan-cot
cot-tan
cosech-sinh

Fig. 2.

the highest number on the log-log scale (E) was 10. It will be seen that common logarithms can be read directly from these scales; all that is necessary, if the number is greater than 10, being to add the appropriate characteristic. If the number is less than unity the characteristic will be negative, or the true negative logarithm can be obtained from the corresponding reciprocal scale (F). If it be desired to raise any number to any power, one proceeds in the ordinary way; but if the result is greater than 10 it cannot be read on the log-log scale. All that is necessary, however, is to use the other end of the slide, when the logarithm of the result can be read on the log scale (A), and by dropping the first figure or characteristic of this logarithm the result can be read on the log-log scale opposite the remaining mantissa on the log scale, it being, of course, necessary to move the decimal point in accordance with the characteristic dropped. Thus, the range of the log-log scale is indefinitely extended in the upward direction, and of the reciprocal scale in the downward direction, by the exercise of no greater mental effort than is required to remember the position of the decimal point in ordinary calculations with the slide rule.

I have found it advantageous to put in the decimal points of the numbers on the upper log scales (A & B). This does not interfere with ordinary multiplication or division, which is nearly always performed on the lower log scale (C & D), and, besides making the common logarithms direct reading, it has several other advantages which will appear later. One of them can indeed be seen now, as there is no longer any doubt as to which end of the scale should be used for the extraction of square roots.

Having got the log-log scales into the length of the rule in a single part without limitations, I found that, though functions of the form e^{ax} were easily dealt with, those of the form ae^x, which are, perhaps, of more frequent occurrence, entailed more labour, and were more liable to error because of the necessity of transferring an intermediate result from one scale to another. I accordingly placed along the middle of the slide an equally divided scale (G) the numbers on which were the natural logarithms of the numbers opposite them on the log scale, so that e^x and log x were direct reading, and ae and log ax could be obtained with a single setting of the slide. It will be seen that by relating this equally divided scale to the upper log scale (B) on the slide, on which the position of the decimal point has already been indicated, a very useful scale is obtained, which can be made twice its apparent length by numbering on both sides, since $e^{4 \cdot 6}$ is very nearly equal to 100.

Now since we can either multiply by e^x, or take the natural logarithm of any number on the upper log scale, it follows that any function reading on this scale can be similarly treated. I have therefore rearranged the ordinary sine and tangent scales (Fig. 2), which are as usual placed on the back of the slide, and have added other scales for the hyperbolic functions so that they all read on the upper log scale in front.

The scales for the circular functions might have been divided into radians or into degrees and minutes as is usual, but as conversion from one into the other by means of the slide rule gives and requires degrees and decimals of a degree, I have found it most convenient to divide the scales in this way, while I have also marked each tenth of a radian on these scales, so that conversion is often not necessary at all.

The arrangement of all these scales on the back of the slide is such that the values of the functions are in every case read on the fixed portion (A) of the upper log scale in front if they are less than unity, and on the sliding portion (B) of that scale if they are greater than unity, so that the position of the decimal point is always given. From this and from what has already been said about the equally divided scale (G) along the middle of the front of the slide it will be seen that the values of all such functions as $e^x \sin x$ and $\log a \cosh x$ can be read direct from the rule, and generally with a single setting of the slide and cursor, so that by these means the power of the slide rule is considerably extended, and it becomes a much more valuable mathematical instrument.

These scales on a 25 cms. rule give sufficient accuracy for all ordinary purposes, and it is hardly practicable to increase the length of a straight rule sufficiently to get another decimal place in the results, but several attempts have been made to increase the power of subdividing the scales. There is the well-known magnifying cursor, which enables the subdivision to be more accurately estimated by the eye, and there is also the Goulding Cursor, by means of which the subdivision can actually be measured, though I have not found it more accurate than the unaided eye. The disadvantage of the ordinary magnifying cursor is that the semi-cylindrical lens gives such distortion near its edges that it is difficult to tell exactly which part of the scale is being magnified unless one of the figures upon the scale is also in view. To obviate this disadvantage I have made a lens which is as wide as an ordinary cursor, and the upper surface of which is curved so as to give sufficient magnification in the middle and so little distortion towards the edges, that everything on the scale within the width is easily seen.

As such a lens is raised off the scale to a distance within its focal length the magnification increases, and I have made use of this fact for the purpose of subdividing the scale. On the under side of the lens I place a finely subdivided division, which can be fitted to any division of the scale by simply raising the lens until the apparent size of the two is the same, so that another decimal place is easily and accurately obtained.

But I have found it impossible to get a really satisfactory lens manufactured, and the compromises I have had made are altogether too expensive to form part of the cursor of a commercial slide rule.

DISCUSSION.

Mr. F. J. W. **Whipple** expressed his appreciation of the Paper. He pointed out to the author that there was a slide rule issued by the Meteorological Office in which the degrees were divided decimally. Could a little more information be given of the significance of the figures. The Paper, as it stood, was a little difficult to follow.

Dr. **Vincent** suggested a modification of the cross-section of the rule so that four of the scales could be readily marked off on paper for graphical purposes. This was the principal use that many workers had for these scales, and on the ordinary rule none were easily transferable.

Dr. **Eccles** asked how the accuracy of the new log log scale compared with the ordinary folded one.

The **Author** said he was interested to hear of the Meteorological Office rule. Dr. Vincent's suggestion could probably be better carried out on a special rule for the purpose he mentioned.

VI.—*The Current Density in the Crater of the Carbon Arc.*

By N. A. ALLEN, *B.Sc.*

RECEIVED MAY 27, 1920.

(COMMUNICATED BY D. OWEN, B.A., D.Sc.)

ABSTRACT.

In this work the Y-arrangement of carbons due to Forrest was used. This facilitated accurate measurement of crater area. The carbons used were solid special electra of diameter 6 mm. For currents up to 10 amperes the crater area was directly proportional to total current, thus maintaining a constant current density. This was 0·746 ampere per square millimetre.

THE object of the present research is an attempt to eliminate the uncertainty attending an investigation of the crater area, and the current density in the crater, of the carbon arc.

Mrs. Ayrton* found that a straight line law approximately connects crater area and current, but did not form any equation expressing the law, on account of possible errors in her measurements.

Current in amperes.			4	7	10	15	20
Projected arc lengths in mm.	1	Ayrton	7·56	13·85	14·2	23·3	32·2
		Rasch & Tornberg	7·74	12·25	16·55	23·76	30·97
	2	Ayrton	11·35	13·85	17·7	24·6	32·2
		R. & T.............	9·19	13·66	17·95	25·25	32·47
	3	Ayrton	9·89	13·85	—	22·5	32·2
		R. & T.............	9·9	14·39	18·7	25·88	33·08
	4	Ayrton	9·89	16·65	18·9	26·4	34·2
		R. & T.	10·35	14·86	19·17	26·42	33·59

Rasch and Tornberg† obtained similar results to those of Mrs. Ayrton, and the two series of measurements are tabulated below, areas in square millimetres.

Mrs. Ayrton used cored positives, Rasch cored negatives.

The lines representing these figures approximately, are shown in Fig. 3, for comparison with the results of the present work, which was done with arc lengths of 1 to 10 mm., the arc being always perfectly silent. Solid carbons were used.

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* "The Electric Arc," p. 151, *et seq.*

† "Electric Arc Phenomena," p. 143, *et seq.*

The chief difficulties met in a measurement of crater area are as follows :—

1. The crater cannot be viewed directly.

2. The area must be calculated from an oblique measurement of crater diameter, assuming the crater to be truly circular, which is by no means certain.

3. The edges of the crater may not be well defined.

4. The crater is curved inwards, so that the true area is greater than that observed.

5. The presence of fumes and carbon vapour in the crater may cause error due to refraction.

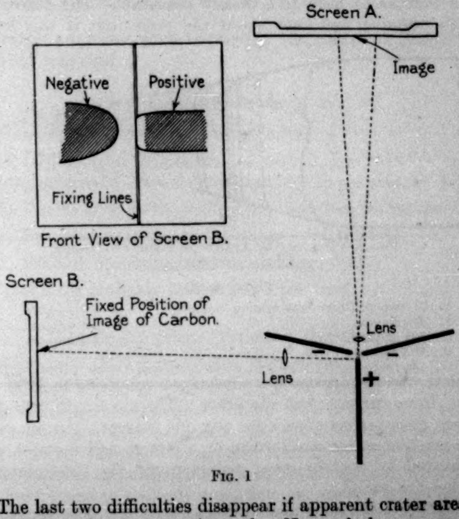
Screen A.
Image
Negative
Positive
Fixing Lines
Front View of Screen B.
Screen B.
Fixed Position of Image of Carbon.
Lens
Lens
–
–
+

FIG. 1

The last two difficulties disappear if apparent crater area is required, as in photometric work. None of these causes of inaccurate results, however, are found with the Y-arrangement of carbons originated by Forrest,* and used in the present work.

Messrs. Paterson, Walsh, Taylor and Barnett† assumed that the positive crater covers the whole of the end of the carbon for all currents when the current density in the actual carbon exceeds 0·2 amperes per square millimetre. Figures since communicated, however, show that for currents of 100 to 220 amperes the true density in the crater was sensibly constant, which agrees with the results of the present work, which was carried out with much smaller currents, the true density in their case being about 0·45 amp. per sq. mm.

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* "Electrician," 1913, Vol. LXXI., pp. 729 and 1007.

† "Proc." Inst. El. Eng., Vol. LVIII.

Methods Adopted.

The arc was always maintained absolutely silent, and before readings were taken, 10 or 15 minutes was always allowed for the carbons to burn to a steady condition. The current in each negative was kept about the same, but it was found that the positive crater was unchanged when one took 50 per cent. more current than the other. The arc length was that which gave perfect silence for any one current. An image of the positive crater was thrown on a screen, and the area was measured by making many points round the circumference with a fine pencil.

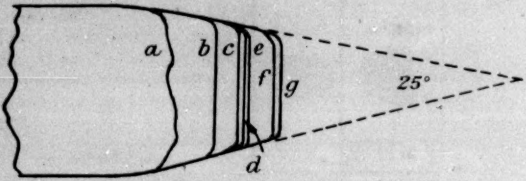

Fig. 2.—Tracing of Image of Positive Carbon.
(Magnification = 5·2 times.)
Crater Face.

a. 12 ampere (unsteady, hissing violently).
b. 9·0 ampere.
c. 7·0 ampere.
d. 6·5 ampere.
e. 6·0 ampere.
f. 4·8 ampere.
g. 4·5 ampere.

Since with the apparatus in use, an error of 1 mm. backward or forward in the position of the positive crater would cause an 8 per cent. error in the area, the arrangement shown in Fig. 1 was devised. A screen B was placed at the side of the arc, and an enlarged image of the positive carbon focussed upon it. Two fine lines were drawn on this screen parallel and vertical, and the end of the image must just show in between for the carbon to be correctly disposed.

The magnification of the image on screen A was measured by placing a series of cylindrical gauges having flat polished ends, with their faces in turn in the correct position, and measuring the image of the gauge when brilliantly illuminated.

Calibration of Magnification.

Gauge.	Diameter in mm.	Magnification.
1	4·782	31·72
2	3·210	31·49
3	2·739	31·73
4	1·680	31·56

Distortion being thus negligible, the mean magnification of 31·62 was used.

Limits of Accuracy.

Possibility of inaccuracy was as follows :—

(*a*) Due to error in position of carbon, less than 0·1 mm. causing area to be subject to an error of 0·75 per cent.

(*b*) Due to error in magnification ; less than 0·1 per cent.

(*c*) Due to error in measuring image —0·6 per cent.

(*d*) Error in observing current —0·25 per cent.

Total error probably well within 1 per cent.

Results (*a*) *Shape of Carbon and Crater.*

The crater was always a true circle. It is suggested that this is due to the mutual attraction of parallel streams of current flowing in the same direction. It was found impossible to maintain two separate arcs on the end of the same positive.

A tracing was made at several currents of the projection of the positive carbon. This showed that in all cases when the arc has burned to a steady condition, the carbon tapers down uniformly at an angle of about 25 deg. (*see* Fig. 2), the crater occupying the whole of the reduced end of the carbon. The crater was flat and quite smooth, and perpendicular to the axis of the carbon, as evidenced :—

1. By the appearance of the carbon when examined, and
2. By the fact of the whole crater being in focus on the screen when projected.

Results (b) Crater Area and Current Density.

On Fig. 3 are plotted all the observations made, and for comparison results by previous investigators are given also.

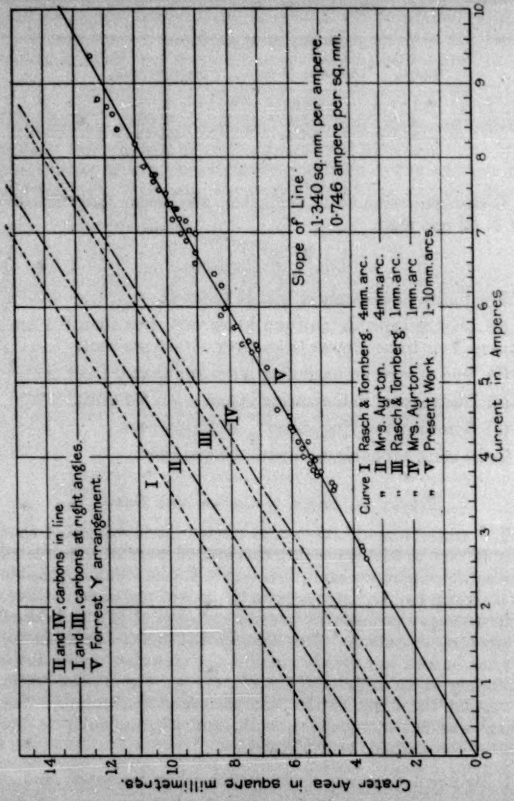

FIG. 3.

The current densities have been worked out for Fig. 4. The maximum deviation from the mean on any observation in a

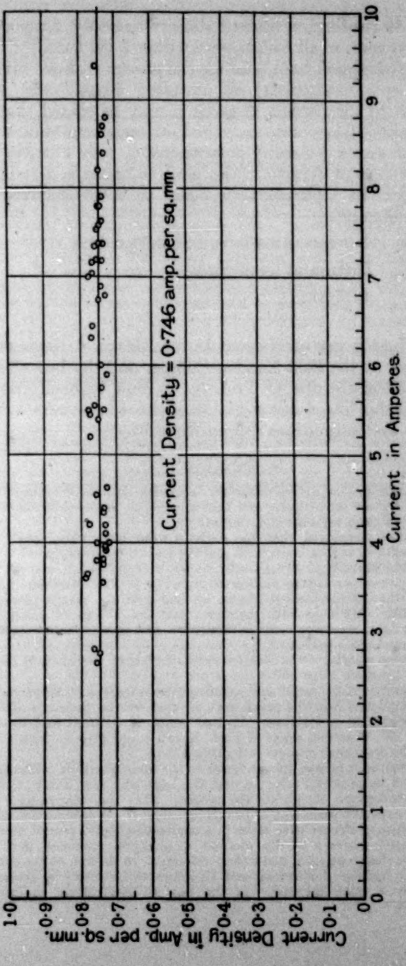

Fig. 4.—The Current Density in the Crater of the Carbon Arc.

series of 60 readings of current density was only 5 per cent., and 70 per cent. of all results were within 2 per cent.

The carbons used were solid special electra carbons marked [----------ELECTRA] of diameter 6 mm., and as a conclusion to the present research it may be stated that for currents up to 10 amperes, using the Y-arrangement of carbons :

(*a*) Crater area is directly proportional to total current, and

(*b*) The current density in the positive crater is a constant and, correct to 1 per cent., is equal to 0·746 amperes per square millimetre.

If A = crater area in square millimetres,

I = current in amperes,

then $A = 1{\cdot}340\,I$.

This research was carried out in the Electrical Engineering Laboratory at the East London College, and the best thanks of the author are due to Prof. J. T. MacGregor-Morris for suggesting this line of investigation, and for some very helpful criticisms and suggestions during the work.

DISCUSSION.

Prof. Morris read a portion of a letter from Mrs. Ayrton, who considered the Paper of great importance, and that the results were what she would have expected from accurate determinations.

The speaker mentioned that the carbons used were very pure. The effect of quality of carbon on current density had to be investigated.

The late Mr. Forrest, of whose work this was a continuation, showed that the candle-power per square millimetre was 172 to 174. Combining this result with that of the present Paper, we find that the candle-power per ampere is 232. He suggested, therefore, that the Y-type of arc gives a standard of light, requiring only an ammeter and good quality carbons, constant to at least 1 per cent.

Prof. Rankine asked if the result was unaffected by the mass of carbon carrying off the heat.

Prof. Morris said the result was certainly true from 6 to 10 amperes.

Dr. Bryan asked how the result was affected by the presence of areas of special brightness or darkness. He had frequently noticed such areas.

Mr. F. J. W. Whipple asked if Prof. Morris could give an idea of the physics of the constancy of current density.

Dr. D. Owen said he was going to refer to the same question. The crater was stated to be perfectly circular, and the edge was very sharp, judging from the photograph shown on the screen. This was surprising. One would have expected some sort of penumbral region. A comparison of the ionic conditions in the arc with those in a conducting liquid was of interest. neu = constant where n is the number of charged particles per unit volume, e the charge on each, and u their velocity. In the arc, as the current increases the voltage diminishes, and therefore the velocity u decreases. Consequently n must increase. In the case of electrolytic conduction n is constant.

Mr. J. W. T. Walsh (communicated): I much regret that another engagement prevents my being present when Mr. Allen's very interesting and valuable Paper is read.

The results obtained are most surprising in their regularity and the accuracy with which the crater area remains proportional to the current for all the arc lengths employed. The current density found is much higher than that generally recognised hitherto as the limits of steady burning for arcs of large currents, say, from 100 to 200 amperes. It would be most valuable if the author could see his way to extending his researches to include currents of such magnitude as this. The work done at the National Physical Laboratory was practically confined to large current arcs, and the difficulties experienced in obtaining steady burning conditions with these show that the smaller current arcs behave much more regularly, and are, therefore, probably capable of smooth running at much higher current densities than the larger arcs. In this connection I should like to express my thanks to Prof. Morris, who very kindly gave me an opportunity of seeing Mr. Allen's apparatus working at East London College. The smoothness of running of the arc, and the uniformity of brightness of the crater were remarkable and I feel sure that everyone who has worked on the carbon arc will be more than interested in the results presented this evening, and will look forward to further work by the author on similar lines.

The **Author** communicated the following: In reply to Prof. Rankine, the carbons used were all the same size—6 mm. diameter.

The effects mentioned by Dr. Bryan, which are due to inequalities in the carbons, do not affect the results, as when a bright or dark area is present the arc is unsteady, and one has been careful to avoid taking readings under unstable conditions.

Dr. Owen assumed that as the current increases the voltage diminishes. This is true when the length of the arc is kept constant, but in the case of these silent arcs, length is approximately proportional to current, hence as the current increases, the voltage remains fairly constant. Then in the equation used by Dr. Owen ($n\,e\,u = \text{const.}$), since the voltage is constant, therefore the velocity is constant, hence the number of ions or charged particles is constant.

Another interesting result following from the relation between current and length for silence is as follows: Mrs. Ayrton ("Electric Arc," p. 158) suggested that the equation connecting arc current and area was

$$A=\alpha L+\beta I. \qquad (A=\text{area},\ L=\text{length},\ I=\text{current.})$$

If $L=\gamma I$,

then $A=\text{constant}\times I$, as the present work shows.

Mr. Walsh mentions that the current density obtained was higher than he would have expected. This high current density appears to be due largely to the reactions of the arc streams upon each other, forcing up the current density to a high value.

VII. *A Sodium Vapour Electric Discharge Tube.* *By* F. H. Newman, *M.Sc., A.R.C.Sc., University College, Exeter.*

Received September 6, 1920.

Abstract.

A simple method of producing a bright source of sodium radiation is described. An electric discharge tube of quartz, having a pool of sodium-potassium alloy as one electrode, and an iron rod as the other electrode, is utilised. The sodium vapour is proeduced by gently heating the lower part of the tube, containing the alloy, over a ring gas burner concentric with the tube. When an electric discharge is passed, a bright yellow light is emitted. This light consists almost entirely of the *D* lines. Any gases present in the tube are absorbed by the alloy while the discharge is passing. No continuous pumping is necessary to keep the tube exhausted, and the experiment works best when the temperature of the sodium vapour is 250°C.

1. Introduction.

In order to stimulate the resonance radiation in a bulb of sodium vapour, the exciting light must consist of very narrow bright *D* lines. A Bunsen flame to which salt has been added is not a suitable source, since the centre of the line is dark. The lines themselves are very broad, and the exciting radiation is not confined to the actual region of the spectrum required. When a sodium arc in vacuum is substituted for the flame, this difficulty is overcome. Lord Rayleigh * has constructed and exhibited a sodium vacuum arc in quartz, which is similar in principle to the mercury lamp. The sodium lamp is more troublesome to construct and work than the mercury lamp, and requires continuous pumping to keep it exhausted while the lamp is running. Iron electrodes are unsuitable, since they fuse and drop off after the lamp has been in use for an hour or two. Tungsten, which seems to withstand the action of sodium vapour, is used instead of iron.

The apparatus described below works well, and although it does not give as brilliant a source of light as the sodium arc, yet it can be quickly constructed, and requires no continuous pumping while the electric discharge is passing.

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* Hon. R. J. Strutt, "Proc." Roy. Soc., A, Vol. XCVI. (1919).

2. Apparatus.

The form of discharge tube used is shown in Fig. 1, and the electric discharge through sodium vapour is used. The tube is of quartz; one of the electrodes (*B*) is a pool of the sodium-potassium alloy; the other electrode is of iron, and an iron wire is used to make electrical connection with the alloy. The latter is introduced into the tube through a side tube (*D*). Throughout an experiment (*D*) is closed by means of a quartz plate. When an electric discharge is passed, the scum which is formed at the surface of the alloy by oxidation, disappears, and a clean surface is obtained. The alloy absorbs all gases in the tube except argon and helium, when the pressure is low, and any gas liberated, while the electric discharge is passing, is at once absorbed by the alloy. As a result the pressure of the gas in the tube is always very low. To obtain the sodium vapour the alloy must be heated to 250°C. to 300°C., as the heat generated by the electric discharge is not sufficient to produce the vapour. A small gas ring is suitable as a source of heat. When a condensed discharge passes through the sodium vapour, a brilliant yellow light is produced. The light is intensified by making a constriction in the tube at (*C*). When examined with a

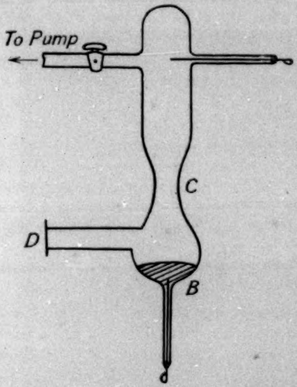

FIG. 1.

spectroscope, the potassium spectrum is very faint, almost all the light coming from the *D* lines of sodium. The faintness of the potassium spectrum is due to the vapour pressure of potassium being less than that of sodium at the temperature of the experiment.

While the discharge tube is running, the sodium and potassium vapours condense on the colder parts of the tube. This is not a serious disadvantage, and the condensation can be eliminated by enclosing the tube within an electrical heater. The iron electrode is not affected by the sodium vapour, even after the tube has been in use for several hours. No continuous pumping is required while the discharge is passing, as, after the initial exhaustion of the tube, the action of the alloy keeps the vacuum high. This is a great advantage, as hydrogen is always freely emitted from the sodium-potassium alloy, when the latter is heated. With the temperatures stated above there is no reduction of the silica. In practice, the alloy should be made the cathode, as it absorbs gas at a greater rate when made the cathode than when it is the anode. The light cannot be viewed through the window at (*D*), as the vapour condenses on it after a short time.

Using this discharge tube, the light from it was focussed on a glass bulb containing sodium vapour. To vary the density of the vapour, the glass bulb was enclosed in an electric heater. At a temperature of 250°C., brilliant fluorescence of the sodium vapour was stimulated. The fluorescence was photographed (Fig. 2). The light patches shown are due to reflected lights. As the temperature of the glass bulb was raised, the fluorescence across the tube decreased, owing to the increased density of the vapour. Heating the side of the bulb with a Bunsen flame the fluorescence moved away from the locality heated. The discharge tube is a suitable source of sodium radiation for interferometer experiments.

DISCUSSION.

Dr. **Bryan** commented on the quality of the silica tap in the author's apparatus, which had evidently remained at a high vacuum for a considerable time.

Mr. **Newman** said the tap had probably been manufactured by luck, as some previous ones he had had were very bad. The grease he used was made of bees wax and vaseline.

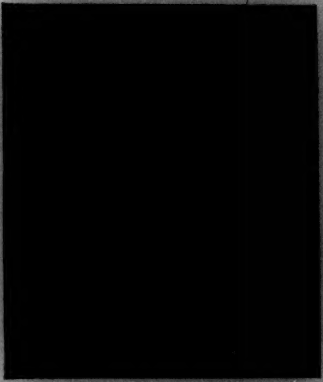

Fig. 2.—Fluorescence of Sodium Vapour. Exciting Light from the Sodium Vapour Electric Discharge Tube.

[*To face* p. 72.

VIII. *Absorption of Gases in the Electric Discharge Tube.*

By F. H. NEWMAN, *M.Sc., A.R.C.Sc., University College, Exeter.*

RECEIVED JULY 20, 1920.

ABSTRACT.

Various elements were deposited in a pure condition on the cathode in an electric discharge tube. To obtain a clean surface of the element, the latter was distilled in vacuum. An electric discharge was passed through nitrogen gas contained in the tube. Measurements were made to compare the amount of gas absorbed by the element in the tube with the quantity of electricity passing in the circuit. Potassium, sodium, mercury, cadmium, antimony, magnesium, calcium, zinc, tin, phosphorus, sulphur and iodine were tested in this way. The rates of absorption were very great with the last three elements. Hydrogen gas was also used in the tube, and absorption occurred with phosphorus, sulphur and iodine. The absorption of the gases is due principally to chemical action, and is attributed to the nitrogen and hydrogen assuming active modifications on the passage of an electric discharge, the gases being at low pressure.

1. INTRODUCTION.

THE present investigation is a development of the experiments described in "Proc." Roy. Soc., A., Vol. XC. (1914), and Proc. Phys. Soc., Vol. XXXII. (1920). In these Papers it has been shown that sodium, potassium and the liquid alloy of sodium and potassium absorb nitrogen when these metals are used as electrodes in the electric discharge tube. The gas is absorbed at a greater rate if the metal is used as the cathode. The quantity of gas absorbed, per unit quantity of electricity passing through the discharge tube, was measured at various pressures of the gas. No absorption occurred unless the surfaces of the metals were quite clean. Sodium and potassium were distilled in vacuum in order to obtain surfaces which were not contaminated. Other elements, in the molten condition, had been tested without success; this was traced to the contamination of the surfaces, and these elements are re-examined in the present work. The same apparatus, as described in the above Papers, was used, but the form of the discharge tube was changed, and is shown in Fig. 1. The electrodes *A* and *B* were of aluminium. To the end of the cathode *B* was attached a small disc of platinum foil. The element under examination was placed at the bottom of the tube, and by distilling it in vacuum, a thin layer of the element was formed on the foil.

2. Experiments.

A clean surface of the element having been formed on the cathode, a known quantity of nitrogen was admitted into the discharge tube. The method of measuring this amount of gas has been described in the previous Papers. The electric discharge was then passed through the gas, and the amount of hydrogen liberated in the voltameter gave a measure of the quantity of electricity passing in the circuit. The absorption of gas during the passage of the discharge was noted. In some cases it was necessary to maintain the discharge tube at a low temperature, by placing it in an enclosure surrounded with a freezing mixture. The nitrogen used in the experiments was prepared by allowing air to stand over phosphorus for two or three days. The hydrogen was obtained by electrolysis. Both gases were examined at various pressures, and the results obtained are shown in Tables I. and II.

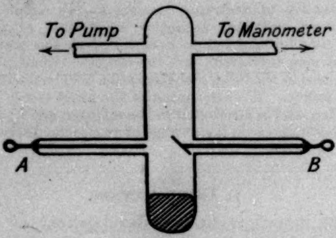

FIG. 1

3. Absorption of Nitrogen.

Lead, thallium and arsenic did not absorb nitrogen. The elements were tested with the gas at different pressures and temperatures.

TABLE I.—*Absorption of Nitrogen.*

Element.	Pressure in mms. Hg.	Temperature.	Volume of N_2 absorbed in c.c. (A)	Volume of H_2 liberated in Voltameter in c.c. (B)	Ratio $\frac{A}{B}$	Pressures at which absorption takes place in mm.s Hg.
Mercury	1·3	−13°C.	0·40	2·21	0·2	0·5—1·5
	0·9		0·36	0·92	0·4	
	0·6		0·33	0·51	0·6	
Cadmium	1·1	30°C.	0·18	0·13	1·4	1·1—1·8
	1·8		0·29	0·21	1·4	
Arsenic Lead Thallium	No absorption					
Antimony	1·8	−15°C.	0·65	0·45	1·4	1·2—1·9
	1·5		0·31	0·22	1·4	
Magnesium	3·4	50°C.	0·07	0·11	0·6	1·0—4·3
	2·0		0·17	0·19	0·9	
	1·9		0·31	0·16	1·9	
	1·3		9·68	0·24	2·8	
Calcium	1·8	−12°C.	0·14	0·12	1·2	1·1—1·9
	1·2		0·19	0·14	1·3	
Zinc	1·7	−12°C.	0·13	0·10	1·3	1·2—1·8
	1·5		0·20	0·14	1·4	
Tin	1·7	0°C.	0·28	0·57	0·5	0·1—2·0
	1·2		0·14	0·23	0·6	
	0·9		0·17	0·17	1·0	
Phosphorus......	13·3	−12°C.	0·37	0·03	12·3	X-Ray stage to 20 mm.s
	10·1		0·27	0·03	9·0	
	7·3		0·29	0·04	7·2	
	4·0		0·14	0·05	2·8	
	2·5		0·15	0·07	2·1	
	1·0		0·27	0·13	2·1	
Sulphur	11·6	−12°C.	0·23	0·04	5·8	0·1—12·0
	7·9		0·15	0·04	3·8	
	3·2		0·13	0·04	3·2	
	1·6		0·08	0·03	2·7	
Iodine	16·4	−20°C.	0·44	0·08	5·5	1·0—18·0
	12·2		0·29	0·07	4·1	
	8·3		0·27	0·08	3·4	
	4·1		0·31	0·11	2·8	
	1·9		0·23	0·11	2·1	
Bismuth	1·7	−14°C.	0·43	0·36	1·2	1·1—1·8
	1·4		0·22	0·17	1·3	

TABLE II.—*Absorption of Hydrogen.*

Element.	Pressure in mm.s Hg.	Temperature.	Volume of H, absorbed in c.c. (A)	Volume of H, liberated in voltameter in c.c. (B)	Ratio $\frac{A}{B}$	Pressures at which absorption takes place in mm.s Hg.
Phosphorus ...	12·8	−12°C.	0·28	0·03	9·4	X-ray stage to 18·0 mm.s
	11·2		0·25	0·03	8·3	
	8·1		0·29	0·05	5·8	
	3·7		0·26	0·09	2·9	
	2·4		0·23	0·11	2·1	
Sulphur	12·1	−12°C.	0·37	0·05	7·4	0·5−12·0
	8·3		0·13	0·03	4·3	
	3·1		0·37	0·11	3·4	
	1·4		0·29	0·12	2·4	
Iodine	14·6	−20°C.	0·39	0·09	4·3	1·0−15·0
	11·8		0·40	0·12	3·3	
	7·3		0·23	0·07	3·3	
	3·1		0·25	0·09	2·8	
	2·3		0·27	0·11	2·5	

Mercury absorbed nitrogen when the pressure of the gas was between 0·5 mm. and 1·5 mm. of mercury, if the tube was kept at −13°C. The amount of gas which could be absorbed at a time was very small. When the discharge was stopped, and the tube heated to about 20°C., all the gas which had disappeared was re-liberated. After the surface of the mercury had been exposed to the atmosphere, no absorption occurred, which showed the importance of having a clean surface of the element. Stark* has noted that mercury, used as an electrode in the discharge tube, absorbs nitrogen and oxygen.

Cadmium absorbed the gas at ordinary temperatures, providing the pressure was between 1·1 mm. and 1·8 mm. of mercury. Most of it was re-liberated on heating the tube. The effect in this case may be due to occlusion of the gas within the metal. On the other hand, there may be a chemical compound of cadmium and nitrogen produced, which compound dissociates easily on the application of heat. The total amount of gas which was absorbed at a time was very small.

Antimony absorbed the gas at low temperatures if the pressure varied from 1·2 mm. to 1·9 mm. of mercury. The amount of gas which disappeared before it was necessary to prepare a fresh surface was only 0·2 c.c., and the gas absorbed was evolved when the tube was heated to 50°C. Antimony resembles cadmium in this respect.

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* "Phys. Zeit.," Vol. XIV., p. 417 (1913).

When magnesium is heated to a red heat it combines with nitrogen, forming a nitride of magnesium. This fact was utilised by Rayleigh* in the separation of nitrogen and argon. At ordinary temperatures there is no such combination. When magnesium was used as the electrode in the discharge tube, nitrogen was absorbed if the pressure of the gas was between 1·0 mm. and 4·3 mm. of mercury. At low pressures the rate of absorption was very high. This absorbed gas was not re-liberated when the tube was heated to 300°C. After about 4 c.c.s of the gas had disappeared, the contents of the tube were mixed with caustic soda, and boiled. Ammonia gas was evolved, showing that magnesium nitride had been formed in the tube. The absorption in this case is due to chemical action. It was possible to absorb about 4 c.c.s of gas at atmospheric pressure before the action ceased.

Soddy† has shown that when calcium is first heated in vacuo, gaseous compounds of carbon, hydrogen and oxygen are given off; if the heating be continued, absorption of the gases begins, accompanied by volatilisation of the calcium. Experiments were made by him on the power of calcium, to absorb different gases, and he found that all gases, except the argon group, were rapidly absorbed. In the discharge tube I found that nitrogen was absorbed at 12°C. if the pressure of the gas was between 1·1 mm. and 1·9 mm. of mercury. It was possible to absorb only 0·1 c.c. of gas at a time, but this small amount was due to the very thin layer of calcium deposited on the cathode. On heating the tube to 300°C. none of the gas was evolved. This suggests that chemical action accounts for the absorption in this case.

Zinc absorbed nitrogen at —12°C., providing the pressure of the gas varied from 1·2 mm. to 1·8 mm. of mercury. The total amount of gas which disappeared was very small, and most of it was re-liberated when the tube was heated.

Tin does not combine with nitrogen when it is heated in that gas, but the gas was freely absorbed by the metal in the discharge tube if the pressure was within the range 2·0 mm. —0·1 mm. of mercury. The rate of absorption was uniform, and was large compared with that obtained with other metals.

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* "Phil. Trans.," 186 (1895).

† "Proc." Roy. Soc., Ser. A., 78, p. 429 (1907).

It was possible to absorb 0·3 c.c. of gas at a time. When the tube was heated to 300°C. no gas was evolved from the metal, and the presence of a nitride of tin was found by the method described in the case of magnesium. This absorption of nitrogen by tin has not been observed previously.

Strutt* has shown that phosphorous absorbs nitrogen and hydrogen when an electric discharge is passed through a bulb containing this element. The gases were not re-liberated on heating, and he attributed the absorption to the formation of chemical compounds. The rate of absorption of these gases, compared with the quantity of electricity passing through the tube, is very great; and if phosphorus is placed in a discharge bulb the vacuum is kept very high. In the present experiments the discharge tube was maintained at —12°C., in order to minimise the production of phosphorus vapour. The actual rate of absorption was about 4 c.c.s (at atmospheric pressure) in 15 minutes; this rate decreased as the discharge continued, and the amount of gas which could be absorbed depended on the thickness of the phosphorus layer used in the experiment. As shown by Strutt, the gas was not re-liberated from the phosphorus when the tube was heated, and it appears that a chemical compound of nitrogen and phosphorus is formed.

With sulphur, a large rate of absorption was observed, and the absorption occurred over a range of pressures 12·0 mm.—0·1 mm. of mercury. By means of the discharge the pressure of the gas in the tube was lowered rapidly, and reached the X-Ray stage, provided a clean surface of sulphur was used on the cathode. When the sulphur was exposed to the atmosphere, the rate of absorption of the gas was less, and it was not possible to obtain such a final low-pressure in the tube. No re-liberation of the gas occurred when the contents of the tube were heated to 100°C. Of course the sulphur vaporised, but on allowing the contents to cool, the original volume was attained. This showed that chemical combination had occurred between the gas and sulphur. The absorption of the gas was greater if the film of sulphur on the cathode was formed by condensation of the vapour, than if the layer was produced by allowing melted sulphur to flow over the cathode.

The results obtained with iodine resembled those noted in the case of phosphorus. No absorption occurred if the pressure of the gas was below 1 mm. of mercury. On account of the sublimation of iodine, the tube was maintained at —20°C. After the surface of the substance had been exposed to the atmosphere, the rate of absorption decreased. There was no re-liberation of the gas when the contents of the tube were heated to 100°C., which indicated that the phenomenon was due to chemical action, and not to mere occlusion of the gas within the iodine.

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* "Proc." Roy. Soc., Ser. A., 87 (1912).

The results obtained with bismuth were similar to those found with antimony. The discharge tube was maintained at —14°C., and absorption occurred when the pressure was between 1·8 mm. and 1·1 mm. of mercury. Only 0·18 c.c. of gas was absorbed by any one surface, and all of the gas absorbed was evolved when the tube and its contents were heated to 40°C. The effect in this case appears to be due to occlusion.

4. ABSORPTION OF HYDROGEN.

As shown in Table II. the only elements which absorbed hydrogen were phosphorus, sulphur and iodine. The amount of hydrogen which could be absorbed at a time by phosphorus, and the rate of absorption, both increased when the tube was warmed. In this case it seems that the chemical action between the solid and the gas is increased as the temperature rises. It is known that phosphoretted hydrogen is produced when phosphorus is heated in a stream of hydrogen. If the discharge was stopped, and the tube warmed, some of the gas absorbed was re-liberated. In this case, therefore, the absorption is due partly to occlusion, and partly to chemical action.

Sulphur absorbed well, providing the pressure of the gas was not below 0·5 mm. of mercury. Below this pressure the effect disappeared. When the contents of the tube were heated above the temperature of melting sulphur, and then allowed to cool, about one-sixth of the gas that had been absorbed was re-liberated. In this case, therefore, chemical action and occlusion account for the absorption.

The amounts of gas absorbed by iodine were similar to those found with phosphorus. There was no absorption when the pressure of the gas was below 1 mm. of mercury. About one-fifth of the gas which had been absorbed in an experiment was evolved when the contents of the tube were heated to 100°C.

5. CONCLUSIONS.

When considering the absorption of gases in the electric discharge tube, the part which the glass of the tube plays must be taken into account. Willows* has shown that in a Crooke's tube there is absorption of the gas, and that it is due probably to chemical action between the gases and the glass. He found that, with a volume of 108 c.c., and a pressure of 1 mm. of mercury, a current of $10{\cdot}8\times10^{-4}$ ampere passing for 30 minutes reduced the pressure nearly one-eleventh. This is a very small absorption compared with that found in the present work. The effect of absorption by the glass can thus be neglected in the present experiments. Willows and George† found that large quantities of hydrogen were absorbed by a quartz bulb, if the discharges were alternated with those in air. They supposed that chemical actions occur with the air, giving oxidation products; these are reduced by hydrogen. This would account for the large quantity of gas which can be absorbed. In the present work, as no oxygen was used, this effect is negligible. Thus practically all the absorption of gases shown in Tables I. and II. is attributed to the elements on the cathode.

The absorption of nitrogen is not an occlusion effect, but is due to chemical action. In several cases, the amount of gas absorbed is many times greater than the quantity of hydrogen liberated in the voltameter. This indicates that the phenomenon is not due to the positive ions within the tube, unless these ions are attached to neutral molecules of the gas. The nitrogen must assume an active modification when the discharge passes, and since the rate of absorption increases as the temperature falls, this modification must be intensified by cooling. Strutt‡ showed that pure nitrogen, subjected to the jar discharge, produces an active modification of nitrogen, and that it is not due to the ions in the discharge tube. He found that the modification was weakened by heating, and intensified by cooling. It acted on ordinary phosphorus and other elements.

Table I. shows that phosphorus, sulphur and iodine absorb nitrogen at a greater rate than other elements. In addition to the absorption by the cathode, there is chemical action occurring between the vapour of the elements and the gas.

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* "Phil. Mag.," 1 (1901).

† Proc. Phys. Soc., Vol, XXVIII. (1916).

‡ "Proc." Roy. Soc., Ser. A., 85 (1911).

This view is strengthened when we consider that the three substances have low boiling points. Some elements do not absorb nitrogen, and this is due, probably, to the absence of suitable conditions (as regards pressure and temperature) in the discharge tube.

Hydrogen is absorbed by sulphur, phosphorus and iodine. With this gas the effect appears to be due to two causes—occlusion of the gas within the substance, and chemical action. As the amount of gas occluded is very small, the more important factor in the absorption is chemical action. The results suggest that hydrogen assumes an active modification in the discharge tube, and that this modification is not due to gaseous ions. Recently, and while the present work was in progress, Wendt and Landauer* have shown that an active modification of hydrogen may be obtained by various methods, all dependent on gaseous ionisation; by α-rays from radium emanation, by electrical discharge under reduced pressure, and by the high-potential corona at atmospheric pressure. They state that this active hydrogen reduces sulphur, arsenic, phosphorus, mercury, nitrogen and both acid and neutral permanganates. It is unstable, and reverts to the ordinary form in about a minute. The activity is not due to gaseous ions, and they suppose that the stable hydrogen molecules undergo dissociation into free atoms, when an electron is removed during the process of ionisation; these atoms then attach themselves to neighbouring neutral molecules to form triatomic systems.

The disintegration of the cathode in a vacuum tube causes a film of metal to form on the adjacent glass. In this way a clean surface of metal is produced. When an electric discharge is passed, the conditions are favourable for the absorption of the gas by the metal. Considering the disappearance of the gas in a vacuum tube it is necessary to take into account the absorption of gas by the film of metal, and also the absorption by the glass walls of the tube. It is probable that the film of metal accounts for the greater part of the effect. Brodetsky and Hodgson showed that the disappearance of gases in vacuum tubes is a problem of some complexity, and that with metallic electrodes the effect is due principally to absorption of gas by disintegrated metal; it may also depend partly on the liberation of the alkali metals by electrolysis of the glass, and chemical combination of these metals with the gas.

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* "Am. Chem. Soc. Journ.," 42, May (1920).

DISCUSSION.

Dr. **Goucher** said that work bearing closely upon Mr. Newman's experiments on the disappearance of gas in the electric discharge had been carried on lately in the Research Laboratory of the General Electric Company. Some of it had already been published ("Phil. Mag.," XL., 585, Nov., 1920). It had led to conclusions precisely opposite to those of Mr. Newman. It did not appear to him that Mr. Newman had brought any direct evidence in favour of absorption being due to chemical action; the only direct evidence of that view would be the establishment of the "law of constant proportions," and of a constant ratio between the amount of gas disappearing and that of something else disappearing at the same time. The absorption of gas in the presence of phosphorus had been studied with especial care, and it had been found that there was no simple relation between the amount of gas disappearing and the amount of phosphorus used. Undoubtedly chemical action plays some part in the disappearance. For instance, the discharge through CO converts the gas into CO_2, and the relation between the current and the amount of gas transformed obeys accurately Faraday's law in suitable conditions. If, as in the discharge tubes which Mr. Newman used, ionisation is accompanied by recombination, it is not to be expected that Faraday's law should be obeyed. But even in such a case where a chemical action takes place, that action is not usually directly responsible for the disappearance of the gas. Chemical action will not cause complete absorption unless the products of the reactions are solid. Such compounds might be formed between nitrogen and the metal of the electrodes, but there is no evidence whatever that they can be formed between nitrogen and phosphorus or sulphur. The main cause of absorption of gas is, in the great majority of cases, adhesion to the walls of the vessel. This adhesion is greatly favoured by the deposition over the adhering gas of a solid layer of some other substance. The solid may be metal spluttered from the cathode or it may be, in the case of phosphorus, red phosphorus produced by the discharge from phosphorus vapour. All the work to which he referred indicated that actions of this kind were the main cause of the disappearance of gas in the electric discharge, except, possibly, when very active metals were used as cathodes.

Dr. H. **Borns** inquired whether formation of hydrides had been observed. If the alkali metals absorbed hydrogen also, as he understood the author to say, alkali hydrides might be formed, which were crystalline compounds like common salt, not in the least resembling metals. Quite recently Nernst and K. Moers ("Zeits. f. Elektrochem," Aug. 1, 1920) had electrolysed fused lithium hydride to see whether the hydrogen would really go to the anode in this case, as the metallic lithium would go to the cathode. Quantitatively the difficult experiments had not been conclusive, but there was evidence of hydrogen at the anode and hence experimental evidence confirming the possibility of the existence of both positive and negative hydrogen ions.

The **Author**, in reply to Dr. Goucher, said he did not think the results he mentioned were conclusive against chemical action. Lord Rayleigh had also attributed the effect to chemical action.

In reply to Dr. Borns, he could not say whether hydrides were formed or not.

IX. *Magnetic Separation of Neon Lines and Runge's Rule.*

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RECEIVED DECEMBER 8, 1920.

ABSTRACT.

The Paper contains the results of an investigation of the Zeeman effect for Neon lines. The departures from Runge's Rule—that the magnetic separation of the lines are aliquot parts of the separation of the normal triplet—are discussed. It is concluded that such discrepancies are due to variations of the ratio e/m.

THE investigation of the Zeeman effect for neon lines by Lohmann[*] leads to the interesting result that the magnetic separations of most of the lines are aliquot parts of a normal triplet. This rule deduced by Runge[†] must, however, be looked upon only as an approximation, for most of the lines do not exactly conform to the rule.

Let e be the charge of an electron, m its mass at rest, H the magnetic field, and c the velocity of light, then the constant a given by

$$a=\frac{e}{m}\cdot\frac{1}{4\pi c}$$

should have a value $4{\cdot}698\times10^{-5}$ gauss^{-1} cm.^{-1}, corresponding to $\frac{e}{m}=1{\cdot}771\times10^{7}$ E.M.U. The last figure in a and $\frac{e}{m}$ may differ by small amounts according to different experiments. This is, however, not fulfilled in most of the neon lines; take, for instance, the lines 6717, 6266, 5852 and 5820, which are all assumed to be separated into triplets. The values of $10^5\times a$ for these lines as given by Lohmann are respectively 4·70, 4·40, 4·74 . . . .; while Takaminé and Yamada[‡] give 4·82, 4·72, 4·84, 4·47 respectively for the above constant. As to the measurement of the field strength, the experiments of the last two mentioned authors were conducted with great care, the amount of separation of different lines being always compared with that of the helium line 6678. Thus the separation of the characteristic neon line 5852 is 1·5 per cent. greater, that of 5820 about 5 per cent. less than the normal value. As to the complex separations, if we express aliquot parts of a by p/q, the values of p and q are to some extent indefinite, as we can closely approximate to the rule by taking tolerably large numbers for p and q. The question is whether a is to be assumed as constant, or as variable within certain limits.

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[*] Dissertation, Halle (1907).

[†] "Phys. Zeit.," VIII., p. 232 (1907).

[‡] "Proc." Tokyo Math. Phys. Soc., VII., p. 277 (1914).

Various reasons can be offered to explain the discrepancy. The theory of relativity would at once suggest that the discrepancy is a consequence of the high velocity of the radiating electrons, but to account for 5 per cent. difference, the velocity of electrons must be equivalent to a third part of the velocity of light, which is probably too great. In addition to this, relativity cannot account for the values of a smaller than the normal value. In a broad sense, we can attribute the difference of a from the normal value to the difference in the surrounding electromagnetic conditions, as the electrons in an atomic configuration are under mutual influence and also subject to attraction towards the nucleus. The inadequacy of the theories of the Zeeman effect may be the principal cause of these difficulties, but there are other doubts as to the experimental results from which these values are deduced.

In the first place, the amount of separation may not be proportional to the strength of the magnetic field, and hence may lead to discordant values. In the second place, the photographic exposure in former experiments was tolerably long, so that the uncertainty of the field strength by heating, the displacement of the echelon grating spectra even by small variations of temperature, may become sources of error in the photograms obtained by long exposure, sometimes extending to several hours.

In the present experiment, the photographic plate was sensitised by means of dicyanin, which made the plate more sensitive to red rays than panchromatic plates, so that the magnetically separated lines of neon in the yellow and red part of the spectrum can be photographed with comparatively short exposure, especially in strong fields. The constancy of magnetic field during this interval can be easily controlled, and by keeping the edge of the echelon grating horizontal, the red and yellow portion of the spectrum can be photographed on one plate, so that the separation of the lines can be compared generally for about 20 lines all in the same field. The line which is most approximate to the normal triplet and lies nearly in the middle of the said portion of the spectrum is 6266. The intensity of the line is neither too strong nor too weak; consequently the separation of the line can be accurately measured on all the plates, and was taken as the standard for gauging the separation of other lines. No exact measurement of the field was considered necessary, as the previous experiment of Takaminé and Yamada on 6266 gave the value $a=4{\cdot}72\times10^{-5}$ by comparison with the helium line 6678, the separation of which was accurately determined by Paschen and Back*. The line 5852 was not used as a standard as the deviation from the normal was greater than for 6266, and the intensity was too strong to admit of accurate determination of the mean position in plates for faint lines.

A neon tube having the gas at about 6 mm. pressure was placed between the poles of an electromagnet, having the pole pieces in the form of a double cone of semi-vertical angle of about 60 deg.; the gap between the pole pieces was about 7 mm., and the capillary portion of the tube was at the apex of the cones. As the tube was liable to crack at the electrodes and especially the capillary tube between the poles owing to local heating, the electrodes and the capillary tube were water-cooled. By this means, the tube in the mean field of 13 kilogauss was kept for more than 200 hours by passing discharge current from an induction coil with a Leyden jar of about 0·001 mfd. capacity in the circuit. The arrangement of water-jacketing the electrodes can be applied with advantage in other works with vacuum tubes, when it is required to run the tube for many hours with discharge current.

With the water-jacketing arrangement, the disintegration of the electrodes is greatly diminished. The capillary tube of the vacuum tube after long use turns brownish, and there is much absorption at the portion placed in strong field, that different parts of the capillary must be used to obtain good illumination. With one tube, the part long exposed to strong magnetic field was broken, and when examined by microscope was found to show a corroded appearance, ramified in several parts in the direction perpendicular to the field. As is evident from the intense light emitted from the portion so exposed, it appears that the ionised gas atoms move with great velocity and corrode the glass tube. A somewhat diffuse appearance of lines in strong fields is probably due to Doppler effect.

The light from the neon tube was focussed by a lens at the slit of the collimator of an echelon spectroscope with horizontal slit. After passing through the echelon grating of 35 plates, each of which was 0·935 cm. thick, the light was brought to a focus by an achromatic lens and the echelon spectrum was projected by a Rudolph's microplanar of 7·5 cm. focal length, on a collimator of a prism spectroscope, and then photographed with an objective of 10 cm. aperture and 75 cm. focal length. The plate was sensitised with dicyanin, which shortened the exposure more than any other sensitiser. At first the spectroscope was provided with a grating, but on account of the feebleness of light it was replaced by a 60 deg. prism of flint glass having faces of 5 cm. square. The spectrum generally extended from 6717 to 5852, and several exposures for the same field were taken. The magnetising current was kept constant by a regulating rheostat, and the pole pieces cooled by a steady stream of water circulating in the axis of the cones, which was perforated with the exception of the portion near the poles. Some of the spectrograms are shown in Plate I.

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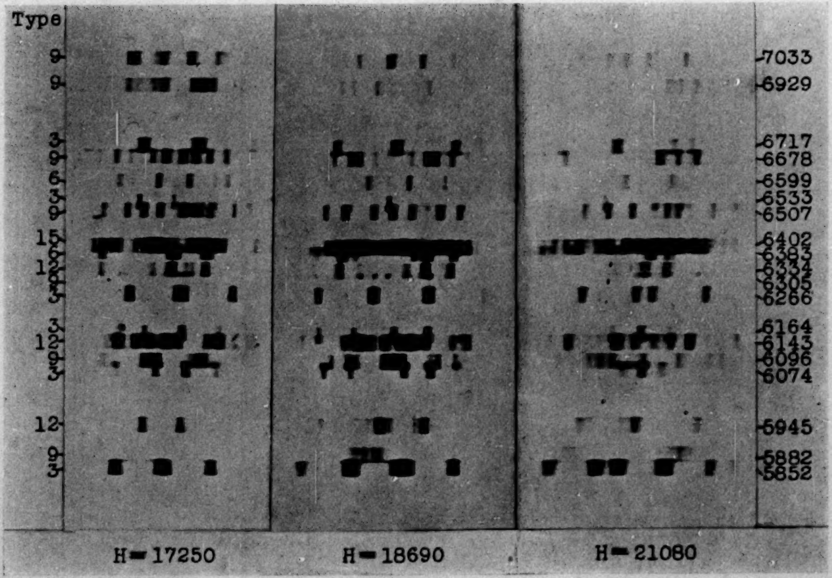
* "Ann. d. Phys.," XXXIX., p. 897 (1912).

The spectrograms were measured by means of a micrometer screw specially polished and accurate to about 1μ, and provided with a microscope of low magnifying power. As the trace of the separated lines sometimes measured several tenths of a millimeter, it was unnecessary to push the measurements to the order of a micron. Referring the separation to that of 6266, the reduction can be easily effected and the strength of the field calculated from the known value of a for 6266.

The following table contains the value of $\frac{\delta\lambda}{\lambda^2 H}\cdot 10^{13}$, the types of separation by the author and by Takaminé and Yamada, the calculated value of $10^5 \times a$, and the value of $\frac{e}{m}10^{-7}$ calculated from a. The bracketed values of px and of $\frac{e}{m}10^{-7}$ refer to results obtained from the normal components only.

The comparison of the types of separation will show the indefiniteness of Runge's rule for complex separation. The deviations from the rule are usually large for parallel components, for which the displacement is tolerably small. They may be due either to the smallness of the displacements or to the peculiarity of the parallel components. The normal components are more regular, and most of the deviations may be ascribed to errors of measurements. There is some doubt, if it is not proper in some cases to use the different values of q in the calculation of $10^5 \times a$ for the normal and the parallel components.

In the table, the values of px are more approximate to the observed values for the normal than for the parallel components. The values found from the normal components only for complex separation are given in brackets for the sake of comparison, as likewise the values of $\frac{e}{m}$ derived from the same numbers, while those obtained from Takaminé


Type
9
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12
9
3
12
9
3
7033
6929
6717
6678
6599
6533
6507
6402
6383
6334
6305
6266
6164
6143
6096
6074
5945
5882
5852
H = 17250
H = 18690
H = 21080

PLATE I.

[*To face* p. 86.

λ in Å.U.		$\frac{\delta\lambda}{\lambda^2 H}10^{13}$	Type (T.Y.)	Multiples of x.	$e/m.10^{-7}$	(T.Y.)	(L.)
6717......	p	0	0	......	1·821	(1·816)	(1·772)
	n	4·83	a				
6678......	p	0	0	......	1·774	(1·857)	(1·756)
	p	1·23	$\frac{3}{12}a\left[\frac{1}{4}a\right]$	$3x=1·172$			
	n	5·09	$\frac{13}{12}a\left[\frac{4}{4}a\right]$	$13x=$ 5·079 (5·079)	(1·768)		
	n	6·27	$\frac{16}{12}a\left[\frac{5}{4}a\right]$	$16x=$ 6·251 (6·251)			
	n	7·40	$\frac{19}{12}a\left[\frac{6}{4}a\right]$	$19x=$ 7·423 (7·423)			
6599......	p	1·47	$\frac{4}{13}a$	$4x=$ 1·432	1·760	(1·755)	(1·785)
	n	5·02	$\frac{14}{13}a$	$14x=$ 5·012 (5·005)	(1·754)		
	n	6·43	$\frac{18}{13}a$	$18x=$ 6·444 (6·435)			
6533......	p	0	0	......	1·831	(1·802)	(1·769)
	n	3·24	$\frac{2}{3}a$				
6507......	p	0	0	......	1·798	(1·769)	(1·769)
	p	1·57	$\frac{5}{15}a$	$5x=$ 1·590			
	n	3·84	$\frac{12}{15}a$	$12x=$ 3·816 (3·816)	(1·800)		
	n	5·42	$\frac{17}{15}a$	$17x=$ 5·406 (5·406)			
	n	6·98	$\frac{22}{15}a$	$22x=$ 6·996 (6·996)			
6402......	p	0	0		1·766	(1·774)	(1·727)
	p	0·80	$\frac{1}{6}a$	$x=$ 0·780			
	p	1·59	$\frac{2}{6}a$	$2x=$ 1·560			
	n	4·69	$\frac{6}{6}a$	$6x=$ 4·680 (4·680)	(1·763)		
	n	5·45	$\frac{7}{6}a$	$7x=$ 5·460 (5·460)			
	n	6·25	$\frac{8}{6}a$	$8x=$ 6·240 (6·240)			
	n	7·02	$\frac{9}{6}a$	$9x=$ 7·020 (7·020)			
	n	7·78	$\frac{10}{6}a$	$10x=$ 7·800 (7·800)			
6383......	p	3·82	$\frac{9}{11}a\left[\frac{10}{13}a\right]$	$9x=$ 3·840	1·765	(1·782)	(1·724)
	n	2·94	$\frac{7}{11}a\left[\frac{9}{13}a\right]$	$7x=$ 2·986 (2·990)	(1·767)		
	n	6·86	$\frac{16}{11}a\left[\frac{19}{13}a\right]$	$10x=$ 6·826 (6·834)			

λ in Å.U.		$\frac{\delta\lambda}{\lambda^2 H}10^{13}$	Type (T.Y.)	Multiples of x.	$e/m.10^{-7}$	(T.Y.)	(L.)
6334……	p	1·76	$\frac{3}{8}a\left[\frac{4}{11}a\right]$	$3x=$ 1·777	1·785	(1·814)	(1·752)
	p	3·50	$\frac{6}{8}a\left[\frac{8}{11}a\right]$	$6x=$ 3·554			
	n	3·50	$\frac{6}{8}a\left[\frac{8}{11}a\right]$	= 3·554 (3·560)	(1·789)		
	n	5·41	$\frac{9}{8}a\left[\frac{12}{11}a\right]$	$9x=$ 5·331 (5·340)			
	n	7·15	$\frac{12}{8}a\left[\frac{16}{11}a\right]$	$12x=$ 7·108 (7·120)			
	n	8·86	$\frac{15}{8}a\left[\frac{20}{11}a\right]$	$15x=$ 8·885 (8·900)			
6305……	p	0	0		1·794	(1·742)	(1·637)
	p	1·08	$\frac{2}{9}a\left[\frac{1}{4}a\right]$	$2x=$ 1·056			
	n	4·75	$\frac{9}{9}a\left[\frac{4}{4}a\right]$	$9x=$ 4·752 (4·752)	(1·792)		
	n	5·81	$\frac{11}{9}a\left[\frac{5}{4}a\right]$	$11x=$ 5·808 (5·808)			
	n	6·87	$\frac{13}{9}a\left[\frac{6}{4}a\right]$	$13x=$ 6·864 (6·864)			
6266……	p n	0 4·720	0 a		1·779	(1·779)	(1·657)
6217……	p	0	0	……	1·772	(1·780)	(1·767)
	p	3·86	$\frac{9}{11}a\left[\frac{5}{6}a\right]$	$9x=$ 3·840			
	n	……	… $\left[\frac{4}{6}a\right]$	……			
	n	7·22	$\frac{17}{11}a\left[\frac{9}{6}a\right]$	$17x=$ 7·253 (7·253)	(1·769)		
	n	11·12	$\frac{26}{11}a\left[\frac{14}{6}a\right]$	$26x=$ 11·093 (11·093)			
6164……	p	0	0	……	1·796	(1·823)	(1·714)
	n	6·35	$\frac{4}{3}a$				
6143……	p	1·32	$\frac{3}{11}a\left[\frac{3}{12}a\right]$	$3x=$ 1·253	1.738	(1·849)	(1·835)
	p	2·57	$\frac{6}{11}a\left[\frac{6}{12}a\right]$	$6x=$ 2·506			
	n	4·60	$\frac{11}{11}a\left[\frac{11}{12}a\right]$	$11x=$ 4·595 (4·591)	(1·731)		
	n	5·82	$\frac{14}{11}a\left[\frac{14}{12}a\right]$	$14x=$ 5·848 (5·843)			
	n	7·10	$\frac{17}{11}a\left[\frac{17}{12}a\right]$	$17x=$ 7·101 (7·095)			
	n	8·34	$\frac{20}{11}a\left[\frac{20}{12}a\right]$	$20x=$ 8·357 (8·347)			

λ in Å.U.		$\frac{\delta\lambda}{\lambda^2 H}10^{13}$.	Type (T.Y.)	Multiples of x.	$e/m.10^{-7}$	(T.Y.)	(L.)
6096......	p	0	0		1·755	(1·771)	(1·769)
	p	0·84	$\frac{3}{16}a\left[\frac{1}{7}a\right]$	$3x=$ 0·875			
	n	5·28	$\frac{18}{16}a\left[\frac{8}{7}a\right]$	$18x=$ 5·250 (5·250)	(1·759)		
	n	6·13	$\frac{21}{16}a\left[\frac{9}{7}a\right]$	$21x=$ 6·125 (6·125)			
	n	6·95	$\frac{24}{16}a\left[\frac{10}{7}a\right]$	$24x=$ 7·000 (7·000)			
6074......	p	0	0		1·742	(1·734)	(1·709)
	n	6·93	$\frac{3}{2}a$				
5976......	p	0	0		1·794	(1·771)	(1·759)
	p	2·47	$\frac{1}{2}a$	$x=$ 2·370			
	n	—	— $[\,a\,]$	—			
	n	7·07	$\frac{3}{2}a$	$3x=$ 7·110 (7·095)	(1·785)		
	n	9·49	$\frac{4}{2}a$	$4x=$ 9·480 (9·460)			
5945......	p	1·08	$\frac{3}{15}a\left[\frac{2}{10}a\right]$	$3x=$ 0·953	1·804	(1·782)	(1·763)
	p	1·92	$\frac{6}{15}a\left[\frac{4}{10}a\right]$	$6x=$ 1·906			
	n	5·12	$\frac{16}{15}a\left[\frac{11}{10}a\right]$	$16x=$ 5·082 (5·077)	(1·796)		
	n	6·03	$\frac{19}{15}a\left[\frac{13}{10}a\right]$	$19x=$ 6·035 (6·029)			
	n	7·00	$\frac{22}{15}a\left[\frac{15}{10}a\right]$	$22\,x=$ 6·988 (6·981)			
	n	7·89	$\frac{25}{15}a\left[\frac{18}{10}a\right]$	$25x=$ 7·941 (7·933)			
5882......	p	0	0		1·781	(1·737)	(1·700)
	p	0·77	$\frac{1}{6}a\left[\frac{2}{12}a\right]$	$x=$ 0·788			
	n	6·32	$\frac{8}{6}a\left[\frac{17}{12}a\right]$	$8x=$ 6·304 (6·304)	(1·782)		
	n	7·08	$\frac{9}{6}a\left[\frac{19}{12}a\right]$	$9x=$ 7·092 (7·092)			
	n	7·88	$\frac{10}{6}a\left[\frac{21}{12}a\right]$	$10x=$ 7·880 (7·880)			
5852......	p	0	0		1·843	(1·824)	(1·786)
	n	4·89					

and Yamada's and Lohmann's measurements are also given side by side. From these results, we find that $\frac{e}{m}$ obtained from n-components are somewhat nearer to the normal value. Some of the values of p and q are artificial, especially when they are large. Thus different values of p and q were given by the experimenters. In addition to this, we sometimes meet with doubtful cases, such, for example, as the separation of 5852; it is here simply taken equal to a, but if large values of p and q are admissible, they may be 23 and 22 respectively, which exactly corresponds to the normal value. This is probably inadmissible, and many cases of complex separation seem likely to have been made to conform to Runge's rule, by assuming large values of p and q. The physical reasons for taking these values are not yet clear, and must be considered only as empirical, so that much weight cannot be given to these numbers. The good coincidence of some of the components is indeed wonderful, and the theory of the Zeeman effect will in the future be able to elucidate the present problem, which must have an intimate connection with the configuration of electrons in the atomic structure.

The question, whether the separation is proportional to the field or not, was examined on about 70 photograms, with the result that between the fields of 8500 to 21500 gauss the linear relation holds true.

The mean value of $10^{-7}\,\frac{e}{m}$ found for the same range of fields from p and n components of 19 different lines is 1·784, and from n-components only 1·782. As the value of the standard line is still subject to some experimental error, I cannot place much weight on the absolute value, but it is quite clear that the mean value nearly corresponds to that of the normal triplet.

The inspection of the table will at once show that the value of $\frac{e}{m}$ is not a constant, which at first sight seems a little paradoxical. From the point of view of the electron theory, the discrepancies in the value of $\frac{e}{m}$ may arise from various causes, which affect the electron configuration in the atom. The value of $\frac{e}{m}$ usually given in the electron theory is deduced on the assumption that an electron is at rest or moves slowly in an infinite space without any external electromagnetic disturbance. This condition is never satisfied in the atomic configuration. If the speed of the radiating electrons is comparable with that of light, the correction due to relativity will evidently enter into the result, even if the electron be moving singly with uniform motion. In the case of neon, it may be argued that the radiating electron cannot satisfy the said condition, so that the mass of the electron would be different from that of the electromagnetic mass usually treated in the electron theory. The case of electrons moving at a fixed distance from each other was treated by Nicholson,* from which it is evident that the mass of two or more electrons moving together is greater than that of electrons taken separately; the consequence is that the value of $\frac{e}{m}$ should be generally smaller than the normal value, which is approximately $1{\cdot}77 \times 10^7$ E.M.U. This is fulfilled in many of the lines above cited, and may either be ascribed to the relativity correction or to the motion of the electron groups. Perhaps the latter explanation is the more probable. In some cases the value of $\frac{e}{m}$ is greater than the normal value, and some explanation must be given.

The effective magnetic field H is assumed to be exactly the same as that given by the electromagnet. If we admit the existence of rings of electrons rotating about the nucleus, the above assumption cannot be justified. The action of the external magnetic field will be superposed on the atomic field, and what is felt in the Zeeman effect will be the resultant of these fields. That the value of $\frac{e}{m}$ is greater or less than
the discrepancy in $\frac{e}{m}$ must be traced to other causes.
the normal value may imply the fact that the effective field is smaller or greater than the external. It is difficult to estimate the magnitude of the atomic field, so that the value of H in deducing Runge's formula is different from that of the electromagnet. If this is really the case, the linear relation of the amount of separation with the strength of the field will be but approximately fulfilled, so that the curve representing the above is no longer straight. As above stated, the

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* Proc. Phys. Soc., Vol. XXVII., p. 217 (1915).

deviation from linearity was within experimental error. Whether the field strength is not great enough to bring out the effect, or the atomic field is negligibly small compared with the external field, is rather difficult to answer. In the latter case,

If we consider the effect of the magnetic field on the radiation as due to the minute change of configuration in the electronic configuration within the atom, the quasi-elastic force acting on the radiating electron must be subject to a change, which may not be too small to detect by means of the delicate spectral test. The effect was examined for triplets and nonets, but with no positive result. This important problem remains still unsolved for the most of lines, but if the change is appreciable, it will lead to important consequences as regards electronic configuration within the atom. The method similar to the examination of the so-called pole effect will lead to the final solution of the problem.

The electronic configuration will be slightly altered by the external magnetic force, and the action of the electric field on the electrons will be correspondingly affected. This will account for the alteration of quasi-elastic force acting on the vibrating electrons, but the atomic electric field may act somewhat like the Stark effect, and cause the displacement of the lines. Probably the change in the configuration will not cause disturbances in the electric field as to produce appreciable displacement, except for anomalous magnetic separations as observed in the mercury line 5790, the satellites of the helium lines 5876 and 4713, and the oxygen triplets in the ultra-violet.

As to the complex separations of the neon lines, the theory of coupling by Voigt can explain nearly all the types here given. The sextets can be ascribed to two coupled electrons, the nonets to three, and so on; and if we consider the radiating electrons as moving together, the value of $\frac{e}{m}$ will evidently be open to question, as in the problem proposed by Nicholson.

The formula for the mass of coupled electrons shows an increase, so that $\frac{e}{m}$ must decrease. For the line 6402, which is separated into 15 components, $\frac{e}{m}$ is very near to the normal value, and shows no decrease in spite of the number of coupled electrons. This seems at first sight contradictory, but various causes may be acting simultaneously, so that in the above example m may increase, but the effective field may decrease, so that different effects compensate each other and give the normal value to $\frac{e}{m}$.

From the mechanical analogy of the Zeeman effect, it seems quite likely thatt he sextet corresponds to the case of two equal roots of the equation giving the natural frequency of the vibrating electron, and nonets to three equal roots, and similarly for other complex separations. Tentative experiments were tried to see if the lines giving rise to sextets, nonets, &c., are not multiple lines having minute differences among themselves. This was tested by bringing the high resolving power of the Fabry-Perot interferometer having thick air plate to bear on the neon lines, but all the lines were apparently simple and showed no signs of multiplicity.

According to Aston,* neon consists mostly of isotopes of atomic weight 20 and 22, while an isotope intermediate between the two may exist in small quantity. Each of the isotopes will give rise to spectrum lines which are nearly coincident. The constant difference in the frequency of lines, of which there are two very definite values, according to Burns, Meggers and Merrill,† seems too large to account for the isotopes, but the complexity of the Zeeman effect in neon may have some connection with the existence of isotopes, and especially the observed fact that the types of separation are all multiples of three.

The various causes above cited will, to some extent, affect the displacement of the lines due to magnetic separation, but we have at present no means of discriminating and assigning the discrepancies to these causes.

A more thorough study of the atomic structure will be necessary to give weight to these different causes, but there may be other causes not yet mentioned, in which the shape of electrons forms a principal feature. The supporters of ring-shaped electrons may find some clue to the elucidation of Runge's rule, and the discrepancies to which it is subject. One of the departures in this direction is the indeterminateness of the effective magnetic field, for the ring electron has its own field, and by making gyrostatic motion about its own axis, the field may be increased or decreased.

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* "Phil. Mag.," Vol. XXXIX., p. 449, 611 (1920).

† "Bull." Bureau Stand., Vol. XIV., p. 765 (1919).

Another suggestion is that, if the ring be contractile, the electro-magnetic mass may change, consequently different values of $\frac{e}{m}$ are to be expected, as already indicated by the experiment. Notwithstanding the various aspects under which the assumption of a ring electron is capable of solving the present problem, we have as yet no definite idea to account for the various types of separation, which is so characteristic of the neon spectrum.

Borrowing a mechanical analogy, we can consider a ring electron as a vortex ring in an incompressible liquid. The circulation about the ring corresponds to magnetic flux, and the vortex to the rapidly moving charge. In this aspect the ring electron is no more than the vortex atom electronised. Such vortex rings are susceptible of radial vibrations and subject to flutings. The Zeeman effect is evidently analogous to flutings caused by the change of circulation about the ring, and is due to small changes of frequency by the magnetic flux in the circuit of the ring electron. We must, however, suppose that such flutings are controlled by the central nucleus, and the types of magnetic separation will be determined by the atomic configuration of the electrons.

Other singular properties of neon may have some connection with the present investigation, but as no thorough study of the separation of lines in the more refrangible part of the spectrum has as yet been made, any hasty conclusion from the investigation of the red and yellow lines may lead to erroneous conclusions.

It will not be out of place to give a mechanical analogy of small oscillations for illustrating the Zeeman effect and Runge's rule. Following Lorentz,* suppose the displacements of the electrons in a radiating atom to be determined by the normal co-ordinates, p_h, so that the kinetic energy, T, and the potential energy, V, are given by

$$T=\tfrac{1}{2}\sum_{1}^{n}A_h\dot{p}_h^2$$

$$V=\tfrac{1}{2}\sum_{1}^{n}B_h p_h^2.$$

By the action of the magnetic field, the system will be slightly disturbed, and the normal co-ordinates cease to be

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[*] "Ann. d. Phys. u. Chem.," LXIII., p. 278 (1897); Theory of Electrons (1909).

exactly normal; consequently the energies in the disturbed system will be given by

$$T=\tfrac{1}{2}\,\Sigma(A_h+a_h)\dot{p}_h^2+\Sigma a_{hk}\dot{p}_h\dot{p}_k,$$
$$V=\tfrac{1}{2}\,\Sigma(B_h+b_h)p_h^2+\Sigma b_{hk}p_hp_k,$$

where h is different from k, a's and b's are small, compared with A's and B's. Further we suppose that a's and b's are proportional to the magnetic field. The new force brought into play by the action of the magnetic field is given by the vector product of the velocity of the electron into the magnetic force. The force acting on the electrons by applying the magnetic field is given by

$$P_h=\Sigma c_{hk}\dot{p}_k.$$

By the condition that P_h is perpendicular to $\dot{p}_h$,

$$c_{hk}=-c_{kh}.$$

Thus the equations of motion, of which there are n, are of the general form

$$(A_h+a_h)\ddot{p}_h+\Sigma a_{hk}\ddot{p}_k+(B_h+b_h)p_h+\Sigma b_{hk}p_k-\Sigma c_{hk}\dot{p}_k=0.$$

Put $p_h=u_he^{i\nu t}$, and find the equation for determining the frequency ν. By substituting in the equation of motion, we get n equations of the form

$$\{(B_h+b_h)-\nu^2(A_h+a_h)\}u_h-i\nu\,\Sigma c_{hk}u_k+\Sigma(b_{hk}-\nu^2a_{hk})u_k=0.$$

Eliminating $u_1, u_2, \ldots, u_n$, we obtain the determinantal equation for finding the frequencies

$$\begin{vmatrix} (B_1-\nu^2A_1)+(b_1-\nu^2a_1), & -i\nu c_{12}+b_{12}-\nu^2a_{hk}, & \cdots \\ -i\nu c_{21}+b_{21}-\nu^2a_{21}, & (B_2-\nu^2A_2)+(b_2-\nu^2a_2), & \cdots \\ \cdots\cdots, & \cdots\cdots & \end{vmatrix}=0$$

It is to be remarked that terms in small letters are very small compared with those given by the capital letters.

In the undisturbed system, let us suppose that a certain number, say the first k natural frequencies $\sqrt{\dfrac{B_h}{A_h}}$ $(h=1, 2, 3 \ldots k)$ have a common value N, and let us seek values of ν satisfying the above condition and nearly equal to N. In that case, we can suppose $N^2-\nu^2$ to be a small quantity nearly equal to $2N\delta N$, where δN is the difference of N from ν, so that the additional small quantities in the determinant cannot be generally neglected, but owing to the presence of the factor, N, $2N\delta N$ is large compared with δN, which is very small.

Dividing the determinant by $A_1, A_2, \ldots$, and putting $\frac{a}{A}=\alpha$, $\frac{b}{A}=\beta$, we find that the result is approximately given by

$$(N^2_{k+1}-\nu^2)(N_{k+2}-\nu^2)\ \ldots\ (N^2_n-\nu^2)\,.\,\Delta=0,$$

where

$$\Delta=\left|\begin{matrix} N^2_1-\nu^2+\beta_1-\nu^2\alpha_1, & -i\nu\gamma_{12}+\beta_{12}-\nu^2\alpha_{12}, & \ldots \\ -i\nu\gamma_{21}+\beta_{21}-\nu^2\alpha_{21}, & N^2_2-\nu^2+\beta_2-\nu^2\alpha_2, & \ldots \\ \ldots & \ldots & \ldots \end{matrix}\right|$$

For finding the frequencies little different from the common value

$$N_1=N_2=\ldots\ldots=N_k=N\ ;$$

the determinantal equation changes to

$$\Delta=\left|\begin{matrix} N^2-\nu^2+\lambda_1, \mu_{12}, \mu_{13}, & \ldots \\ \mu_{21}, N^2-\nu^2+\lambda_2, \mu_{23}, & \ldots \\ \mu_{31}, \mu_{32}, N^2-\nu^2+\lambda_3, & \ldots \\ \ldots\quad \ldots\quad \ldots & \end{matrix}\right|=0,$$

where for simplicity

$$\lambda_1=\beta_1-\nu^2\alpha_1,\ \ldots$$
$$\mu_{12}=-i\nu\gamma_{12}+\beta_{12}-\nu^2\alpha_{12},\ \ldots$$

Evidently α_{hk} and β_{hk} give rise to the disturbances in the normal co-ordinates of the initial system. We suppose the change due to the magnetic field to be very small; then α_{hk} and β_{hk} will be generally small compared with $N^2-\nu^2$, so that $\mu_{hk}=-i\nu\gamma_{hk}$ nearly, and is proportional to the magnetic field.

In $\lambda_h=\beta_h-\nu^2\alpha_h$, we remark that β_h is of the same order of small quantities as δN, and most probably $\nu^2\alpha_h$ would be of similar order. This point is still questionable, as ν^2 is a large quantity. Under the above supposition the equation turns out to be

$$\left|\begin{matrix} N^2-\nu^2 & \mu_{12} & \mu_{13} & \ldots \\ \mu_{21} & N^2-\nu^2 & \mu_{23} & \ldots \\ \mu_{31} & \mu_{32} & N^2-\nu^2 & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{matrix}\right|=0,$$

which is a skew determinant, since

$$\mu_{hk}=-\mu_{kh}.$$

The determinant gives rise to an equation of the form

$$(N^2-\nu^2)^k+C_2(N^2-\nu^2)^{k-1}+C_4(N^2-\nu^2)^{k-2}+\ . \ . \ .=0$$

If $\nu^2 a_h$ is not negligibly small, the right-hand side is equal to a small quantity, which is nearly constant. When the position of the lines remains unaltered, $\nu^2 a_h$ must be very small, and the change in kinetic energy due to magnetic force is negligible. This condition is nearly satisfied in the neon spectrum. The above consideration therefore shows that the change in kinetic and potential energies by the action of the external magnetic force of the electromagnet cannot be detected by the experiments.

When k is odd, the last term contains $N^2-\nu^2$, giving the root $N=\nu$, which shows that the initial position of the line remains unaltered. By cancelling the common factor $N^2-\nu^2$, we obtain an equation which holds for the case of an even number of equal roots. If the roots for natural frequencies in the disturbed system are of the form

$$N^2-\nu^2=\pm R_1,\ \pm R_2,\ \pm R_3,\ .\ .\ .\ ,$$

we get by expansion

$$\nu_h=N\pm\frac{R_h}{2N},$$

where R_h is directly proportional to the magnetic field to a first approximation. Thus the mechanical analogy of the magnetic separation of the neon lines is demonstrated. I remark that the separation is symmetrical about the initial position, as has already been observed, indicating the evanescence of the influence of the magnetic field, since the effect of a_h and b_h affected with H is immeasurably small.

Whether the external magnetic force is not strong enough to bring out the expected result, or the resolving power of the interferometers is not high enough to separate the closely packed lines into the components, awaits further experiments.

From these considerations, we find that the change in frequency depends principally on c_{hk} and hence on γ_{hk}. Since this quantity arises from the force $\frac{e}{c}$[velocity $\times H$], the bracket signifying that it is a vector product, it is evident that γ_{hk} is proportional to $\frac{eH}{4\pi mc}$, which may be written aH, following the notation at the outset. Experiment further shows that the roots R_1, R_2, . . . are proportional to the field. In the case of complex separation with the electric force either parallel or normal to the magnetic field, we find the relation that the differences between the roots of the determinantal equation are equal, or $R_2-R_1=R_3-R_2= \dots$, which is again an aliquot part of the separation of a normal triplet. It can only be conjectured that the rule is a consequence of some quantum relation.

How far Runge's rule holds true is still a problem, but the present experiments and mechanical analogies show that small terms of correction are generally negligible except γ_{kh}, depending on the field and the ratio $\frac{e}{m}$. Thus the result is but approximate, and probably no exact coincidence in the values of $\frac{e}{m}$ can be expected, though further accuracy in the measurements can be attained. From the preceding discussion, it is also evident that the change in atomic configuration by an external magnetic force of 20 kilogauss is not sufficient to affect appreciably the kinetic and potential energies of the system.

The present investigation was made at the Institute for Physical and Chemical Research with the view of utilising the magnetic separation of neon lines for surveying heterogeneous magnetic fields. The photograms were mostly taken by Mr. T. Mishima and Mr. T. Okazawa, and the micrometric measurements and reductions were made by Mr. S. Sakurai, all assistants in the Institute, to whom my best thanks are due.

DISCUSSION.

Prof. J. W. **Nicholson** said the Paper was of the very greatest value. The experimental results were of high importance. He did not, however, think that the explanation of the discrepancies advanced by the author was adequate. Personally, he did not believe in the so-called series electron, *i.e.*, an electron very much more remote from the nucleus than any others in the atom. The modern conception of the normal helium atom assumed that one electron pursued a nearly circular elliptic orbit at one focus of which the nucleus $2e$ was situated. The orbit of the other electron was a smaller and more elongated ellipse intersecting the first The arrangement was such that although the second electron would sometimes be quite near the nucleus, at other times it could be as far away as the first, and the simple theory of the series electron could not be expected to hold. The effect of a magnetic field was to change the character of the orbits. There were various possibilities. He had worked out a possible case for helium some years ago. It consisted of two concentric circular orbits, the electrons moving in such a way as to keep as far apart as possible. The ratio of the radii of these orbits was about 9·8 : 1. He felt sure that the magnetic separation did not depend only on e/m, but was also a function of this ratio of the orbits, and that values such as he had indicated would probably put right the discrepancies referred to in the Paper.

Mr. M. N. Saha (of Calcutta University) said, in regard to Prof. Nicholson's remarks, that the difficulty in extending Bohr's theory to the problem of helium radiation was that a complete solution had not yet been obtained of the problem of three bodies. Poincaré had succeeded in giving a solution assuming the masses of two of them to be infinitesimally small compared with that of the central body. He had tried to apply Poincaré's method to the helium atom, but so far without success. With regard to the size of the orbits, he referred to a Paper by Mr. S. N. Basu in the "Phil. Mag.," November 1920. He deduced Rydberg's Law from Bohr's theory by assuming the central attracting system to be composed of a net unit positive charge plus an electric doublet of definite moment. The fact that the ordinary lines of an element could be grouped into series with the Rydberg constant N showed clearly that the lines are produced by the quantum changes of orbit of the series electron, the nuclear charge and the remaining electrons merely determining the central attracting system. The size of the natural orbits of the electrons could be deduced from a corollary of Basu's theory, viz., that the radius is inversely proportional to the ionisation potential. For helium this is 25·4 volts, and, taking the natural radius of the electron in the H-atom as $0{\cdot}532 \times 10^{-8}$ cm., the radius of the outer electron of the He-atom is $0{\cdot}28 \times 10^{-8}$ cm. When this electron is lost the next one describes a circular orbit of $0{\cdot}134 \times 10^{-8}$ cm. (taking the second step ionisation potential as 54·6 volts), the ratio being about 2 : 1. In the neutral atom the orbit of the inner electron will be pushed inwards, but he could not see how the ratio could ever be so high as 9 to 1, as suggested by Prof. Nicholson.

Prof. Rankine asked Prof. Nicholson how much the ratio of the orbits differed before and after radiation took place.

Prof. Nicholson replied at length to the various questions raised. In the type of orbit he had mentioned the ratio was never less than about 7. As regards the Rydberg constant necessitating the series electron being far away he pointed out that when this was so they obtained the Balmer formula involving N/m^2. If it was less far away this was replaced by $N/(m+\mu)^2$, and if still less by $N/(m+\mu+\frac{\alpha}{m^2})^2$. All these corrections to the denominator were simply in order to preserve the constant N in the numerator. They could get equally satisfactory series formulæ by introducing the corrections in the numerator instead.

X. *A Method of Demonstrating the Retroactive Property of a Triode Oscillator.* By E. V. Appleton, *M.A., M.Sc.*

Received September 24, 1920.

ABSTRACT.

The author, following Vallauri, gives an approximate treatment of the conditions which gives rise to retroaction between the grid and anode circuits of a triode valve, and describes an arrangement of circuits whereby the property can easily be demonstrated to a large audience.

The method of using triode retroaction for the amplification of wireless signals is now well known. Of the various examples of the use of this method the arrangement described by Armstrong * may be regarded as typical. Here the usual oscillatory circuit L_1C (*see* Fig. 1) is connected to the grid and filament of a triode, retroaction being obtained by means of the coil L_2 in the anode circuit.

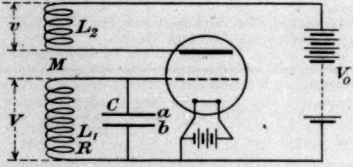

Fig. 1.

If the mutual inductance between L_1 and L_2 is of the correct sense and magnitude it is possible to cause the variations of the anode current, produced by a free or forced oscillation in L_1C, to induce favourably timed electromotive forces in the oscillatory circuit and so partly or wholly compensate for Joulean and other losses. In the case of total compensation, of course, the oscillation is undamped.

The following approximate quantitative treatment, suggested by the work of Vallauri,† seems to make the matter clearer. Consider a free oscillation in the circuit L_1C. Ignoring for the moment the effect of L_2 we have for the ordinary expression of the electromotive forces in the circuit

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* E. H. Armstrong, "Proc." Inst. Rad. Eng., 3, p. 215, 1915.

† G. Vallauri, "L'Elettrotecnica," 3, p. 43, 1917.

$$L_1\ddot{Q}+R\dot{Q}+\frac{Q}{C}=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where Q is the charge on the plate (a) of the condenser at any instant.

If we now take into account the mutual inductance M between L_1 and L_2, and also assume that the resistances of both coils are small compared with their reactances for the particular frequency considered, we have approximately

$$\frac{V}{v}=\frac{M}{L_1},$$

where V and v are the induced anode and grid voltages respectively. Also the value of v at any instant is $\frac{Q}{C}$. If we now assume that the anode current changes in the tube are proportional to the anode and grid voltage changes involved, we have, if I is the value of the anode current,

$$I=av+bV+c,$$

where a, b and c are constants.

On substituting for V and v this becomes

$$I=a\frac{Q}{C}+\frac{bMQ}{L_1C}+c,$$

that is

$$M\dot{I}=\frac{aM\dot{Q}}{C}+\frac{bM^2\dot{Q}}{L_1C}.$$

Now $-M\dot{I}$ is the value of the E.M.F. induced in the oscillation circuit by the anode current variations. In the retroactive case we must, therefore, rewrite (1) as follows:

$$L_1\ddot{Q}+\left(R+\frac{Ma}{C}+\frac{bM^2}{L_1C}\right)\dot{Q}+\frac{Q}{C}=0 \quad . \quad . \quad . \quad . \quad (2)$$

For retroaction in the circuit we are considering it is well known that M must be negative and thus the effective resistance of the circuit may be written as

$$R-\frac{Ma}{C}+\frac{M^2b}{L_1C},$$

where M represents the numerical value of the mutual inductance. We may make this effective resistance as small as we please by increasing the numerical value of M, as in a practical case $\frac{bM}{L_1}$ is small compared with a. This reduction in the effective resistance of the oscillatory circuit makes, of course, a corresponding reduction in the damping. Now the time required for a free oscillation to decay to one-hundredth of its value in a non-retroactive circuit is

$$\frac{9{\cdot}212 L_1}{R} \text{ seconds}$$

(L and R being in a consistent set of units). In a retroactive circuit we must write this

$$\frac{9{\cdot}212 L_1}{R - \frac{Ma}{C} + \frac{bM^2}{L_1 C}} \text{ seconds.}$$

It is possible to demonstrate the difference between these two times in circuits where L_1 is very large and the signals are of acoustic frequency. A simple circuit for this is shown in Fig. 2. The condenser C is charged by putting the change-over switch in the (A) position and then discharged through the inductance L_1 by changing to the (B) position. The oscillation in the L_1C circuit is heard in the telephone T. By performing this experiment first with the coils L_1 and L_2 very loosely coupled, and afterwards with the value of M adjusted so that the effective resistance is appreciably diminished, it is possible to show to a large audience that a free oscillation, which, without retroaction, sinks to inaudibility in a fraction of a second, can, by means of retroaction, be prolonged for 4 or 5 seconds.

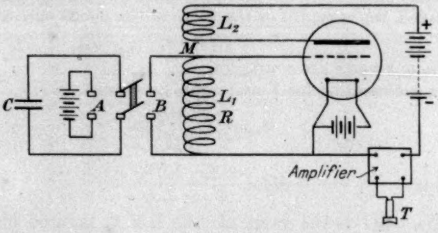

FIG. 2.

If a vibration galvanometer is used instead of a telephone a quantitative determination of the effective resistance in any particular case is possible.

[*Note added Jan. 26th.*—A slightly more satisfactory arrangement than that shown in Fig. 2 is obtained when the oscillatory circuit is impulsed electrically by means of a neighbouring aperiodic circuit.]

DISCUSSION.

Prof. W. **Eccles** said the method would be very useful as a lecture experiment. Those who were accustomed to high-frequency work were familiar with these phenomena, but students were often mystified by the perfect silence of high-frequency circuits. The author's arrangement for demonstrating the effect at audio frequencies was therefore valuable. It reminded him of one apparatus, which Mr. Jordan and himself had shown, in which a tuning fork took the place of a condenser and inductance, the mass standing for the inductance and the elasticity for the capacity. The arrangement was very portable. The author's treatment of the conditions of the experiment was a very useful amplification of Vallauri's analysis.

Prof. **Howe** said he presumed the reason why the time during which the oscillations remained audible could not be extended beyond a few seconds was that harmonics had to be taken into account, whereas the analysis assumed a single sine wave.

Dr. D. **Owen** said that it was quite possible to hear the note produced in a shunted telephone included in an oscillatory circuit of audible natural frequency, on charging or discharging the condenser. By use of tuning fork and monochord the frequency could be measured, and thus the oscillatory nature of the discharge, and the laws connecting the frequency with inductance and capacity could be verified by the individual student. He had tried to multiply the effect by the aid of a tuning-fork interruptor, but not with much success. The author's method made the demonstration possible to an audience, though at the sacrifice of simplicity. It was to be noted also that the reaction of the mutual inductance would have the effect of slightly raising the frequency of the original circuit. This rise of frequency appeared to be a general consequence of supplying energy for the production of maintained vibrations in an oscillatory system.

XI. *The Quickness of Response of Current to Voltage in a Thermionic Tube.*

By D. OWEN, *B.A., D.Sc., and* R. M. ARCHER, *B.Sc., M.I.E.E., Birkbeck College, London.*

RECEIVED NOVEMBER 24TH, 1920.

ABSTRACT.

STEADY voltages were applied between the hot and cold electrodes of a thermionic tube, for intervals of time which could be varied from 0·00001 second to a minute or longer. The mean current during the interval was measured by the Wheatstone bridge, using a null ballistic method.

Two types of thermionic tube were employed, one at a comparatively high gas pressure, the other at a far higher degree of exhaustion. It is found that on applying the voltage the initial rise of current to its maximum is followed by a fall, the rate of which diminishes with time. In the tube at the high gas-pressure the effect is considerable, and the final value of current may be less than half the initial value. In the case of the tubes at lower gas pressure the fall, though sensible, is far less pronounced, say, three or four per cent.

An experiment is described showing that whatever the change of conditions causing the observed fall may prove to be, this fall is not attributable merely to the high temperature of the filament, but is conditional on the thermionic current being permitted to flow.

The results point to the practical conclusion that in order to avoid phase difference between current and applied voltage, and consequent distortion, at telephonic frequencies, of signals transmitted through a thermionic tube, the vacuum should be as high as possible.

INTRODUCTORY.

The thermionic tube is now very widely used as an amplifier or relay in telephonic and other circuits. The accuracy of reproduction of the incoming signals is clearly dependent on the absence of any time-lag, or equivalent effect, between the thermionic current flowing in the valve and the voltage impressed on its terminals at the same instant. It is often assumed that owing to the extremely low inertia of the electron, which is the vehicle of the current, the response of current to voltage must be practically instantaneous, and the two therefore necessarily in phase. Thus, for a field of one volt per centimetre the time required for an electron to cover a distance of 1 cm. from rest is less than a thirty-millionth of a second.

There are, however, other factors possibly at work ; secondary ionisation of residual gas or of gas liberated by bombardment of the electrodes, or a change in the surface of the hot electrode, may be supposed to be capable of affecting the time required for the attainment of the equilibrium value of the current between grid and filament.

The present Paper contains a description of experiments made to ascertain whether such a time-effect can be placed in evidence. It is shown that in every case tried the answer is in the affirmative. The effect is considerable in tubes containing a platinum (lime-coated) filament, the residual gas pressure being moderately high—well above 0·001 mm. of mercury. In tubes in which the heated filament is of tungsten, and the degree of exhaustion of the bulb far greater, the effect is small, though quite appreciable. In every case the effect observed is a *fall* of current with time ; it is thus incorrect to describe it as a time-lag, though it is evident that the outcome is none the less to produce a phase-displacement, and so a distortion of signals in a telephonic or telegraphic circuit in which the thermionic valve is employed.

EXPERIMENTAL ARRANGEMENTS.

The measurements were carried out by the ballistic method described in a previous Paper.* Consider a Wheatstone bridge connected as in Fig. 1. The grid and anode of a triode valve are put in direct metallic connection, and the space between them and the negative leg of the filament inserted as the unknown resistance in the arm *CD* of the bridge. The battery circuit is closed for a definite period, which can be

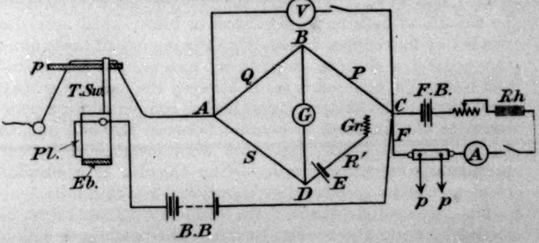

FIG. 1.—DIAGRAM OF ELECTRICAL CIRCUITS. *T.Sw.*=TIME-SWITCH ; *Pl.*=STEEL TARGET ; *Eb.*=EBONITE INSULATION ; *B.B.*=BRIDGE BATTERY ; *F.B.*=BATTERY SUPPLYING FILAMENT ; *p p*=LEADS TO POTENTIOMETER ; *Gr.*=GRID ; *E*=ACCUMULATOR IN SERIES WITH THERMIONIC TUBE.

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* Proc. Phys. Soc., Vol. XXIX., p. 33 (1916).

varied from 0·00001 second to any desired upper limit, the galvanometer being permanently in circuit. The bridge is balanced when the integral flow of electricity through the galvanometer is zero, this condition being attained, as in the ordinary use of the bridge, by variation of the resistance of the adjustable arm.

A difficulty obviously presents itself at the outset when the bridge is applied to the case of a thermionic tube, namely, that even when the battery circuit is open a current will flow from grid to filament, so producing a deflection of the galvanometer. With tubes containing a platinum filament this effect is quite small, and it can be readily shown that observations made with the slightly disturbed zero give sensibly the same results as though the effect were non-existent. With the higher temperatures holding in tubes with tungsten filaments, on the contrary, this effect is quite large. The most convenient way of eliminating it was found to be by inserting an accumulator cell in series in the arm containing the tube; this, of course, involves a change in the subsequent calculation of the resistance at balance, as shown below.

The durations of closure of the battery circuit usually employed were 0·00001, 0·000025, 0·0001, 0·00082, 0·005, 0·05, 1 and 30 sec. The very shortest periods were obtained by the use of balls or rings of steel or brass, which were suspended by fine copper wires. On drawing one of these out of the vertical a definite distance by means of a silk thread, and letting go, collision is made between the ball and a heavy steel target; the battery circuit is thus completed for a period equal to the duration of contact between the ball and the target. These times of contact were ascertained by supplementary measurements involving the use of a standard condenser and a resistance of known and suitable value.

The intermediate times, from 0·005 to 0·02 sec., were obtained by using a pendulum switch. The pendulum, with an attached tongue of hard copper strip, is drawn out a noted distance, then released and allowed to glide over one of two brass strips, one narrow, one wide, let flush into a sheet of ebonite chamfered off to avoid vibratory motion of the tongue. The long closures were made by hand, the balance in these cases being not ballistic, but continuous.

The mean resistance of the space between the cold and hot electrodes, and the current across this space, are calculated (for fuller treatment reference may be made to the Paper already quoted) from the following relations, which hold at balance :—

$$R'=\frac{P}{Q}\,.\,S=\frac{v'}{i}\quad . \;.\;.\;.\;.\;.\;.\;.\;. \quad (1)$$

$$v'=V\,.\,\frac{P}{P+Q}\quad . \;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (2)$$

$$v=v'-E\quad . \;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (3)$$

$$i=\frac{v'}{R'}=\frac{VQ}{S(P+Q)}\quad . \;.\;.\;.\;.\;.\;. \quad (4)$$

$$R=\frac{v}{i}=\frac{(v'-E)(P+Q)S}{VQ}\quad . \;.\;.\;.\;. \quad (5)$$

where

P and Q denote the resistances of the ratio arms of the bridge,
S the resistance of variable arm at balance,
R' the apparent resistance of the arm CD (including the thermionic tube in series with the accumulator cell),
R the resistance of the tube alone (hot to cold electrode),
V the voltage applied across the bridge (A to C),
v' the voltage across the arm containing the tube (D to C),
v the voltage across tube only,
E the E.M.F. of accumulator in series in arm containing the tube,
i the current through the tube.

Precautions were taken to ensure the elimination of possible sources of error. It is essential that the voltmeter used in measuring the voltages V should be disconnected during the actual ballistic test, in order to avoid the disturbing effect arising from its inductance. It appeared possible that the inductance of the galvanometer (of the Broca type, resistance 1,000 ohms) might not be without a similar effect. To test this point the connections between B and D were arranged as shown in Fig. 2. A resistance, w, is placed in series with the galvanometer, and in shunt with the two is arranged a $\frac{1}{3}$ mf. mica condenser in series with a resistance, W. The condenser branch exactly compensates the inductive branch if $W=G+w$, and $L/W=KW$, where L denotes the inductance of the galvanometer and G its resistance. With this arrangement the voltage between the points B and D at any instant in the variable state is the same as if these points were connected by a non-inductive resistance W. It was found on making the trial that the balance-point was entirely unaffected by this change, showing that the inductiveness of the galvanometer as simply connected was without sensible effect on the results.

The bridge circuits and the heating circuit were insulated, both from one another and from earth.

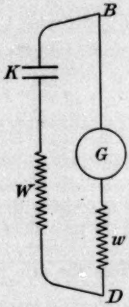

FIG. 2.—ALTERNATIVE GALVANOMETER CIRCUIT.

Whenever necessary, extreme constancy of the current through the filament was obtained by the aid of a potentiometer, to which the resistance PS in the heating circuit could be connected.

EXPERIMENTAL RESULTS.

1. *Tube with Platinum (Lime-coated) Filament.*—In this type of tube, as already pointed out, the residual gas pressure is comparatively high. Successive values of voltage from zero to 50 volts were applied between the electrodes of the tube. The observations are represented by the curves of Figs. 3, 4, 5, and 6. Fig. 3 contains the resistance- and current-characteristics over the range 1 to 5 volts, for $t=1$ sec. and $t=0{\cdot}00082$ sec. In Fig. 4 are incorporated the resistance-characteristics over the whole range of voltage used, and for times of application of voltage between the limits of 1 sec. and 0·000025 sec.; the corresponding current-characteristics are given in Fig. 5.

The current falls with time, the effect being very marked except at the higher voltages. Thus at 5 volts the current at the end of 1 sec. is only about two-fifths of its mean value during the first 0·000025 sec. At a voltage of 25 the dependence of current on time has almost disappeared, and this independence persists up to the maximum value of 50 volts. It will be observed that with this tube saturation is far from being attained at that voltage.

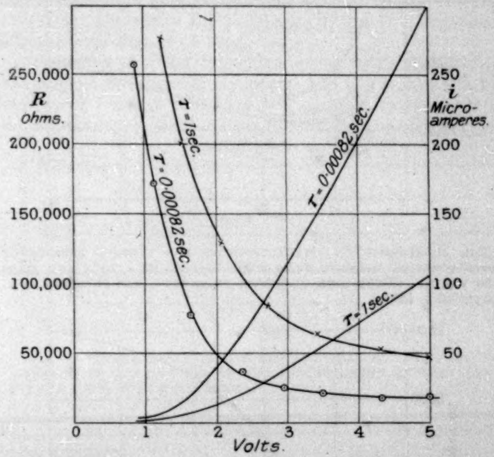

FIG. 3.—RESISTANCE—AND CURRENT-CHARACTERISTICS BELOW 5 VOLTS, FOR $\tau=1$ SEC. AND $\tau=0·00082$ SEC. (TUBE WITH PLATINUM FILAMENT. COMPARATIVELY HIGH GAS PRESSURE.)

Within the range employed the current approximates to a limit as the time of application of voltage is continuously reduced. This may be seen by plotting the values of current against time abscissae, or rather, for convenience of scale, against log t, as shown in Fig. 6. This leads to the inference that, within the time-range employed, the current falls with time according to a law of an exponential type, with a time-constant of 0·0065 sec. The maximum and minimum limits of current indicated are 0·9 and 0·25 milli-ampere.

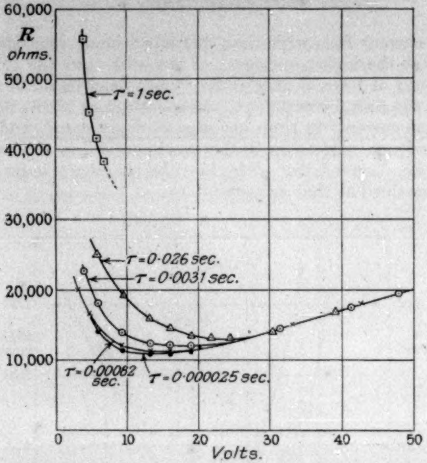

FIG. 4.—RESISTANCE CHARACTERISTICS FOR VARIOUS PERIODS OF APPLICATION OF VOLTAGE FROM 0·000025 SEC. TO 1 SEC., AND FROM 0–50 VOLTS. (TUBE WITH PLATINUM FILAMENT: GAS PRESSURE COMPARATIVELY HIGH.)

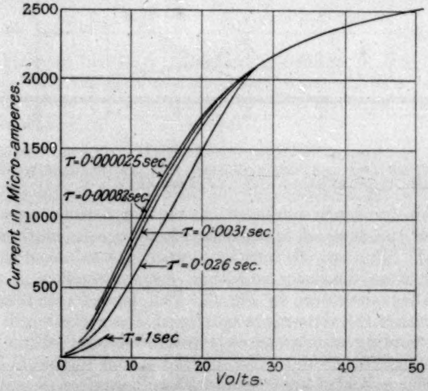

FIG. 5.—CURRENT CHARACTERISTICS. (TUBE WITH PLATINUM FILAMENT.) DERIVED FROM FIG. 4.

2. *Tube with Tungsten Filament.*—These tubes were of the R-type supplied by the General Electric Company. By the courtesy of Mr. C. C. Paterson we are able to state that the residual gas pressure does not exceed one ten-thousandth of a millimetre of mercury, and may be distinctly below that value.

The results may be summed up in a single curve, that of Fig. 7. Three of the characteristics only are shown, for periods of application of voltage equal to 0·0001, 0·005, and 1 sec. respectively. The curve for $t=0·000025$ sec. is indistinguishable from that at 0·000025 sec. Similarly the curve expressing the results for $t=30$ sec. is almost inappreciably different from that at $t=1$ sec.

It is evident that the time-effect is very small in this type of tube, though it unmistakably exists. As before, the effect is a decrease of current with time. The whole variation in this case amounts only to 3 or 4 per cent. A curve of the type of Fig. 6 is also found, a time-constant of the order of 0·0008 sec. being indicated.

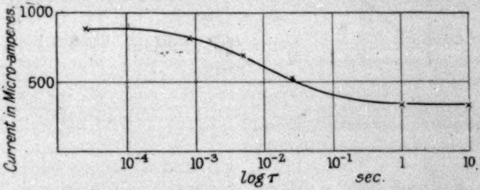

FIG. 6.—THERMIONIC CURRENT PLOTTED AGAINST LOG τ, SHOWING APPROACH TO LIMITING VALUES FOR τ VERY GREAT OR VERY SMALL. (TUBE WITH PLATINUM FILAMENT.)

3. *Experiments on the Influence of the Heating Current.*—It is easy to demonstrate by the experimental method of the present investigation that the cause of the change of condition within the tube which leads to the observed variation of current with time is not to be found in the mere action of the heating current, apart from an actual flow of electrons from it to the cold electrode. Fig. 8 represents the results of a test which extended over 20 hours on a tube with platinum filament. The portion *ABC* gives values of current obtained by ballistic measurement, the period of flow permitted being 0·00082 sec.; the grid at the intervening times

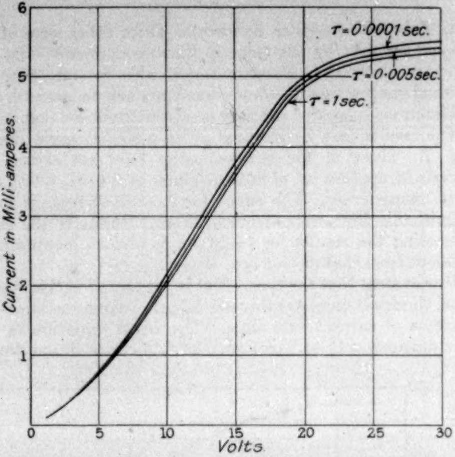

FIG. 7.—CURRENT CHARACTERISTIC FOR THREE PERIODS (τ) OF APPLICATION OF VOLTAGE. (TUBE WITH TUNGSTEN FILAMENT. GAS PRESSURE COMPARATIVELY LOW.)

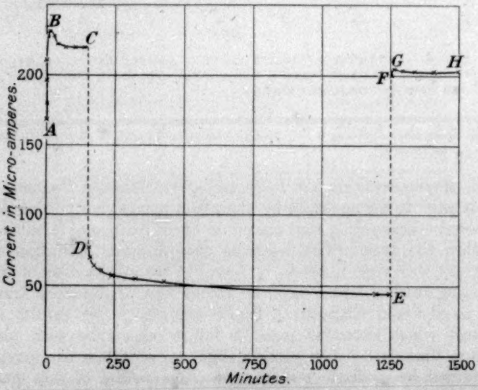

FIG. 8.—CURVE SHOWING THAT THE TIME-VARIATION OF CURRENT FROM GRID TO FILAMENT IS THE RESULT OF A CHANGE OF CONDITIONS PRODUCED BY THE FLOW OF THIS CURRENT.

being at —2 volts with respect to the filament, the thermionic flowing was practically zero except during the brief interval of the actual test. The curve indicates the usual course followed by a characteristic during the initial period after switching on the heating current; long before *C* is reached it has "settled down" to its steady value. On reaching *C* the voltage (4·39 volts) was permanently applied, and the balance-point obtained about a minute later is represented by the point *D*. The current has already fallen to less than half its value at *C*; it continues to fall steadily, and even at *E*, after 15 hours' steady application of the voltage, the rate of fall is quite appreciable. At *E* the voltage was removed, and readings again taken every few minutes by short-period applications of the voltage; thus the portion *FGH* was obtained, showing that the effect of the prolonged flow of thermionic current quickly disappears, though the initial value is not entirely recovered.

CONCLUSIONS.

These experiments prove that a sensible time is required to establish the equilibrium value of the current between the hot and cold electrodes in a thermionic tube when a steady voltage is suddenly applied. In every case the current *falls* with time; although the duration of application of voltage was cut down to the limit of 0·00001 sec., in no case was any observation of the initial rise secured.

The cause of the change is assignable to some variation of conditions within the tube, which comes into play only when the thermionic current is allowed to flow. In the type of tube containing a lime-coated platinum filament, for which the residual gas pressure is probably well over 0·001 mm. of mercury, the value of the current after a lapse of some seconds may be as low as one-third of its initial maximum.

In tubes containing a tungsten filament, exhausted to pressures of the order of 0·0001 mm., the total reduction of current amounts only to 3 or 4 per cent.

It is thus probable that the magnitude of the variation of thermionic current with time is determined by the value of the pressure of residual gas in the tube, being continuously reduced as this pressure is reduced.

The distortion of signals of telephonic frequency would probably be quite appreciable in tubes in which the gas pressure is above 0·001 mm., but inappreciable when the pressure is as low as 0·0001 mm.

The earlier work (on the platinum filament type of tube) was carried out in 1915–1916 at the instance of the Department of Wireless Telegraphy, H.M.S. Vernon, Portsmouth.

The authors are indebted to the courtesy of Sir Robert Hadfield for the target of rustless steel, which proved a considerable convenience in the ballistic tests.

DISCUSSION.

Mr. E. V. **Appleton** pointed out as a possible cause of the effects that the filament is always cooled when the anode current is put on. This means that the electronic emission is reduced. Dr. van der Pol and he had recently had to explain the form of some oscillograms of a triode generator in this way. The condition of the filament before and after closing the anode circuit was quite different, and the effect of the latent heat of electronic evaporation and the heating effect of the anode current had to be taken into account in the latter case. It had been found that because of this the final emission in a certain case was only 86 per cent. of the original amount. Regarding things in this way it is possible to account for the exponential fall of the current with time observed in the present experiments, and if a filament of typical size is considered the time constant comes out about the right value. In soft tubes anything might happen, but he expected that with the filament voltages used a cooling would also be obtained.

Prof. **Eccles** said he had himself thought of this as a possible explanation. There were also other things which might act in a similar way. If there is a very little gas in the valve a deposit of molecular thickness is formed on the filament and evaporated from it as changes of the anode current occur. This layer greatly alters the electron emission. Another point which had occurred to him was that the change in the distribution of the electron cloud which must occur when the voltage is applied might be responsible, but a consideration of the times involved ruled this out. Besides resistance these tubes had a species of back E.M.F., and, as far as possible, attempts should be made to separate them in dealing with valve problems.

Dr. **Rayner** said that if the effects were due to cooling of the filament the dimensions of the latter ought to have an important influence.

Mr. F. E. **Smith** pointed out that the straight part of the curve *DE* (Fig. 8) was approximately parallel to *CG*. This seemed to indicate that the steady drop between 250 and 1,250 minutes had nothing to do with the anode voltage being on, but was probably due to fatigue of the filament.

A **Visitor** queried the authors' statement regarding Fig. 2. The inductance of the path *BGD* was unaffected by the shunt circuit *BKD*.

Dr. **Vincent** suggested that it was desirable to indicate by a ring, in Fig. 1, the existence of glass round the valve to show that it was a vacuum tube. His first impression on reading the Paper was that our usual ideas as to characteristics would have to be scrapped. He was relieved to find that the curves of Fig. 7, for the hard valve, were so nearly the orthodox characteristics taken in the usual way.

Dr. G. D. **West** emphasised the importance of the study of the internal action of valves, and congratulated the authors on their work.

Dr. **Owen**, in reply to the discussion, said : The theories of the variation of thermionic current with time which have been put forward by speakers are of great interest. The authors in the present Paper had made it their concern to establish the existence and the magnitude of this effect, in the case of the two classes of valve—namely, the soft and the hard types. They would be able to pursue their further work in the light of the explanations advanced, and perhaps to discriminate between them. With respect to the query as to the arrangement of Fig. 2, the important point was that the *combined* circuit behaved like a perfectly non-inductive resistance connecting the points *B* and *D*, thus avoiding the persistence of an induced E.M.F. acting on the part of the bridge outside *BD*. How this was brought about, and the state of affairs in *BGD* and *BKD*, constituted an interesting though simple problem, which the questioner would be able to solve after a little examination. In reply to Dr. Vincent, he hoped that the fact that valves were enclosed in glass tubes was sufficiently known to render unnecessary the expense incurred in altering the block.

Mr. Archer made some remarks explanatory of the devices used in the production of the short intervals of time used in the experimental work described in the Paper.

On the Mechanical Equilibrium of a Sphere of Gravitating Fluid. *By* CHARLES H. LEES, *D.Sc., F.R.S.* (*Part of the Presidential Address to the Physical Society, February* 13, 1920.)

(*Continued from Vol. XXXII., p.* 270.)

9. IN many cases it is more convenient to define the state of the homogeneous medium at radius r in terms of the pressure p and temperature θ, rather than the pressure and specific volume v of the fluid. We then write

$$\frac{dv}{dr}=\left(\frac{\partial v}{\partial p}\right)_\theta \frac{dp}{dr}+\left(\frac{\partial v}{\partial \theta}\right)_p \frac{d\theta}{dr},$$

or, since in virtue of the pressure equation

$$\frac{dp}{dr}=-\frac{g}{v},$$

where g is the gravitational acceleration at radius r,

$$\frac{dv}{dr}=-\left(\frac{\partial v}{\partial p}\right)_\theta \frac{g}{v}+\left(\frac{\partial v}{\partial \theta}\right)_p \frac{d\theta}{dr}.$$

The condition for stable mechanical equilibrium (6·2) then becomes

$$-\left(\frac{\partial v}{\partial p}\right)_\theta \cdot \frac{g}{v}+\left(\frac{\partial v}{\partial \theta}\right)_p \cdot \frac{d\theta}{dr} > g\beta_\phi,$$

or

$$\left(\frac{\partial v}{\partial \theta}\right)_p \cdot \frac{d\theta}{dr} > g(\beta_\phi-\beta_\theta) \quad . \quad . \quad . \quad . \quad . \quad (9\cdot1)$$

where β_θ is the isothermal compressibility of the medium.

Thus, for stable equilibrium

$$\frac{d\theta}{dr}>-g\frac{\beta_\theta-\beta_\phi}{\left(\frac{\partial v}{\partial \theta}\right)_p}.$$

But

$$\beta_\theta-\beta_\phi=\frac{1}{c_p}\cdot\frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_p^2. \quad . \quad . \quad . \quad . \quad . \quad (9\cdot2)$$

Hence the condition for stable equilibrium becomes

$$\frac{d\theta}{dr}>-\frac{g}{c_p}\frac{\theta}{v}\cdot\left(\frac{\partial v}{\partial \theta}\right)_p,$$

i.e.,

$$-\frac{d\theta}{dr}<\frac{g}{c_p}\cdot\frac{\theta}{v}\cdot\left(\frac{\partial v}{\partial \theta}\right)_p, \quad . \quad . \quad . \quad . \quad . \quad . \quad (9\cdot3)$$

where c_p is the specific heat of the medium at constant pressure expressed in work units.

10. If we apply this condition to a homogeneous ideal gas well away from the centre of the fluid sphere we have for the gas

$$pv=\frac{R}{m}\theta.$$

Hence $$\frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_p=1,$$

and the condition for stability becomes

$$-\frac{d\theta}{dr}<\frac{g}{c_p}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10{\cdot}1)$$

Or, since for the gas

$$c_p=\frac{R}{m}\cdot\frac{\gamma}{\gamma-1},$$

we may put the condition in Kelvin's form

$$-\frac{d\theta}{dr}<\frac{\gamma-1}{\gamma}\,\frac{g}{R}\cdot m. \quad . \quad . \quad . \quad . \quad . \quad (10{\cdot}2)$$

As the specific heats and their ratio are constant for such a gas, we see that while g is constant there will be no tendency for one portion of the gas to become unstable for the same temperature slope before another. The decrease of g as we proceed outwards through the outer and rarer portion of the gas will, however, reduce the margin of stability for a given temperature slope.

For a gas or vapour well away from the centre, to which Van der Waal's equation applies, we have

$$\left(p+\frac{a}{v^2}\right)(v-b)=\frac{R}{m}\theta,$$

where R is nearly the same in value for all gases, and

$$\frac{\theta}{v}\cdot\left(\frac{\partial v}{\partial \theta}\right)_p=1+\frac{1}{v}\left(\frac{2am}{R\theta}-b\right). \quad . \quad . \quad . \quad . \quad (10{\cdot}3)$$

For gases for which the Joule-Kelvin effect is a cooling

$$\frac{2am}{R\theta}-b$$

is positive. For hydrogen at temperatures above 193° absolute it is negative, but becomes positive at lower temperatures. For most gases c_p increases as the temperature rises. Hence the variations of $\frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_p$ and of c_p may oppose each other in their effect on the stability. The effect of gravity is the same as in the case of a perfect gas.

For a homogeneous liquid not near the centre $\frac{1}{v}\left(\frac{\partial v}{\partial \theta}\right)_p$ is of the order 0·001 and increases about 10 per cent. per 100°C. c_p is of the order $1{\cdot}5 \times 10^7$, and increases at a somewhat greater rate. The variation of stability will, therefore, depend mainly on the product $g\theta$, the factors of which may change in the same or in opposite directions as we move radially. Where $g\theta$ is least the stability condition is most exacting.

11. If the fluid is not homogeneous throughout but consists of a number of homogeneous media which do not mutually dissolve each other, the various media will for stable equilibrium be arranged in concentric spherical shells, subject to the equivalent conditions that in each homogeneous fluid shell

$$\frac{dv}{dr} > g\beta_\phi, \quad \frac{d\varphi}{dr} + ve \quad \text{and} \quad -\frac{d\theta}{dr} < \frac{g}{c_p} \cdot \frac{\theta}{v} \cdot \left(\frac{\partial v}{\partial \theta}\right)_p,$$

while at each spherical surface separating two media the specific volume v must not decrease as r increases.

On each side of the separating surface $\frac{d\varphi}{dr}$ must be positive, while at the surface itself φ changes from φ_1 for the first to φ_2 for the second medium. We may, therefore, state the law of change of entropy with r in the form ·—. For stable equilibrium $\frac{d\varphi}{dr}$ at any point must exceed the rate of increase of φ due to change of composition of the fluid at that point.

The fact that ice is in equilibrium on the surface of water shows that the entropy per gram of the upper material need not always exceed that of the lower.

As the conditions for thermal equilibrium at a surface separating two fluids are that the heat fluxes and temperature on the two sides of the surface shall be equal, it is sometimes convenient to write the condition for stable mechanical equilibrium in each fluid in the form

$$-k\frac{d\theta}{dr} < k\frac{g}{c_p} \cdot \frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_p \quad . \quad . \quad . \quad . \quad . \quad . \quad (11{\cdot}1)$$

where k is the rate of transmission of heat per sq. cm. per second, for unit slope of temperature.

Thus the flux of heat outwards per square centimetre per second must be less than

$$k\frac{g}{c_p}\frac{\theta}{v}\cdot\left(\frac{\partial v}{\partial\theta}\right)_p,$$

that is than $k\frac{g}{c_p}\cdot\theta\cdot a_p$ where $a_p=\frac{1}{v}\left(\frac{\partial v}{\partial\theta}\right)_p$, and is sometimes called the coefficient of cubical dilatation.

12. Although it will not often happen that the equilibrium just becomes stable in each of two contiguous fluids at the same instant, it may be mentioned that in such an event there would, in general, be an abrupt change in the value of $\frac{dv}{dr}$ on passing through the surface of contact. If v_1 is the specific volume and β_1 the adiabatic compressibility of the first and $v_2\beta_2$ of the second fluid, we have on one side of the surface of contact $\frac{dv_1}{dr}=g\beta_1$ and on the other $\frac{dv_2}{dr}=g\beta_2$. In the same way the value of $-\frac{d\theta}{dr}$ changes from $\frac{g}{c_1}\cdot\frac{\theta}{v_1}\left(\frac{\partial v_1}{\partial\theta}\right)_p$ to $\frac{g}{c_2}\frac{\theta}{v_2}\left(\frac{\partial v_2}{\partial\theta}\right)_p$ and of φ from φ_1 to φ_2.

Thus in passing upwards from a liquid to a gas or vapour in both of which equilibrium is attained at the same instant, since the adiabatic compressibility of the gas or vapour is much greater than that of a liquid, the rate of increase of the specific volume with height will be much greater in the gas or vapour than in the liquid. Since $\frac{1}{v}\left(\frac{\partial v}{\partial\theta}\right)_p$ for a gas is of the order $3{\cdot}6\times10^{-3}$ and for a liquid of the order 10^{-3}, while the specific heats of gases and vapours are, in general, less than those of liquids, the value of $-\frac{d\theta}{dr}$ in the gas or vapour will be of the order 5 times that in the liquid just below it, if the two attain equilibrium together.

As the condition for thermal equilibrium requires the values of $-\frac{d\theta}{dr}$ to be inversely as the heat transmission rates of the gas and liquid, which are about in the ratio 1 : : 10, as a rule the condition for mechanical equilibrium commencing in the two together will not also give thermal equilibrium.

In the case of dry air at the surface of the earth convective equilibrium exists when $-\frac{d\theta}{dr}$ is 1°C. in 100 metres, in sea water at temperatures between 0°C. and 30°C., when $-\frac{d\theta}{dr}$ is $\frac{1}{200}$°C. in 100 metres. On the other hand, the rate of heat transmission in sea water is about 30 times that of air. Hence "convective" and thermal equilibrium could not occur together in air and sea water.

13. Although the probability of two contiguous layers of fluid attaining the "convective" or neutral equilibrium state together is small, they may subsequently be in stable mechanical and thermal equilibrium with each other. If k_1 is the thermal transmission in the first fluid, the condition for stable mechanical equilibrium in that fluid may be written $-k_1\frac{d\theta_1}{dr} < k_1\frac{g}{c_1}\frac{\theta_1}{v_1}\left(\frac{\partial v_1}{\partial \theta_1}\right)_p$. Similarly, in the second the condition is $-k_2\frac{d\theta_2}{dr} < k_2\frac{g}{c_2}\frac{\theta_2}{v_2}\cdot\left(\frac{\partial v_2}{\partial \theta_2}\right)_p$. The conditions for thermal equilibrium are $\theta_1=\theta_2$, $-k_1\frac{d\theta_1}{dr}=-k_2\frac{d\theta_2}{dr}$ at the interface.

Both mechanical and thermal conditions are satisfied if at the interface the equal quantities $-k_1\frac{d\theta_1}{dr}$ and $-k_2\frac{d\theta_2}{dr}$ are less than the smaller of the two quantities $k\frac{g}{c_p}\cdot\frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_p$ for the two fluids respectively. For air this has a value considerably less than for water. Hence the condition for stability in the air is last to be satisfied.

14. Although convection currents in the sphere while in the unstable state will, in general, ensure the complete mixture of miscible fluids, there may be cases in which two miscible fluids are not completely mixed before the equilibrium becomes stable. In order to examine this point we assume that the fluid is not homogeneous but consists of a mixture in varying proportions of a number of fluids which mutually dissolve each other. We see from §5 that stable equilibrium will only be possible if the specific volume of the mixture is

uniform throughout each thin concentric shell and increases from shell to shell as we proceed outwards.

The equilibrium here referred to is a molar equilibrium, and its existence does not preclude diffusion of the various fluids into each other, which is a molecular, not a molar, motion. The diffusion is superposed on the phenomena we are discussing.

The condition for stable equilibrium is still given by the inequality

$$\frac{dv}{dr} > g\beta_{\phi},$$

in which $\frac{dv}{dr}$ is the complete rate of increase of the specific volume of the mixture with radius and β_{ϕ} is the adiabatic compressibility of the mixture at radius r, but the corresponding conditions involving the entropy and temperature are slightly modified.

15. In the case of the entropy condition we write the compressibility equation in the form

$$\frac{dv}{dr} > -\frac{g}{v}\left(\frac{\partial v}{\partial p}\right)_{\phi . M},$$

where the subscript M indicates that the mass of each constituent in 1 gram of the mixture is constant, and by means of the relation

$$\left(\frac{\partial v}{\partial p}\right)_{\phi . M} = -\left(\frac{\partial \varphi}{\partial p}\right)_{vM} \Big/ \left(\frac{\partial \varphi}{\partial v}\right)_{pM},$$

convert it into

$$\left(\frac{\partial \varphi}{\partial v}\right)_{pM} \frac{dv}{dr} > \frac{g}{v}\left(\frac{\partial \varphi}{\partial p}\right)_{vM}. \quad . \quad . \quad . \quad . \quad . \quad (15{\cdot}1)$$

Now

$$\frac{d\varphi}{dr} = \left(\frac{\partial \varphi}{\partial v}\right)_{pM} \frac{dv}{dr} + \left(\frac{\partial \varphi}{\partial p}\right)_{vM} \frac{dp}{dr} + \sum_{n}\left(\frac{\partial \varphi}{\partial M_n}\right)_{vp} \frac{dM_n}{dr}, \quad . \quad (15{\cdot}2)$$

where M_n is the mass of the nth constituent and $\Sigma M_n = 1$.

On substituting in this equation the condition for stability, and $\frac{dp}{dr} = -\frac{g}{v}$, we get as the entropy condition for stability

$$\frac{d\varphi}{dr} > \sum_{n}\left(\frac{\partial \varphi}{\partial M_n}\right)_{pv} . \frac{dM_n}{dr}. \quad . \quad . \quad . \quad . \quad . \quad (15{\cdot}3)$$

That is the rate of increase of the entropy with radius due to *all* causes must exceed that which would be due to change of composition of the fluid mixture if the pressure and specific volume remained constant.

16. To find for the mixture the modification of the condition of stability expressed in terms of the temperature slope outwards, we again write the general compressibility condition in the form

$$\frac{dv}{dr} > -\frac{g}{v}\cdot\left(\frac{\partial v}{\partial p}\right)_{\phi\ M}.$$

Taking p, θ and the mass M_n of each of the n constituents of the mixture where $\Sigma M_n=1$, as the variables defining the mixture at r we get

$$\left(\frac{\partial v}{\partial p}\right)_{\theta\,.\,M}\frac{dp}{dr}+\left(\frac{\partial v}{\partial \theta}\right)_{pM}\cdot\frac{d\theta}{dr}+\sum_n\left(\frac{\partial v}{\partial M_n}\right)_{p\theta}\frac{dM_n}{dr} > -\frac{g}{v}\left(\frac{\partial v}{\partial p}\right)_{\phi\,.\,M}.$$

But

$$\frac{dh}{dr}=-\frac{g}{v}.$$

Hence the condition for stability becomes

$$\left(\frac{\partial v}{\partial \theta}\right)_{pM}\cdot\frac{d\theta}{dr}+\sum_n\left(\frac{\partial v}{\partial M_n}\right)_{p\theta}\cdot\frac{dM_n}{dr} > \frac{g}{v}\left\{\left(\frac{\partial v}{\partial p}\right)_{\theta M}-\left(\frac{\partial v}{\partial p}\right)_{\phi M}\right\},$$

that is using (9·2)

$$\frac{d\theta}{dr} > -\frac{g}{c_p}\cdot\frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_{pM}-\frac{\sum_n\left(\frac{\partial v}{\partial M_n}\right)_{p\theta}\cdot\frac{dM_n}{dr}}{\left(\frac{\partial v}{\partial \theta}\right)_{pM}},$$

or

$$-\frac{d\theta}{dr} < \frac{g}{c_p}\cdot\frac{\theta}{v}\left(\frac{\partial v}{\partial \theta}\right)_{pM}+\frac{\sum_n\left(\frac{\partial v}{\partial M_n}\right)_{p\theta}\cdot\frac{dM_n}{dr}}{\left(\frac{\partial v}{\partial \theta}\right)_{pM}}.$$

$$< \frac{g}{c_p}\cdot\frac{\theta}{v}\cdot\left(\frac{\partial v}{\partial \theta}\right)_{pM}+\frac{\frac{1}{v}\sum_n\left(\frac{\partial v}{\partial M_n}\right)_{p\theta}\cdot\frac{dM_n}{dr}}{\frac{1}{v}\left(\frac{\partial v}{\partial \theta}\right)_{pM}}.$$

The last term is the quotient of the fractional increase of specific volume with radius which would be due to change of composition at constant pressure and temperature, by the coefficient of cubical dilatation at constant pressure, and represents the rate of change of temperature with radius which at constant pressure would give the same change of specific volume as the actual change of composition of the fluid with the radius would at constant pressure and temperature.

The process of diffusion diminishes the value of this term, and therefore diminishes the stability of the fluid for a given temperature slope. If the diminution is sufficient, the stability may cease, and convection currents begin to facilitate the mixture of the constituents.

17. Although the three forms of the condition for stability for a sphere of gravitating fluid dealt with in the foregoing pages are equivalent, it appears desirable to retain all three. The form $\frac{dv}{dr} > g\beta_\phi$ holds in all cases, the second—that the rate of increase with the radius of the entropy per gram must exceed that due to change of composition only, and the third—that the temperature slope outwards or the flux of heat outwards per square centimetre per second must be less than a certain minimum, are connecting links between the problem of mechanical stability here considered and that of thermal stability, which is of equal importance.

DISCUSSION

ON

THE ABSOLUTE MEASUREMENT OF ELECTRICAL RESISTANCE, AND INSTRUMENTS BASED ON THE TEMPERATURE-VARIATION OF RESISTANCE.

CONTENTS.

PAGE

I.—THE ABSOLUTE MEASUREMENT OF RESISTANCE.

1. "Historical Account of the Absolute Determination of the unit of Electrical Resistance," by Sir RICHARD GLAZEBROOK, K.C.B., F.R.S. 126
2. "Two Proposals: (*a*) Respecting the present System of Electrical Units; and (*b*) on a Lecture Demonstration of the Absolute Measurement of Resistance," by F. E. SMITH, O.B.E., F.R.S. 131

II.—INSTRUMENTS BASED ON THE TEMPERATURE VARIATION OF RESISTANCE.

Resistance Thermometry, &c.

1. "Platinum Thermometers, a Compensated Bridge, a Compensated Bolometer, and the Radio-balance," by Prof. H. L. CALLENDAR, LL.D., F.R.S. 135
2. "Early Work on the Resistance Pyrometer," by C. R. DARLING, A.R.C.S. 138
3. "Resistance Thermometers for Steam Temperatures," by C. JAKEMAN 140
4. "An Electric Transmitting Radiator Thermometer," by EDGAR A. GRIFFITHS 141

"*The Hot Wire Microphone*," by Major W. S. TUCKER... 144

Hot Wire Anemometers.

1. "A Hot Wire Anemometer," by Prof. J. T. MCGREGOR MORRIS 148
2. "The Directional Hot Wire Anemometer," by J. S. G. THOMAS, D.Sc. 149
3. "An Instrument for Use in Measuring Convected Heat," by A. H. DAVIS, B.Sc. 152

The Katharometer.

1. "The Katharometer," by G. A. SHAKESPEAR, D.Sc.... 163
2. "Industrial Applications of the Katharometer," by H. A. DAYNES, D.Sc. 165

"*The Caleometer*," by Prof. LEONARD HILL and Miss HARGOOD-ASH 169

"*The Liquid Depth Gauge* (*Direct Reading type*)," by EDGAR A. GRIFFITHS 171

I.

THE ABSOLUTE MEASUREMENT OF RESISTANCE.

1. "*Historical Account of the Absolute Determination of the Unit of Electrical Resistance,*" *by* SIR RICHARD GLAZEBROOK, *K.C.B., F.R.S.*

Sir **RICHARD GLAZEBROOK** gave an historical outline of some of the chief events in the establishment of the C.G.S. unit of electrical resistance, or rather of its multiple, the ohm, and of the standards which represent this; the ground covered being in the main from the initiation in 1861 of the British Association Committee on Electrical Standards to its dissolution in 1912.

At the date first mentioned a great assortment of arbitrary units of electrical resistance were in use, and the question of a definitely recognised standard was a pressing one. Latimer Clark and Sir Charles Bright read a Paper before the British Association, the outcome of which was the appointment, on the motion of Professor William Thomson, of the Committee referred to above. The work of this committee has proved to be of fundamental importance in the history of electrical measurements.

Wilhelm Weber had in 1851 proposed a system of absolute units in regard to electrical quantities. The principle was at once taken up and developed by Thomson, and in 1863, as the result largely of his advocacy, the C.G.S. system of electromagnetic units was adopted by the Committee. As a practical unit of convenient value, the multiple 10^9 units was adopted; for this the term Ohmad was suggested, which later took the abbreviated form Ohm.

The next step was the production of a concrete standard of resistance to represent the Ohm. This involved the determination in absolute measure of the resistance of a particular coil. The method chosen by the Committee was that of the spinning coil, devised by Thomson, though apparently here too Weber had anticipated it several years before. The experiments were carried out at King's College, London, by Clerk Maxwell, Fleeming Jenkin, and Charles Hockin. Two series of experiments were made, one in 1863, the second in 1864. The resistances of two wire coils were thus determined at specified temperatures. On the basis of these, ten standards—eight wire coils and two mercury tubes—were constructed by Matthiessen and Hockin, to be deposited at Kew Observatory, each having a value equal to one ohm at some specified temperature. As a matter of historical interest the data referring to these coils are reproduced in Table I. The initials in the last column denote Matthiessen and Hockin. The Second Report of the Committee in 1863 contains, in addition to a

TABLE I.—*This Table shows the value of the different copies of the B.A. Units that have been made for preservation at Kew*:—

Material of coil.	No. of coil.	Date of observation.	Temp. at which coil has a resistance 10^9 cm./sec.	Observer.
Platinum-iridium alloy	2	Jan. 4, 1865 June 6, 1865 Feb. 10, 1867	15·5°C. 16·0°C. 16·0°C.	C.H. A.M. C.H.
Paltinum-iridium alloy	3	Jan. 4, 1865 June 6, 1865 Feb. 10, 1867	15·3°C. 15·8°C. 15·8°C.	C.H. A.M. C.H.
Gold-silver alloy ...	10	Jan. 5, 1865 Feb. 10, 1867	15·6°C. 15·6°C.	A.M. C.H.
Gold-silver alloy ...	58	April 10, 1865 June 6, 1865 Feb. 10, 1867	15·3°C. 15·3°C. 15·3°C.	A.M. A.M. C.H.
Platinum	35	Jan. 7, 1865 Aug. 18, 1866 Feb. 10, 1867	15·7°C. 15·7°C. 15·7°C.	C.H. A.M. C.H.
Platinum	36	Jan. 7, 1865 Aug. 18, 1866 Feb. 10, 1867	15·5°C. 15·5°C. 15·7°C.	C.H. A.M. C.H.
Platinum-silver alloy...	43	Feb. 15, 1865 Mar. 9, 1865 Feb. 10, 1867	15·2°C. 15·2°C. 15·2°C.	C.H. A.M. C.H.
*Mercury	I.	Feb. 2, 1865 July 18, 1866 Feb. 11, 1867	16·0°C. 16·0°C. 16·7°C.	A.M. A.M. C.H.
Mercury	II.	Feb. 3, 1865 Aug. 18, 1866 Feb. 11, 1867	14·8°C. 14·8°C. 14·8°C.	A.M. A.M. C.H.
Mercury	III.	Feb. 11, 1867	17·9°C.	C.H.

* The alteration of this coil, observed on Feb. 11, 1867, is due, no doubt, to a defect observed in the glass tube.

description of the experiments of that year, what amounts to a fundamental treatise on electrical units and measurements.

At that time little was known as to the degree of constancy of the resistance of a coil under various physical conditions or with lapse of time. Valuable data were acquired on these points in the years following. The extent of the secular variation in the case of some of the B.A. standard coils may be seen from Table II.

TABLE II.—*Resistances at 16·0°C. in terms of the original B.A. Unit* (1867). (*Values obtained through the two Platinum Coils, D, E.*).

Coil.	Material.	1867	1876	1879-81	1888	1908	Max. diff. B.A.U.
A	Pt Ir	1·00000	1·00077	1·00056	1·00147	1·00122	147×10^{-5}
B	Pt Ir	1·00029	1·00121	1·00080	1·00104	1·00098	92 ,,
C	Au Ag	1·00050	1·00141	1·00101	1·00146	1·00173	123 ,,
D	Pt	1·00092	1·00092	1·00092	1·00092	1·00092	0 ,,
E	Pt	1·00152	1·00152	1·00152	1·00152	1·00152	0 ,,
F	Pt Ag	...	...	1·00016	1·00072	1·00160	144 ,,
G	Pt Ag	1·00022	1·00030	0·99982	1·00025	1·00175	193 ,,
H	Pt Ag	1·00020	...	...	1·00042	1·00044	24 ,,
Flat	Pt Ag	...	...	1·00079	1·00120	1·00125	46 ,,

In the meantime other methods of measuring electrical resistance in absolute measure had been proposed, and some results had been obtained—by Kohlrausch and by Rowland, and from the measurements of Joule—which appeared to show an error in the original measurements of the Committee. As a result the Committee, which had not met since 1870, was reappointed in 1880. In 1881 and 1882 several papers* appeared from the hand of Lord Rayleigh, who made a repetition of the experiments with the original spinning coil used by Maxwell, and also a series of experiments with a new apparatus of the same design. For a third series Lord Rayleigh employed the Lorenz method of measurement. The accounts of these, and a critical account of the principal methods that had been used, aroused intense interest. The result was to show that the B.A. ohm was nearly one and a half per cent. too low, probably owing to an underestimate of the value of the self-inductance of the revolving coil.

In 1881 the first International Congress on Electrical Units was held at Paris; this led up to the adoption of the C.G.S. system of units internationally. At this Conference it was agreed that the term ohm should be given to the resistance of a column of mercury 1 mm.^{2} in section at 0°C., and that an international commission should determine the length of this column. Various determinations were made, and in 1884, when the Congress met again, Mascart presented a table showing a mean length of 106·02 cm. The figure 106 cm. was adopted, and defined the "Legal Ohm" of the Paris Congress.

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* "Proc." Roy. Soc., 1881; "Phil. Trans.," 1882; "Phil. Mag.," 1882.

This was, however, felt to be unsatisfactory; the value 106·0 was not that resulting from the best observations, but represented the mean. In 1890 the Board of Trade Committee on Electrical Standards was established; and basing their decision on the results given in Table III., they adopted the length 106·3 cm. This result was confirmed at the Edinburgh meeting of the British Association in 1892, at which Helmholtz, Lindeck, Kahle, Guillaume, and Carhart were present. Helmholtz made the proposal that in place of specifying the sectional area of the column of mercury, the mass of mercury should be substituted; pointing out that in this way the uncertainty in regard to the precise value of the density of mercury would be eliminated. This suggestion was adopted.

TABLE III.—*Results used in assigning the value of the Ohm. (Edinburgh B.A. Meeting,* 1892.)

No.	Observer.	Date.	Method.	Value of B.A. units in ohms.	Value of ohm in centimetres of mercury.
1	Lord Rayleigh...	1882	Rotating coil...	0·98651	106·31
2	Lord Rayleigh...	1883	Lorenz ...	0·98677	106·27
3	Mascart ...	1884	Induced currents	0·98611	106·33
4	Rowland ...	1887	Mean of several methods	0·98644	106·32
5	Kohlrausch ...	1887	Damping of magnets	0·98660	106·32
6	Glazebrook ...	1882 & 1888	Induced currents	0·98665	106·29
7	Wuilleumeier ...	1890	...	0·98686	106·31
8	Duncan & Wilkes	1890	Lorenz ...	0·98634	106·34
9	Viriamu Jones	1891	Lorenz ...	...	106·31
					106·31 mean

The great superiority of the specification of the International Ohm in terms of a column of mercury, rather than in terms of a coil of wire, may be seen from a comparison of the results of Table IV. with those of Table II. The former Table is taken from figures obtained by F. E. Smith, representing the results of a series of fillings in the reproduction, according to specification, of the International Ohm.

Table IV.—*Values obtained in a series of mercury fillings in reproduction of the International Ohm (by F. E. Smith).*

Glass.	Length at 0°C. mm.	Calibre factor.	Mass of mercury at 0°C.	Calculated resistance in ohms.	Observed resistance in terms of mean of all.	Difference abs.-calc. 1×10^{-5}
Jena ...	1028·879	1·00010$_1$	13·5540$_1$	0·99986$_6$	0·99986$_0$	−0·6
" ...	1058·125	1·00007$_7$	14·3339$_3$	0·99994$_5$	0·99993$_7$	−0·8
" ...	1000·756	1·00011$_6$	12·8274$_9$	0·99956$_7$	0·99957$_9$	+1·2
" ...	929·586	1·00018$_1$	11·0679$_7$	0·99965$_6$	0·99967$_0$	+1·4
" ...	812·187	1·00007$_3$	8·4475$_0$	0·99977$_1$	0·99976$_1$	−1·0
Verre dur	1188·249	1·00002$_6$	18·0776$_2$	0·99976$_3$	0·99976$_9$	+0·3
" ...	1160·500	1·00012$_8$	17·2438$_9$	0·99983$_8$	0·99985$_5$	+1·7
Jena ...	1138·576	1·00002$_9$	16·5965$_0$	0·99986$_1$	0·99985$_8$	−0·3
" ...	800·722	1·00007$_3$	8·2197$_8$	0·99967$_3$	0·99968$_0$	+0·7
" ...	712·299	1·00000$_8$	6·4993$_4$	0·99948$_0$	0·99946$_0$	−2·0
Verre dur	1098·653	1·00007$_1$	15·4645$_7$	0·99916$_5$	0·99917$_1$	+0·6
"	1024·146	1·00007$_0$	13·4291$_3$	0·99987$_9$	0·99986$_5$	−1·4
"	1049·821	1·00010$_0$	14·1116$_4$	0·99984$_5$	0·99985$_6$	+1·1
"	1061·576	1·00013$_3$	14·4321$_4$	0·99967$_6$	0·99967$_3$	−0·3

In 1908 an International Conference was held in London, at which the conclusions of the Edinburgh Conference of 1892 were adopted internationally. This Conference for the first time drew a distinction between Absolute and International Units; the Ohm being 10^9 C.G.S. units, the International Ohm the resistance at 0°C. of a column of mercury of uniform cross-section having a length of 106·300 cm. and a mass of 14·4521 gr. In 1910 this became legalised in this country by Order in Council.

At the B.A. meeting of 1889 the name of Viriamu Jones was added to the Electrical Standards Committee. To him are due important modifications in the design and construction of the Lorenz apparatus, which was by that time generally recognised as offering the greatest promise of extreme accuracy in the absolute determination of the ohm. Jones pointed out the importance of construction on an engineering scale, and the great increase in accuracy that would result from the use of single-layer coils of uncovered wire. The double form of Lorenz apparatus at the National Physical Laboratory was built as a memorial to Viriamu Jones. With this apparatus, designed by Mr. F. E. Smith, and used by him, results of extraordinary accuracy have been attained. A full account of this work will be found in the "Phil. Trans.," Vol. CCXIV. The conclusion of the experiments is that the Ohm is equal to the resistance of a column of mercury, of uniform cross-section equal to that adopted in the specification of the International Ohm, having a length of 106·245±·004 cm., at the temperature of 0°C.

A full description of the earlier work here outlined will be found in the Collected Reports on Electrical Standards, issued by the B.A. Committee in 1912. Reference may also be made to the Fourth Kelvin Lecture ("Proc." Inst. E.E., 1913.).

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2. "*Two Proposals: (a) Respecting the Present System of Electrical Units; and (b) On a Lecture Demonstration of the Absolute Determination of Resistance*," *by* F. E. Smith, *O.B.E., F.R.S.*

My remarks are limited to two proposals, one respecting our present system of units, and the other the need for teachers to demonstrate the ease of measuring electrical resistance in C.G.S. measure.

At the present time electrical quantities can be measured in terms of the centimetre, gramme, and second with an accuracy in excess of all commercial requirements. In the case of resistance, as with other electrical quantities, the unit and standard are distinct. The unit of resistance is the ohm, 10^9 centimetres per second, while the standard of resistance, the international ohm, is the resistance of a carefully defined column of mercury at 0°C. In the past the material standard, mercury, has had an advantage over the unit of resistance, the ohm, inasmuch as the accuracy of reproducing the standard was greater than the accuracy with which the unit could be realised. But this is no longer true. The international ohm can be reproduced with an error of about ±2 parts in 100,000, and this has been done in five countries; but the unit of resistance, 10^9 C.G.S. units, can now be realised with an accuracy equally great. Precise measurements of resistance in absolute measure have not been made in many countries or by many observers, but the following results show the accuracy which has been obtained. It should be noted that only the last two observers claim a probable error less than 1 part in 10,000. It has also to be remembered that the errors in reproducing the international ohm are involved in the differences between the values recorded :—

	Observer.	Ohm in centimetres of mercury.
1882	Rayleigh	106·26 cm.
1882	Glazebrook	106·25 ,,
1883	Rayleigh and Sidgwick	106·24 ,,
1885	Wiedemann	106·16 ,,
1889	Dorn	106·24 ,,
1891	V. Jones	106·31 ,,
1912	Campbell	106·27 ,,
1913	Smith	106·245 ,,
1920	Gruneisen and Giebe	106·246 ,,

At the present time it is practically certain that the ohm (10^9 C.G.S. units) can be, and has been, realised with an error not greater than ±2 parts in 100,000. The question arises : "If the international ohm were displaced, and all measurements were made in terms of the ohm, would there be any practical advantage ?" My answer is, Yes. It is clearly desirable that the same physical quantity measured in different ways should have the same value assigned to it. Now, many quantities can be measured by electrical and by mechanical means. As an example, take the frequency of a tuning-fork. If such a frequency is measured through the medium of electrical quantities, and these quantities are expressed in international measure, the result will differ from the value measured by mechanical means by about 5 parts in 10,000. If, however, the electrical quantities were expressed in terms of the absolute system of units, the results would be identical.

My proposal is, therefore, that all resistances should be expressed in ohms (10^9 cm./sec.), without qualification, such as legal, international, &c. Similarly, with the ampere and volt. It is not possible here to deal fully with the advantages and the possible difficulties of change, but the difficulties will certainly increase with time. At an early date I hope to discuss the question much more fully.

My second proposal I will illustrate by means of an experiment.

Teachers of electricity invariably find it difficult to interest students in absolute measurements, and this I believe is largely due to the poor presentation of the subject in text books and the failure to make measurements in front of students, or to encourage students to make measurements for themselves. I submit that it is easy to measure current in terms of the

C.G.S. unit by means of a simply constructed current balance of two coils, or by the use of one coil and a magnet of known moment. But I have never heard of resistance being measured in C.G.S. units as an experiment for students. I hope to show you that the measurement of resistance in terms of length and time is very easy and very instructive; indeed, every student who knows the principle of the dynamo knows the principle of the rotating coil method of measuring resistance. The apparatus consists of a coil of wire of several hundreds of turns and having a diameter of about 30 cm.; it is convenient to divide the coil into two halves, side by side, so that they may be rotated about a vertical axis and yet enable a magnet to be suspended at their centre without disturbance except by electromagnetic action. The original British Association coil is illustrated in many text books, and a simple apparatus based on this can be constructed very cheaply. The arithmetic of the rotating coil method is very simple. Here it is:—

a=Radius of coil.

A=Area of one turn$=\pi a^2$.

n=Number of turns of wire.

N=Number of revolutions per second.

$\omega=2\pi N$.

θ=Angle the plane of the coil makes with the magnetic meridian. $\theta=\omega t$.

H=Horizontal intensity of earth's magnetic field.

R=Resistance of coil.

Self-inductance is neglected.

At time t the flux through coil$=nAH\sin\omega t$.

Induced E.M.F.=rate of change of flux$=-nAH\omega\cos\omega t$.

i the current at time $t=\dfrac{-nAH\omega\cos\omega t}{R}$.

At time t the magnetic force at right angles to the meridian$=\dfrac{-2\pi n i}{a}\cos\omega t$.

Substituting for i, force$=\dfrac{-2\pi n^2 AH\omega}{2aR}(1+\cos 2\omega t)$.

The first part of this is constant; the other portion is periodic, and may be neglected when the frequency is great.

If φ=angle of deflection of magnet,

$$\tan\varphi=\frac{2\pi n^2AH\omega}{2aRH}=\frac{2\pi^3n^2aN}{R}.$$

$$\therefore\quad R=\frac{2\pi^3n^2aN}{\tan\varphi}.$$

In practice the coil is rotated in the magnetic field of the earth; a current is induced in the coils and the magnet is deflected by the action of the current. The final formula for the resistance of the coil involves the radius of the coil, which is a length, and the number of revolutions per second, which has the dimensions of T^{-1}; *i.e.*, the dimensions of resistance are the same as those of a velocity.

I submit that the theory of the rotating coil method is very simple and the essential parts of the apparatus are easy to make. A measurement for demonstration purposes takes from two to three minutes, and I propose, therefore, that teachers should seriously consider introducing such an experiment to illustrate their lectures.

[Mr. Smith made an absolute measurement of resistance in front of the audience with the original British Association apparatus.]

II.

INSTRUMENTS BASED ON THE TEMPERATURE VARIATION OF RESISTANCE.

Resistance Thermometry, &c.

1. "*Platinum Thermometers, a Compensated Bridge, a Compensated Bolometer, and the Radio-Balance,*" *by Prof.* H. L. Callendar, *LL.D., F.R.S.*

Prof. H. L. Callendar exhibited and explained some platinum thermometers, a compensated bridge, and some instruments for measuring radiation.

(1) *Calorimetric Thermometers.*—A very sensitive platinum thermometer was exhibited, made for Drs. Lea and Gaskell in 1887, for measuring the heat developed by the beating of a frog's heart. This was one of a differential pair, each consisting of 8 ohms of pure platinum wire, a thousandth of an inch in diameter, wound on a very thin plate of mica 3 mm. square, and insulated with thin plates of mica attached on either side with a special cement. This type of thermometer was described and figured in the British Patent No. 14,509, 1887, and proved to be extremely quick and sensitive for delicate calorimetric work at ordinary temperatures. Similar thermometers, of various sizes, with flat coils wound on mica, enclosed in glass, metal or porcelain sheaths, were made for other experiments about the same time, but none of these have survived.

(2) *Compound Thermometers for Linear Scale.*—The experiments with the gas-thermometer (Phil. Trans. A., 1887) showed that the variation of resistance of pure metals with temperature was of a different type to that deduced by Siemens from his experiments with the copper-ball pyrometer. The coefficient of t^2, or the difference-coefficient, being positive for some metals and negative for others, it was possible, as explained in the patent, by combining two metals, to make a compound thermometer giving a linear scale of temperature over an extended range. This device has since proved useful in special cases, but for accurate thermometry it is better to use a single pure metal, such as platinum, for the thermometer, and to reduce the readings to the gas-scale by the difference-formula.

(3) *Compensated Resistance Box.*—The same patent described the compensation of the coils of the resistance box for changes of atmospheric temperature, by means of a series of compensating coils in the opposite arm of the bridge. The box exhibited at the meeting was one made in 1893, and used in the experiments on the variation of the specific heat of water, and in other investigations requiring a high order of precision. The resistance coils were made of platinum-silver, and the compensating coils of platinum, both carefully annealed. Each pair of coils was separately adjusted to a constant difference over the range 10 to 30°C. before being placed in the box. This method of eliminating the correction for variation of box-temperature saves a great deal of trouble in work of precision, and is preferable to assuming a common temperature, or a common temperature-coefficient, for the various coils.

(4) *Compensation of Thermometers for Conduction along the Leads.*—A compensated platinum thermometer was exhibited, which was used at McGill College in 1895 for observing rapid variations of temperature in the cylinder of a steam-engine, in determining the adiabatic index of dry steam, and investigating the phenomena of supersaturation. About a dozen thermometers of this type were made for these experiments.* Each thermometer consisted of two loops of pure platinum wire, 0·001 in. in diameter, fused autogenously to two pairs of thick leads of platinum projecting from a glass-stem, in the end of which they were hermetically sealed. The fine wire was directly exposed to the steam and followed all the variations of temperature with almost incredible quickness. The two loops were connected on opposite sides of the bridge-wire, so that the observation of the balance-point gave the difference of resistance corresponding to the central portion of the longer loop, automatically corrected for end-effects due to conduction along the thick leads. Without this provision it would have been impossible to obtain satisfactory observations with the temperature varying 200 deg. or 300 deg. in each cycle at the rate of 100 revs. per min. The temperature at any point of the cycle was observed by setting a periodic contact to close the circuit of the galvanometer at the desired point. The advantage

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* The fundamental interval of each of these thermometers was adjusted within 2 or 3 parts in 1,000 to 2 ohms, a fine adjustment being obtained by slightly stretching either one loop or the other as required.

of the method for measuring the adiabatic index of steam was that errors due to leakage or condensation on the walls did not affect the relation between pressure and temperature in the middle of the cylinder, whereas they produced marked distortion in the pressure-volume relation shown on the indicator card.

The same method of compensation for end-effects was employed for many similar purposes, *e.g.*, in investigating the heat-losses from heated platinum wires, the melting-point of platinum and other substances with a resistance meldometer, and the variation of the viscosity of gases by the transpiration method at high temperatures. (Proc. R.I., 1899.)

(5) *Compensated Bolometer.*—An important application of the same principle was made in the construction of a bolometer for the absolute measurement of radiation by Kurlbaum's method. In this method the rise of temperature of the grid when exposed to radiation is first observed. The radiation is then cut off, and the increase of current required to produce the same rise of temperature is measured. As applied by Kurlbaum, this method gave a result for the constant of radiation about 7 or 8 per cent. too low. It appeared probable that the error was to be attributed to a difference in the end-effects in the two cases. A compensated bolometer was accordingly constructed with two similar grids of different lengths, used differentially, as described and exhibited at a meeting of the Physical Society in May, 1905. The area exposed to radiation was defined by a rectangular aperture corresponding to the difference in length of the grids. This gave a result free from end-effects, agreeing closely with recent measurements of the constant, and was confirmed by the entirely independent method of the radiobalance.

(6) *The Radiobalance.*—The defects of the bolometric method for the absolute measurement of radiation are, (1) that the electric compensation is not simultaneous, but requires successive observations; (2) that the absorption coefficient of the grid is somewhat uncertain and difficult to determine. In the radiobalance, these objections are met, (1) by simultaneous compensation of the radiation by absorption due to the Peltier effect in a thermo-junction through which a measured current is passed; (2) by using a cup to receive the radiation admitted through a small circular aperture in such a way that the absorption is practically

complete. Four types of radiobalance were exhibited: (1) a laboratory instrument, with water or steam-jackets, for investigating the variation of the coefficients over large ranges of temperature; (2) an instrument with an equatorial mounting for convenience in observing solar radiation; (3) a similar instrument with an altazimuth mounting, which is more convenient for laboratory work, but can be used on the sun without difficulty; (4) a radiobalance in a simple block mounting, which could be used either vertically or horizontally, and was specially adapted for delicate calorimetric experiments, such as the direct measurement of the Peltier effect, or of the heat-emission of small quantities of radium. This instrument was employed at the meeting to demonstrate the heat-emission from a bulb containing about 1·8 mgm. of radium. There is no difficulty in measuring the heat-emission, by balancing it with the heat-absorption due to the Peltier effect, to a tenth of a microwatt, or better than one in a thousand, for a single milligram of radium.

When tested on radiation measurement, all the radiobalances gave results consistent with each other to a similar order of accuracy, which is about the limit to which a given intensity of radiation can be reproduced, either with a focus-lamp, or from a black body electrically heated. The statement made by W. W. Coblentz (Bull. U.S.A. Bureau, 1913) that the method was of no value for absolute measurement, and that two of the results given by the author (Proc. Phys. Soc., XXIII., Dec., 1910) differed by 12 per cent., was very misleading, because the two results quoted referred to separate tests with different focus-lamps at different distances, and were in no way comparable. Unfortunately, the Bureau omitted to send the author a copy of the Bulletin containing this criticism, so that he was unable to correct the misstatement at the time.

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2. "*Early Work on the Resistance Pyrometer*," *by* C. R. DARLING, *F.I.C.*

I DESIRE to refer briefly to the early work of Sir William Siemens, who was the first to measure temperatures by the resistance method. Siemens constructed a simple resistance thermometer of copper wire in 1860, which was used for measuring the rise of temperature in the interior of cables, and described the instrument before the British Association at Manchester in 1861. Subsequently, with a view to extending the method to the measurement of furnace temperatures, Siemens investigated the change in the resistance of platinum with temperature, using mercury thermometers as standards up to 350°C., and a calorimetric pyrometer for higher temperatures. He concluded that the relation between temperature and resistance could be expressed by a parabolic formula, in which zero resistance corresponded with the absolute zero of temperature. A practical form of pyrometer, based on the data thus obtained, was made in 1871, and used to a small extent industrially. The coil of platinum was wound on a pipeclay former, and a third lead was used to compensate for variations in the resistance of the ordinary leads due to varying depth of immersion. Experience in the use of this pyrometer showed that the coil should be adequately protected from furnace gases, as otherwise a serious alteration in the zero resistance occurred. Resistance pyrometers for commercial use have been made by the firm of Siemens & Co. since 1871, three forms of indicating apparatus having been successively used, viz.: (1) A differential voltameter, in which the resistance of the pyrometer coil was deduced from the volumes of gas liberated in two tubes containing dilute H_2SO_4, and each forming part of a differential circuit; (2) an ordinary differential galvanometer circuit, in which the pyrometer coil was balanced against an adjustable resistance; and (3) a Wheatstone bridge arrangement, with direct temperature readings. Of these, the last-named is the only one in use at the present time. The pipeclay former is still employed, and the coil is surrounded by magnesia to prevent access of furnace gases.

(Examples of the three types of indicator mentioned above, and a coil wound on pipeclay, were exhibited and explained by Mr. Darling.)

3. "*Resistance Thermometers for Steam Temperatures*," *by* C. Jakeman (*National Physical Laboratory*).

In connection with experiments upon the properties of steam under pressure it was found convenient to use platinum resistance thermometers with envelopes of steel tube arranged so that the metal envelope could be screwed into a special fitting in the steam pipe. These thermometers were of 1 ohm. "fundamental interval" and the steel envelopes were about 10 mm. in diameter. For a large range of work these thermometers proved to be somewhat deficient because there was considerable time lag in taking up the temperature owing to the mass of the metal in the envelope and its high conductivity, and the size of the thermometers prevented their use in small pieces of apparatus.

An attempt was made to construct a thermometer of similar form and range but with much reduced heat capacity and overall diameter. For the work in hand a temperature of 400°C. was the highest required, and that only occasionally, but at the same time it was necessary to make the thermometer so that its sulphur-point could be determined for calibration. The envelope was constructed of thin brass tube 3 mm. external diameter closed at the lower end with a brass plug silver-soldered in place. The internal diameter of the tube was a little less than 2·5 mm., and the provision of a platinum coil of usual length and diameter would give a bulb of inordinate length. The diameter of the platinum wire used was 0·025mm. and a simple loop containing 1·7 cm. of wire is all that is necessary to obtain a fundamental interval of 1 ohm. The insulation of the leads in the tube was provided by four glass capillary tubes each fitting over one of the four leads. The leads were of copper 0·25 mm. in diameter and the joint between the thermometer leads and the loop of platinum wire was made by autogenously welding the two metals together in a very small oxy-coal gas flame. This requires some care, but with practice a perfect joint can be made with a very small ball of metal at the joint. These joints were then drawn back into the capillary tubes and the glass closed round the metal ball, thus preventing the wires sliding in the glass tubes. (The joint between the copper lead and platinum loop can be made by silver soldering; but it is difficult to make the joint small enough to enter the capillary in this case.) The head was of ordinary construction, the resistance of the pair of copper compensating leads being made equal to that of the pair of thermometer leads and the compensating and thermometer flexible leads being similarly balanced. The insulation thus provided is satisfactory, but there is still considerable space in the tube which allows convection currents to form in the air enclosed. These were prevented by filling the lower part of the envelope with flakes of mica.

The thermometer proved to take up the surrounding temperature extremely rapidly, being better than a mercury in glass thermometer, while the diameter of the envelope enables it to be used in any place in which there is room for a small size mercury in glass thermometer.

Some of these thermometers have been in use in the engineering laboratory of the National Physical Laboratory for some 10 years and have proved very satisfactory. Owing to the method of construction of the loop it has not been found possible to adjust the resistance with any accuracy and the welding of the joints is a somewhat uncertain process, so that the fundamental interval may be ±20 per cent. of that required; and if direct readings are required a specially calibrated bridge must be used. Owing to the small size of the wire the current density must be small and it is usual to use 0·002 amperes. If the current is more than this heating of the loop is apparent in a very few seconds. With a Broca galvanometer of moderate sensitivity, readings can be taken to 0·01°C., which is sufficient for most experimental work.

One of these thermometers was exhibited, and also a model in glass showing the insulation of the leads and the loop resistance

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4. "*An Electric Transmitting Radiator Thermometer*," by EDGAR A. GRIFFITHS.

THE instrument described below may be of interest as it is commonly supposed that resistance thermometers find no application on the modern aeroplane.

This outfit was captured during the war and formed part of the equipment of a bombing machine.

As no published description of the instrument could be found it was taken apart and carefully investigated.

The outfit is used for measuring the temperature of the water in the engine radiators, and two pyrometers were connected to one indicator fixed on the instrument board.

The indicator operates on the same principle as the ohmmeter. The resistance thermometer is made of platinum wire, wound on a strip of micanite. The coil is enclosed in a flat copper tube, forming part of a brass elbow which screws into the radiator. The resistance of the thermometer coil at zero centigrade is about 90 ohms; this high resistance is desirable in view of the fact that no compensation is applied for the change in resistance of the connecting leads and the variation in these leads would be important with a low resistance thermometer. A push-button switch forms part of the equipment, so that the current is only used during the actual observation.

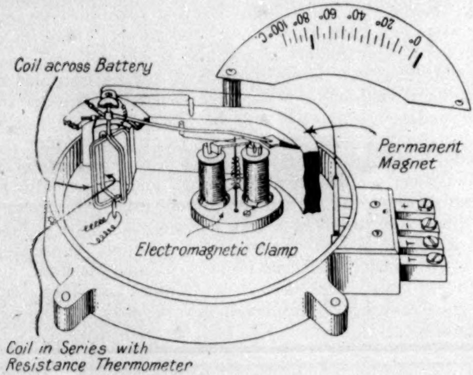

FIG. 1.—VIEW OF INSTRUMENT.

The indicator (Fig. 1) is a miniature form of the well-known simple ohmmeter; it is based on the moving coil principle. The two coils are wound crosswise on the same aluminium frame, and move freely in a powerful magnetic field. One coil is connected through a resistance across the battery (*see* Fig. 2), and circuit connections may be traced from the battery by conductors 1 and 10, resistance 11, conductor 12, coil 13, conductor 7, resistance 8, and conductor 9 to the battery. The other coil is connected in series with the resistance thermometer. Circuit connections may be traced from the battery by conductors 1 and 2, resistance 3, conductor 4, through the thermometer, by conductor 5, coil 6, conductor 7, resistance 8, and conductor 9 to the battery.

The indication is thus the ratio $\frac{\text{electromotive force}}{\text{current}}$, *i.e.*, the resistance of the thermometer circuit. The reading is theoretically independent of the pressure employed, but actually the E.M.F. must be kept within limits: otherwise, if the E.M.F. is too low, friction decreases the sensitivity, and if too high, the current causes appreciable heating up of the resistance thermometer above its surroundings.

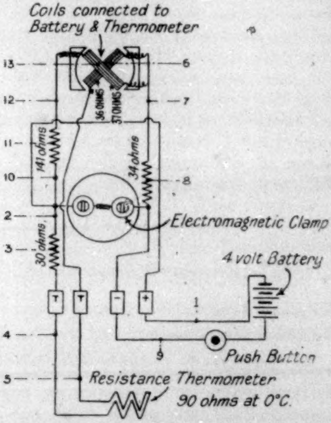
Coils connected to Battery & Thermometer
13
12
11
10
2
3
4
5
6
7
8
36 OHMS
37 OHMS
141 ohms
34 ohms
30 ohms
Electromagnetic Clamp
4 volt Battery
T
T
-
+
1
9
Push Button
Resistance Thermometer
90 ohms at 0°C.

FIG. 2.—DIAGRAM OF CONNECTIONS.

The same rule should, in fact, be used here as in the case of the laboratory platinum thermometer; the current should always have the same value as that used in the calibration of the instrument.

It will be evident from the principle of this instrument that, owing to the absence of any control spring, the pointer always remains at a position somewhere away from the zero, depending on the last reading taken: this apparent reading would prove somewhat deceptive and therefore requires to be eliminated by some form of clamp for setting the pointer back to zero. The clamp in this particular instrument consists of a small arm, pivoted about a vertical axis, and arranged to hold the pointer at zero by the action of a spring. When the current is switched on, the arm is immediately swung round to the top of the scale by a small electro-magnet connected across the battery, and remains fixed while the reading is taken. As soon as the current is switched off again the arm is released and returns the pointer to zero.

This device will be clear from Fig. 2. The scale of the instrument is very cramped, owing chiefly to the large angle subtended by the crossed coils (60 degrees in this case). The graduation is, moreover, not uniform, but closed up in the central part of the scale, making the calibration somewhat troublesome. It would, of course, be an improvement to use a smaller range of temperature for the purpose of controlling the temperature of the water in the radiator (instead of 0° C. to 100° C., as in this instrument).

The Hot-Wire Microphone.

"*Experiments Illustrating the Function of Convection Currents in Determining the Sensitivity of the Hot-Wire Microphone,*" *by Major* W. S. TUCKER, *R.E., D.Sc.*

THE hot-wire microphone is essentially an instrument for recording alternating air currents by its change in electrical resistance, and a full description of it has been given in a paper on "The Selective Hot-Wire Microphone" read before the Royal Society on January 20 of this year.

It has been found by experiment that when a hot-wire grid is inserted in a diaphragm so that it intercepts progressive sound waves, it is too insensitive to give satisfactory electrical indications unless the sound is very intense. For this purpose, however, the air must be very still, as the smallest pressure difference on the two sides of the diaphragm causes air currents which completely spoil the effect.

If, however, the rear side of the diaphragm be a closed cavity, and if, furthermore, the diaphragm aperture and the cavity together be made to constitute a Helmholtz resonator, the new arrangement serves not only to protect the wire from draughts, but if correctly tuned, will give considerable magnification of the sound received. The tuning of this resonator is effected by alteration of orifice or volume of the cavity as expressed by the well-known relation—

$$n = \frac{a}{2\pi}\sqrt{\frac{c}{S}}$$

where a is the velocity of sound, S the volume of the cavity and c a quantity called by Rayleigh the "conductivity of the orifice."

The resonance curve for the completed microphone, consisting of Helmholtz resonator and hot platinum grid, is fairly sharp, depending on the nature of the aperture and the material of the resonator.

The resonators of the microphones hitherto employed give a magnification of from 50 to 100.

The effect of convection currents on the sensitivity of the hot wire for responding to oscillatory air currents can best be illustrated by a study of what may be termed the "characteristic curve" of a hot wire.

This curve gives the relation between resistance and convection current for a hot wire in which the electric current is kept constant.

Such a curve was obtained by Capt. Paris, employing air currents of velocity less than 6 cm. per second and with a wire of diameter 0·0006 cm.

For observations of alternating air currents the above quantities are most suitable, but as they employ different conditions to those employed by L. V. King[*] or J. S. G. Thomas,[†] it was thought desirable to take new measurements.

The current of air in the orifice was controlled by an aspirator with suitable stop-cock, and the velocity of the imposed current, which is the average over the microphone aperture, could be calculated. The curve shown, which is already contained in the Paper above mentioned, shows a typical "characteristic curve." The positive values of V, the air velocity, oppose in direction the convection due to the wire.

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[*] L. V. King, "Phil. Trans.," A. Vol. CCXIV., pp. 373-432.

[†] J. S. G. Thomas, "Phil. Mag.," Vol. XXXIX., pp. 505-534.

At Q the temperature of the wire indicated by its resistance is the same as at O, so that the value OQ is twice the velocity of the convection current. At N when $ON=\frac{1}{2}OQ$ the temperature acquires its maximum at a point where the air in the orifice is at rest. As V extends beyond the limits of O and Q the curve is extended and proves to be very nearly parabolic for small values of V.

The sensitivity of the hot wire for a given sound is measured by $\frac{\partial(\delta R)}{\partial V}$, and it is obvious that this increases approximately with the velocity of flow. This can be shown by experiment. It involves opening the walls of the Helmholtz cavity to admit a current of air. Such an aperture alters the pitch of the resonator and the volume must be adjusted to correct for it.

Since the resonance curve for the resonator is so sharp, it is essential that a preliminary test should be made to find out if the pitch of the resonator is altered by passage of the air current. No such alteration was observed.

If now an air current be passed through the resonator while it is responding to a steady source of sound, variation of sensitivity is shown in agreement with the characteristic curve.

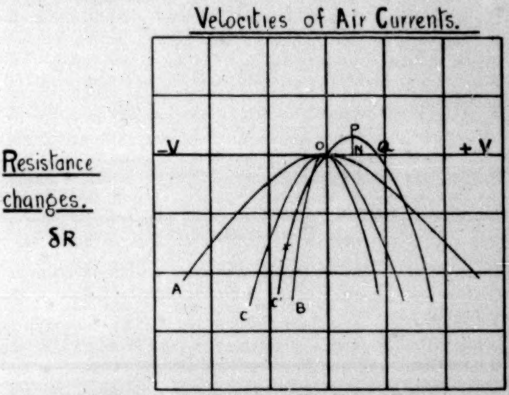
There is, however, a limit to the magnitude of the superimposed air current, so that magnification cannot be produced indefinitely. This may be demonstrated by passing the microphone current through an amplifier, such as that employed by the Army. If the effect be observed by a telephone the smooth sound of the tuning fork source, which may be employed, becomes broken up and gives a crackling sound. This effect is probably due to the production of swirls round the hot wire, and it is noticeable that these swirls are produced by air currents which are comparatively small.

The amount of magnification obtainable may be measured by a vibration galvanometer tuned up to the frequency of the tuning fork, and it is then seen that the deflection actually diminishes again after the superimposed current has attained a critical value. This may be due to the fact that the swirls in the region of the wire protect it from the vibratory motion of the air in the orifice.

The manner in which the hot-wire interprets the alternating air current movements depends on its condition as represented by a point on the characteristic curve. Thus, at

R an effect of the same periodicity is observed. At *P* the hot wire acts as a " rectifier," giving only a cooling effect whether the velocities of the vibrating air particles be positive or negative. It should be noted that the fluctuation in microphone current has twice the periodicity of the sound which produces it, so that, if the conditions of the hot-wire be indicated by *P* the octave will be reproduced.

If the condition of the hot-wire be represented by a point near *P* the sound reproduced will contain the fundamental and the octave together when the intensity of the sound is sufficiently large.



A—Coarse wire. B—Fine wire. CC′—Medium Wire.

Curve *C* in the diagram would be obtained for a microphone wire in the horizontal plane, and the alternating air currents which it measures have displacements in the same direction as the convection currents.

If, however, the resonator be gradually rotated until the hot wire is brought into a vertical plane, it is only the resolved part of this convection current which is effective in increasing the sensitivity and the curve gradually passes from condition C_1 to *C*. With no superimposed convection current, therefore, the octave of the original note is reproduced and not the fundamental.

"Characteristics" for wires of different diameters are difficult to compare. It should be laid down that the maximum temperatures and the total resistance at these maximum temperatures should be the same and that they should be similarly mounted in the same orifice.

Characteristic curves will always be symmetrical about the axis of resistance, and it can be shown that the curvature for coarse wires will be less than for fine ones. (Curves *A* and *B*.)

It is expected, therefore, that variation of the superimposed convection current will produce less effect on the coarser than the finer wires.

No attempt is made to compare by means of these curves the sensitivities of the different wires for a given sound under the same convection conditions, since account has to be taken of the thermal capacities and thermal lag of the wires. This is a much more difficult problem, which is now receiving consideration.

Hot Wire Anemometers.

1. *"A Hot Wire Anemometer," by Prof.* J. T. McGregor Morris, *M.I.E.E.*

Prof. Morris demonstrated the action of the instrument devised by him, an account of which may be found in the following Papers:—

(*a*) "The Electrical Measurement of Wind Velocity." Dundee B.A. "Engineering," Dec. 27, 1912.

(*b*) "The Distribution of Wind Velocity in the Space Surrounding a Circular Rod in a Uniform Current of Air." "Engineering," Aug. 8, 1913.

(*c*) "A Portable Direct-Reading Anemometer for the Measurement of Ventilation in Coal Mines." Cardiff B.A. "Electrician," Aug. 27, 1920.

2. "*The Directional Hot-Wire Anemometer,*" *by* J. S. G. THOMAS, *D.Sc.* (*Senior Physicist, South Metropolitan Gas Company, London*).

THE laws of convection of heat from fine heated wires are such as to indicate the hot-wire type of anemometer as more especially the type of instrument for the precise measurement of the velocity of a slow moving stream of fluid. In the region of low velocities, difficulties arise in the application of the types of hot-wire instruments devised by King* and Morris† respectively, owing to the disturbing influence upon the convection effect due to the stream, produced by the free convection current arising from the heated wire exposed to the cooling action of the stream. The effect of such free convection current in the region of low impressed velocities of the stream is readily seen from Fig. 1, in which are given the respective forms of the calibration curves obtained by employing a hot-wire anemometer of the Morris type in the investigation of the flow of air in a brass pipe of diameter 2·04 cms. The hot wire was approximately 0·1 mm. diameter, and the heating current 1 000 ampere. The calibration curves were determined for both upward and downward flows of the impressed air stream, and in each case calibrations were made with the shielded and unshielded wires respectively leading. It will be seen from the figure that the calibration curves for velocities greater than about 5 cms. per second were slightly different according as the shielded or unshielded wire was the first to meet the flow. This result was probably due to turbulent conditions of flow produced when the stream was first incident upon the unshielded wire. It is not proposed to discuss this matter further here, but it may be remarked that the results indicate the power of the hot-wire instrument in the investigation of turbulent conditions of flow—a matter to which attention has been drawn by King.‡

It will be seen from Fig. 1 that whereas the respective calibration curves for upward and downward flow of the impressed air stream, with the same wire leading in each case, are identical for impressed velocities greater than about 20 cms. per second, a marked difference in the calibration curves exists for velocities below this critical value in the present instance. This is most clearly seen by a consideration of the portions of the respective calibration curves corresponding to impressed velocities between 0 and 6 cms. per second. Whereas in the case of an upward flow, the deflection is positive and increases with increase of the impressed velocity between these limits, the deflection in the case of the downward flow is initially negative and increases numerically to a maximum value, thereafter diminishing to its initial zero value. Employing the instrument for the investigation of the velocities of stream at various inclinations to the horizontal

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* "Phil. Trans." Roy. Soc., A.520, 214, pp. 373-432 (1914). Eng. Pat. 18,563/1914.

† British Association, Dundee, 1912. "Electrician," Oct. 4, 1912, 1056. Eng. Pat. 25,923/1913.

‡ "Phil. Trans.," *loc. cit.* p. 407.

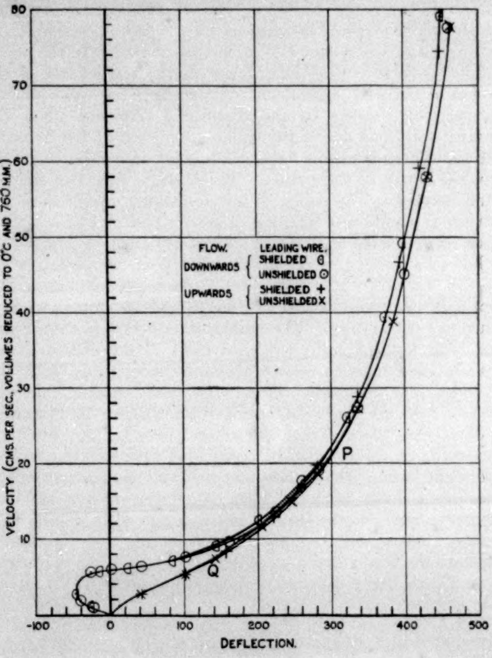

FIG. 1.

using the heated wire in a horizontal position throughout, various forms of the calibration curve may be obtained in the region of low velocities, such forms being enclosed within the two corresponding to upward and downward flow shown in Fig. 1. Various other peculiarities of the calibration curves for the hot-wire type of instrument referred to have been investigated by the author, and a type of directional instrument devised in which the difficulties originating in the free convection current have been almost entirely eliminated. In the directional type of instrument, two fine platinum wires are mounted parallel, and one behind the other in close juxtaposition, transversely to the direction of flow of the gas in the pipe or channel. The wires are inserted in a Wheatstone bridge in the usual manner, and the indications of the instrument enable both the magnitude and direction of a slow-moving stream of fluid to be readily ascertained. The characteristics of such a type of hot-wire instrument, and the influence of the free convection-current upon the indications of the hot-wire amemometer have been discussed by the author in the following series of Papers :—

(1) "Hot Wire Anemometry: Its Principles and Applications." Journ. Soc. Chem. Ind., 1918, XXXVII., 165T-170T.

(2) "The Hot Wire Anemometer: Its Application to the Investigation of the Velocity of Gases in Pipes." "Phil. Mag.," 1920, XXXIX., 505-534.

(3) "A Directional Hot Wire Anemometer of High Sensitivity, especially applicable to the Investigation of Low Rates of Flow of Gases." Proc. Phys. Soc., 1920, XXXII. (III.), 196-207.

(4) "The Directional Hot Wire Anemometer: Its Sensitivity and Range of Application." Phil. Mag., 1920, XL., 640-665.

(5) "The Thermal Effect produced by a Slow Current of Air flowing past a Series of Fine Heated Platinum Wires, and Its Application to the Construction of Hot Wire Anemometers of High Sensitivity, especially applicable to the Investigation of Slow Rates of Flow of Gases." Phil. Mag., 1921, XLI., 240-258.

(6) "A Null-Deflection Constant-Current Type of Hot Wire Anemometer for Use in the Determination of Slow Rates of Flow of Gases, together with an Investigation of the Effect of the Free Convection Current upon such Determinations." Phil. Mag., 1921.

A type of hot wire anemometer utilising the effect referred to in the fifth of the series of Papers given above has been described by H. Gerdien and R. Holm,* who likewise describe an obstructive device ("stauscheibe") for insertion in the flow tube, whereby the instrument may be employed for the measurement of large velocities of gas streams.

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* "Wiss. Veröffentlichungen aus dem Siemens-Konzern," Bd. I., Heft I., pp. 107-121 (1920).

3. "*An Instrument for Use in Measuring Convected Heat*," *by* ALFRED H. DAVIS, *B.Sc.* (*National Physical Laboratory*).

INTRODUCTION.

HEAT losses by convection depend not only on the area of the surface of the body, but also on its form and orientation. If the theoretical calculation were possible in a few ideal cases, in actual practice it would still often be necessary to determine the heat loss by other means. Even in ideal cases the laws of convection are not yet accurately known, and it is desirable to ascertain the circumstances of air temperature and velocity in the vicinity of the cooling body. In this connection the present apparatus was devised to trace convection losses in the air itself.

If air, of density ρ and temperature θ, flowing with velocity v, was originally at 0°C., then the air has been supplied with heat which is passing a section normal to the stream at the rate of

$$v\rho s\theta \text{ cals. per sq. cm. per sec.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

s being the known specific heat at constant pressure. The instrument to be described is a hot-wire anemometer measuring $v\rho$,* independent of the temperature of the air, and adapted for use as a resistance thermometer to obtain θ. Thus it determines the rate at which convected heat is passing a given point.

It was constructed in two types, one being a null instrument, the other being direct reading, and so more convenient, but involving an approximation which somewhat restricts its range.

PRELIMINARY THEORY OF HOT-WIRE ANEMOMETER.

King,† in comprehensive experiments with platinum wires, verified his theoretical formula of the form

$$H/\theta = B\sqrt{(v\rho)} + C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

where H=heat loss per unit length of wire.

θ=*Excess* of wire temperature over air stream temperature.

v=velocity of air.

ρ=density of air.

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* In calibrating the instruments, mass flow $v\rho$ was expressed in terms of the velocity v_{17} at which air at 17°C. would have to move to give the same mass flow.

Thus, $v\rho = v_{17}\rho_{17}$; and $v\rho s\theta = 0{\cdot}000289\ (v_{17}\theta)$ cals./cm.^{2}/sec. . . (1A)

† King, "Phil. Trans.," 214, p. 373, 1914.

B and *C* being constant for a given wire and temperature excess, and having but a small coefficient of change with temperature excess.

In his experiments, although the temperature of the wire was altered, that of the ambient air was always about 17°C.; but theoretically there is no reason for appreciable departure from the law through change of actual air temperature, for temperature *excess* is the essential factor. The null instrument was arranged to work at a constant temperature excess, and by experiment in a wind channel to which heated air was supplied, it was found that in these circumstances the relation between H and $v\rho$ was independent of the actual temperature of the air.

DESCRIPTION OF THE NULL INSTRUMENT.

Construction.

The instrument consists essentially of a Wheatstone's net, as is seen from Fig. 1. In the fourth arm of the net *TT* is

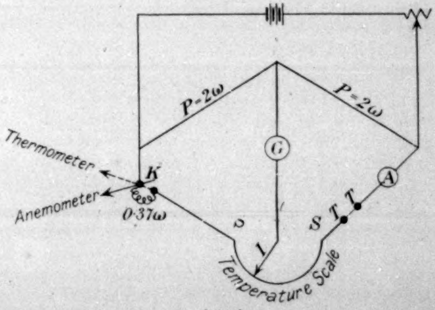

FIG 1.

the 6 mil platinum wire, which is placed in the air stream. An ammeter, *A*, gives the current through the wire. Manganin is the material of the rest of the bridge, as this has no temperature coefficient. *SS* is a slide wire. The third arm of the net consists of two coils, one of which can be short-circuited as desired by the key *K*, which carries an indicating pointer. For convenience, this pointer also controls the sensitivity of the galvanometer, and further makes about a hundredfold difference to the current in the bridge. The connections, however, are not shown.

Method of Use and Theory of Null Instrument.

Setting the indicator at *Thermometer* closes the key K and passes but a small current through the bridge. The instrument is now a resistance thermometer, the temperature of the air being read directly on the bridge wire scale from the position of the pointer I for balance. The slider is fixed in this position.

Setting the indicator at *Anemometer* now passes a larger current through the bridge and opens the key K, thus increasing the resistance of the arm by 0·37 ohms. Balance can only be restored by heating the platinum wire to a definite temperature excess (108°C.), independent of the initial temperature of the air.* The requisite heating current i, which restores balance, is read from the ammeter. If R is the resistance of the wire at its working temperature, we have, using (2),

$$i^2R/\theta=H/\theta=B\sqrt{(v\rho)}+C \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or, since θ is constant,†

$$i^2R=F(v\rho). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus i^2R measures the mass flow of air, and noting that the position of the pointer I already determined along the slide wire gives R, we see that i^2R is readily obtained from one setting of a slide rule, and the mass flow is then known.

Even more conveniently, a three-scale nomogram may be readily constructed in which a thread stretched between the value θ on one scale and i on another cuts the third scale at a point from which the air velocity is read off directly.

Calibration of the Instrument.

As thermometer the instrument was calibrated by direct determination of the ice point and of the steam point, and checked from a knowledge of the constants of the bridge wire and of the platinum wire.

As anemometer it was calibrated in wind channels. For use in measuring convection currents the calibration should extend as far as possible into the region of low speed. Also, when the air speeds become so low that they are comparable with the speed of the natural convection current from the anemometer wire, the instrument should be calibrated under such conditions that the air velocity is in the same direction relative to the vertical, as in the experiment in which the instrument is to be employed.

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* This is exactly true when the resistance-temperature law is linear.

† Rigorous constancy of θ is not essential, for if θ depends on the initial temperature of the wire, the position of the pointer I determines R/θ, and (3) is used instead of (4) without complication.

The actual instrument worked at 108°C. temperature excess, but some preliminary work was done at air temperatures with an anemometer working at nearly three times this excess, so that the natural convection currents should be more pronounced. In this, instead of using vertically upward currents, the instrument was moved downwards through still air by a modification of Attwood's machine. The calibration was found to agree with that for horizontal currents in a wind channel, except, perhaps, for speeds lower than 10 cm. per second, and the relation (2) held down to 15 cm. per second, needing slight correction below.

The actual instrument should be reliable at least to these limits, and it was calibrated for various temperatures of air stream in a specially constructed vertical channel, and for room temperatures it was also calibrated in the horizontal 3 ft. wind channel of the National Physical Laboratory. In the horizontal channel the usual laboratory calibration was used for the higher speeds; however, since the pressure head measurements become insufficiently accurate for speeds less than 100 cm. per second, the lower speeds were obtained by observing smoke puffs through a window, and timing them with a stop watch over a distance of about 1 metre.

The special vertical channel is shown diagrammatically in Fig. 2. The entrance contained layers of wire gauze, and a honeycomb grid to break up eddies and to ensure an evenly distributed and vertical flow of air. The speeds being too slow for direct measurement with the Pitot tube, this was inserted in a subsequent portion of the channel of smaller diameter, reached by a section of approximate streamline shape. For higher temperatures over a range of about 50°C., the air entering this channel was heated by the heater grid. The distribution of velocity across the neck was determined by the Pitot tube, and found to be independent of the temperature of the air stream. It was found that the calibration for ordinary temperatures in the horizontal channel agreed with those for the vertical channel and the hotter air, up to the temperatures reached. The results are shown graphically in Fig. 3.

The relation (2) holds down to 9 cm. per second, and is not found to depend on the temperature of the ambient air.

THE DIRECT READING INSTRUMENT.

Direct-reading instruments have already been used,* in which, for ordinary temperatures, the out-of-balance deflection measures the mass flow of air. In the instrument here described it has been found possible to allow for the effect of temperature very approximately by choosing for each temperature an appropriate current through anemometer wire.† The instrument differs from Fig. 1 only in the absence of the 0·37 ohm coil and its key K. The indicator which carried K

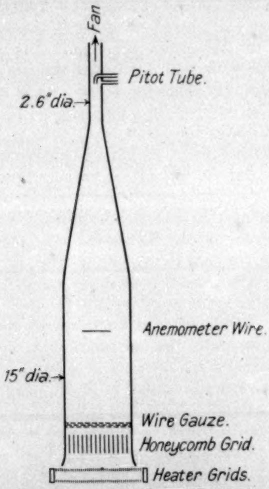

FIG. 2.—VERTICAL HOT-AIR CHANNEL.

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* Morris, " Electrician," October 4th, p. 1056, 1912.

† Since this Paper was first written, Morris (" Electrician," August 27, 1920) has described an instrument for measuring air velocity in mines, in which the conversion of mass velocity to actual velocity is effected for each temperature by appropriate setting of the current. The present Paper shows that such adjustment is needed even to get mass velocities.

remains, retaining its other functions, altering the current in the bridge and the sensitivity of the galvanometer. A simple key enables one millivoltmeter to be used either as A or as G, as required.

Setting the indicator at *Thermometer* sends a very small current through the wire. As before, the air temperature is read directly from the balance position of the slider on the slide wire. At the same time a second scale shows the anemometer current appropriate to the temperature.

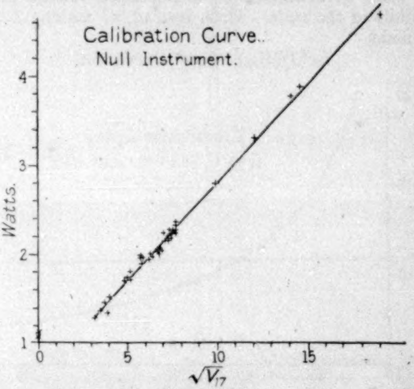


Fig. 3.

Setting the indicator at *Anemometer* sends a heating current through the wire, and adjusting this to the appropriate value, the mass velocity of the air is read directly from the galvanometer scale. The reason for this is clear from the theory of the instrument in the anemometer position.

Let P, P, R, and $(X+L_\theta)$ be respectively the resistances of the four arms of the bridge and let L_0, L_t, L_θ be respectively the resistance of the wire at 0°C., at air temperature t°C., and when heated by the current c to θ°C. above air temperature. From the thermometer setting $R=X+L_t$. Then if g be the current through the galvanometer of *high* resistance G, we have $g/c=L_0a\theta/2G,\ L_\theta=L_t+L_0a\theta.\ c^2L_\theta=\theta.f(v\rho),$ whence $(2G/L_0a)g=L_tc^3/[f(v\rho)-L_0ac^2].$

It will be seen that if c be chosen so that L_tc^3 is constant, and if $[f(v\rho)-L_0ac^2]$ does not vary appreciably for the necessary variations of c, then the galvanometer reading is a function of the mass velocity only. The appropriate current c determined by L_t, depends on the temperature of the air stream only. For a range of 100°C.,only 12 per cent. variation in c is necessary to keep L_tc^3 constant. The slight effect of this variation on $[f(v\rho)-L_0ac^2]$ may be approximately allowed for by choosing the current appropriate to L_t such that the instruments reads quite correct for one standard velocity $(V_s\rho)$ in the middle of the scale. Thus, instead, of making L_tc^3 constant, make

$$L_tc^3/[f(V_s\rho)-L_0ac^2]=\text{constant}.$$

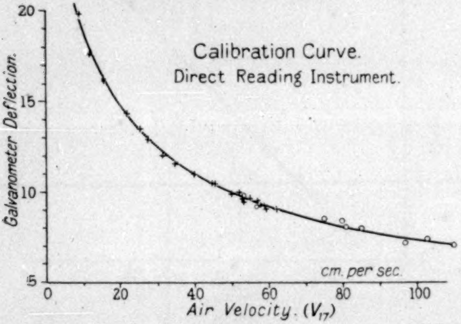


FIG. 4.

Calculations given in Table I. show for two air temperatures, 17°C. and 100°C., the appropriate currents and the galvanometer deflections produced by them for a range of air velocities.

TABLE I.

Velocity.	Galvanometer deflection.	
v_{17} cm./sec.	17°C. $c=1{\cdot}00$	100°C. $c=0{\cdot}929$
9·0	20·1	19·6
42·1	10·7	10·7
100	7·3	7·3

The anemometer current varies but little over the range of temperature and the instrument is quite satisfactory.

The actual instrument was calibrated in the wind channels by means of the previously calibrated null instrument. The results are shown graphically in Fig. 4, where it is seen that the form of the curve agrees excellently with that calculated, and that no dependence on the temperature of the air is revealed. Finally, the scale of equal divisions was removed from the galvanometer, and a direct velocity scale engraved.

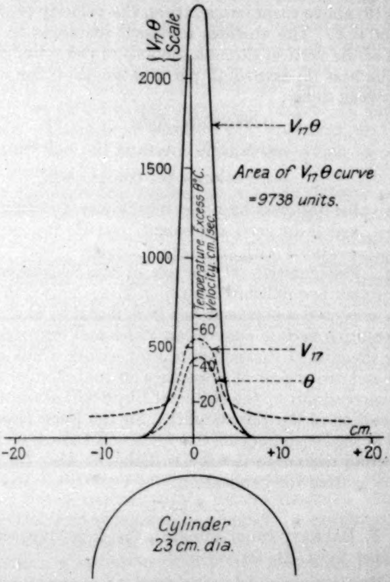


FIG. 5.—CONVECTION CURRENT ABOVE HORIZONTAL CYLINDER.

EXAMPLE OF MEASUREMENT OF CONVECTION.

The heat loss by convection from unit length of a horizontal cylinder was measured by means of the instrument and checked from a knowledge of the actual watts consumed, after allowing for radiation. Keeping the exploring wire always parallel to the axis of the cylinder, the distributions of velocity and temperature were obtained for the air passing vertically upwards through a horizontal plane about 10 cm. above the cylinder.

In Fig. 5 the curves show for the air stream the temperature excess (θ) above room temperature, the velocity (v_{17}), and the product $v_{17}\theta$. The abscissæ represent distances to the right or left of the vertical through the axis of the cylinder. Using (1*a*), the heat (h) carried off per unit length of the cylinder is readily seen to be

$$h = 0{\cdot}000289 \int (v_{17}\theta)\,dx$$

i.e. $= 0{\cdot}000289$ (Area of the $v_{17}\theta$ curve)

$= 2{\cdot}8$ calories per cm. per sec.

The value obtained by other means was 3 calories per cm. per sec., and is in good agreement, considering the sources of error associated with convection measurements using large bodies. Fuller details of the use of the instrument in this manner may be published later.

The combined instrument has been designed and developed for measuring certain convection losses and currents in elucidating the laws of convection of heat—work which has arisen out of and forms part of researches on cold storage insulation being carrried out at the National Physical Laboratory under the direction of Dr. Ezer Griffiths, for the Food Investigation Board of the Department of Scientific and Industrial Research. The author desires to thank Dr. Griffiths also for advice in the preparation for publication.

Dr. T. BARRATT communicated the following remarks on the Paper by A. H. Davis :—

As Mr. Davis remarks the laws of convection are not accurately known, even in ideal cases. One of the greatest difficulties in their determination lies in the fact that the total loss of heat from a heated surface is made up of three parts—conduction, convection, and radiation. As unfortunately there is no perfect insulator of heat, all three causes come into operation in every experiment.

In certain cases I have shown that it is possible to determine separately the losses due to convection and to radiation, the conduction loss being made inappreciable. (Proc. Phys. Soc., XXVIII., December, 1915, and XXXII., August, 1920). "Convection" may be taken as that portion of the heat emitted which is carried away by, and in the direction of the cooling fluid, while "radiation" is emitted equally in all directions from a homogeneous heated surface.

Experiments in which the convection was what is usually termed "free" or "natural" convection showed that:

(1) Out of 100 parts of "total heat" emitted from a wire 1 mm. diameter raised 10°C. or 12°C. above its surroundings at 17°C., 2·5 parts consist of radiation if the wire is uncoated, and 12·6 if it is "black." At 100°C. (the wire being again raised 10°C. or 12°C.) these percentages become 44 and 20·7 respectively. There is an increase of 20 per cent. in "total loss of heat" at the higher temperature, which is caused chiefly by increased convection.

The last sentence does not accord with Mr. Davis's remark that "there is no reason for appreciable departure from the law $H/\theta = B\sqrt{v\rho} + C$ through change of actual air temperature for temperature *excess* is the essential factor." Many measurements have been made of the rate of emission of heat from 1 sq. cm. of surface of wires, &c., for 1°C. excess. Lees (Bak. Lect., 1908) found values 0·000126, 0·000160, and 0·000200 at temperatures −155°C., −64°C. and +23°C., respectively, for rods about 6 mm. diameter. My own experiments on rods of similar dimension gave 0·000224 at 18°C., and 0·000265 at 100°C.

(2) When a cylinder is heated to 100°C., in air at about 20°C. the amount of "free" convection, *after deducting the radiation loss*, is inversely proportional to the square root of the diameter. For a sphere (within certain limits of diameter) this convection loss appears to be inversely proportional to the cube root of the diameter.

The amount of radiation from a very thin wire, such as is used in a hot-wire amemometer, must be a very small proportion indeed of the convected heat. Experiment has indicated that Newton's "First Power Law" of Cooling is very closely applicable to the *convection*, whether "free" or "forced." Where the radiation, as in very thin uncoated wires, is a very small proportion of the total heat emission, the law may safely be accepted for the total loss of heat. This has in fact been tacitly assumed in mathematical analyses in Thermal Conductivity for a century or more. When the cylinder, however, has a diameter of a millimetre or more Newton's law becomes less and less applicable, especially if the heated surface has a high absorptive power.

Results very near the truth can, however, be obtained by applying the First Power Law to the Convection loss alone, and the Fourth Power Law to the Radiation.

Mr. A. H. DAVIS, in reply to Dr. Barratt, writes :—

Since the theory of the hot-wire anemometer is that of forced convection and Dr. Barratt's communication is confined mainly to natural convection it is convenient to notice the difference between the two.

In free convection the hot body itself sets up the motion of the air, which depends upon gravity, and upon the relative decrease in gas density caused by 1 deg. rise in temperature. Although these variables must occur in the theoretical equation for free convection they appear to be negligible in the case of forced. An experimental difference may be noticed. Newton's first power law of cooling appears to have a fairly wide application to forced convection, compared with its application to free convection. In this latter case a $\theta^{5/4}$ law seems to be a better approximation over a wide range, as indicated by the experiments of Dulong and Petit, Peclet and Compan (thermometers, spheres, &c.). Langmuir has obtained results for wires and for a plane disc in which the law seems satisfactory up to 600°C. It is a good approximation for some larger bodies now being used in some convection experiments at the National Physical Laboratory.

It is not, of course, directly possible to predict a temperature effect with forced convection from free convection data, particularly as the density change coefficient referred to above depends markedly on temperature. The following is in amplification of remarks in the Paper on this effect. In the formula $H/\theta = B\sqrt{(v\rho)} + C$, for a given wire B and C depend only on the physical properties of the fluid of the stream, and these of course, change with temperature. A calculation was made of the probable effect of a change of 100°C. in actual temperature, and it appeared that the effect was within limits of experimental error met in using the instrument. (The calculation followed the general lines of a Paper now in the press ("Phil. Mag.") correlating data for cylinders in a stream of fluid.) The subsequent experimental work in the vertical channel with hot air up to 60°C. did not reveal that any vital factor had been overlooked. It would be necessary to introduce a correction with high temperature gases, but the complications do not appear to be great. As Dr. Barratt indicates, the radiation loss from a thin anemometer wire working at moderate temperature excess in a stream of air is practically negligible, and this fact is utilised in the design of the instrument.

The Katharometer.

1. "*The Katharometer*," *by* G. A. SHAKESPEAR, *M.A., D.Sc.*

THE Katharometer in its most usual form consists of a cylindrical block of copper about 4 cm. long and 2·5 cm. diameter. Parallel to the axis of the block are drilled two holes (about 0·6 cm. diameter) extending from one end nearly to the other. In each of the cells thus formed is a helix of platinum wire about 0·001 inch thick. One end of each helix is in electrical contact with the block, the other end of each being connected with a lead which passes out through an insulating air-tight plug. One of the cells communicates with the outer air through three small holes drilled through the end of the block, the other cell is hermetically sealed. In another form of the instrument two other holes are also drilled parallel to the axis of the block and each of these communicates with one of the cells through two or three small apertures, the gases to be examined are passed through these subsidiary cells or pipes. This form is called a double-flow katharometer and is very useful for comparing two gases of slightly differing composition.

The two helices form two arms of a Wheatstone bridge, the other two arms usually being of manganin. These bridge resistances may either be fixed or adjustable as required, and the whole may be contained in the "head" of the katharometer. The main current in the bridge is usually 0·1 amp., which suffices to maintain the platinum helices about fifteen centigrade degrees above the temperature of the block, their heat escaping mainly by conduction through the surrounding gas. Hence a small change in conductivity of the gas in one cell (the "exposed" cell) produces a change in temperature of the wire therein and the balance of the bridge is upset. Readings may be made of the galvanometer with fixed bridge resistances or of slide wire reading with balanced bridge.

The sensitiveness of the instrument is remarkable. For example, it may easily be made to give 100 scale divisions deflexion for 1 part of hydrogen in 1,000 of air, by volume. Hence, by estimating to 0·1 scale division 1 part in a million could be detected. The capacity of the cell being about 0·5 cm.^{3} the actual volume of hydrogen used is 5×10^{-7} cm.^{3}. Of course, for such refinements of measurement, disturbances must be guarded against. During the war we applied the katharometer to a variety of uses, such as the measurement of permeability of balloon fabrics to hydrogen and other gases, the detection of leakages in balloons and the improvement of the seams where the fabric is joined, the detection of hydrogen in air and of air in hydrogen. For this last purpose a special unsymmetrical form of instrument was developed by Dr. H. A. Daynes (for reasons which will be apparent on reading his Paper in Proc. Roy. Soc., Vol. XCVII., p. 273, 1920). I have also used a katharometer in conjunction with a "thread-recorder" as a recording hygrometer for continuous records of humidity for many months—it can be made to give the pressure of water vapour in the air directly. If the comparison cell be kept saturated with water vapour (instead of dry) the instrument records the difference between saturation and existing vapour pressures, *i.e.*, the "dryness" of the air. The double-flow form of instrument is extremely useful for measuring small traces of impurity in certain cases. There are many obvious applications of the instrument, some of the more important of these will be described by Dr. H. A. Daynes. The theory of the instrument has been worked out by Dr. Daynes in his Paper "On the Theory of the Katharometer" (Proc. Roy. Soc., Vol. XCVII., p. 273, 1920). Various communications on the subject by Shakespear and Daynes will be found in the Reports of the Advisory Committee on Aeronautics, 1916-18.

2. "*Industrial Applications of the Katharometer,*" *by* Dr. H. A. Daynes (*The Cambridge & Paul Instrument Co., Ltd.*).

The katharometer described by Dr. Shakespear has a great many applications to the measurement of the composition of industrial gases, and I will confine myself to discussing this field of application. It will be as well to summarise the points which favour the use of this method of analysis in controlling manufacturing processes.

(1) The instrument gives a continuous reading, and does not need intermittent withdrawals of samples of gas and subsequent analysis.

(2) The reading of the instrument does not depend on the rate of gas flow through it, so no regulation is necessary for this.

(3) The instrument is very quick in action. The sensitive element itself has a half value period of only about 15 seconds in the case of hydrogen in air, and about 1 minute for CO_2 in air, varying for other gases according to the diffusion constant of the mixture.

(4) The zero of the instrument is reasonably independent of variations of temperature.

(5) The zero is also very nearly independent of the current flowing through the bridge.

(6) The thermal conductivity on which the operation of the instrument depends, is very nearly indeed independent of atmospheric pressure, there being very little loss of heat from the wire by convection. It is not necessary therefore to reduce the gas to normal pressure before measurement or to make any corrections.

(7) The instrument can be made quite robust; in fact, quite as robust as many industrial thermo-couples.

(8) Being ultimately an electrical measurement calibration and standardisation are easy, parts can be made completely interchangeable, and any number of sensitive elements can be connected as in distance thermometry to a single recorder or indicator.

Against these advantages we have to set three disadvantages:—

(1) The chief limitation of the method is that it can give no qualitative analysis, and can only give quantitative analysis in

certain special cases. Very complicated mixtures are almost impossible to deal with.

(2) The sensitiveness depends slightly on temperature. This is inseparable from the nature of the instrument, because the thermal conductivity increases with temperature at the rate of about one-fifth of 1 per cent. per 1°C.

(3) The sensitiveness depends very largely on the current in the bridge, being nearly proportional to the cube of the current. Adjustment is only necessary about once a day.

The chief field for the method is in cases of binary mixtures of known constituents. The graph will show the relative sensitiveness to various gases mixed with air. The deflections are those given on a double pivoted galvanometer of robust construction as supplied for industrial pyrometry, which gives about 30 deg. per millivolt. It will be seen that hydrogen is the most effective, as might be expected. The katharometer has been used for detecting dangerous mixtures in the neighbourhood of hydrogen generating plants and in airship sheds. It has also been used in research on the leakage of hydrogen and helium through balloon fabrics, and for the detection of leakage from inflated airships and balloons; also for the measurement of the purity of hydrogen in hydrogen generators and in the envelopes of balloons and airships. The measurement of hydrogen and nitrogen in mixtures during the synthesis of ammonia is an interesting and important case. Nitrous oxide, carbon dioxide, methane and water vapour in air are all mixtures which lend themselves readily to analysis by this method.

The analysis of ternary and complex mixtures is only possible in certain cases. The analysis of producer gas, blue water gas, coke oven and blast furnace gases has not yet been attempted for this reason, but it is hoped to deal with these in time.

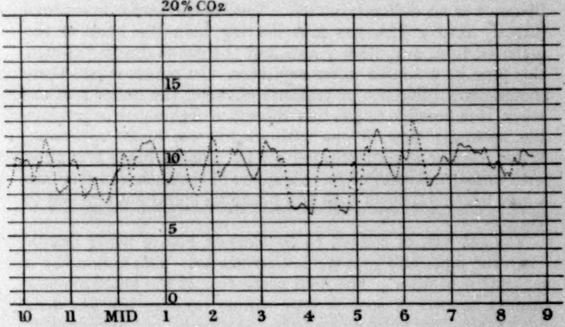
(1) When the percentages of all other gases but the one to be measured are small and the gas to be measured can be readily absorbed without affecting the others, the double flow method is used, in which both sides are exposed to the gas, one before and one after absorption of the gas to be measured.

(2) The effect of water vapour can always be counteracted automatically. If the gas is known to be saturated the other platinum arm can also be exposed to saturated water vapour, and in any case it is quite easy to saturate the gas being

measured, so that this compensation is always possible. It is much easier than drying the gas before measurement.

(3) When all the other constituents but the one of interest are very nearly equal to one another in thermal conductivity there is no difficulty.

All these three cases are illustrated in the one which promises to be the most important of all, and which has already been developed for industrial use—that is, the measurement of CO_2 in flue gases. The gases present are nitrogen, oxygen, carbon monoxide, carbon dioxide, hydrogen, methane and water vapour. Of these, nitrogen, oxygen and carbon monoxide have very nearly the same conductivity. The residual of hydrogen and methane in flue gases is extremely small, and the gas is always saturated at the temperature of the instrument, so that water vapour can easily be compensated for. Another illustration of a complex mixture is that of water vapour in blast furnace gases. Here there are other gases present, 3 or 4 per cent. of hydrogen and 10 or 12 per cent. of CO_2. These in themselves would give very much larger deflections than would be given by the water vapour, but the effect of these gases is eliminated by exposing both platinum coils to the gas, the only difference being that on one side the gas has to diffuse through a chamber containing phosphorus pent-oxide to absorb the water vapour.



Reproduction of record from Cambridge Electrical CO_2 Recorder.

For measuring the percentage of CO_2 in boiler flue gases, the katharometer is mounted in a head which carries the two manganin bridge coils and a series coil in the galvanometer circuit which makes all the katharometers give the same E.M.F. for a given current in the bridge, and so makes them completely interchangeable, the galvanometers where recorders or indicators are also standardised to the same resistance and sensitiveness. This makes the addition of extra points on to an existing outfit very simple. The bridge coils of the katharometer being entirely in the head, the leads do not come into the arms of the bridge, so that it is not necessary for the leads to be balanced. Four-way leads are taken from the katharometer to the indicator or recorder wherever it may be placed, distance is no difficulty at all. The katharometer is screwed directly into a ¾ in. gas pipe leading from the back of the boiler or from any other point, and the gas is drawn through this pipe by a simple water aspirator. In plants where there is sufficient difference of pressure between the boiler and the stack, it is possible to use this difference of pressure to produce the necessary flow of gas without the use of an aspirator. Water vapour is compensated for by a small brass box filled with water, and screwed on to the side of the katharometer into a hole which indicates with the platinum spiral not exposed to the gas.

The gas has, of course, to be cleaned before being drawn down the pipes and past the katharometer. This is done by a separate soot filter. The final protection of the katharometer against deposit of soot is very simple because the gas does not have to flow actually into the instrument, but only past it, so it is possible to put a glass wool filter in between the katharometer and the pipes through which the gas is flowing without causing any appreciable lag due to the diffusion of the gas from the pipe to the katharometer.

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"Further Applications of the Katharometer." G. A. Shakespear and H. A. Daynes.

PROCEEDINGS OF THE ROYAL SOCIETY, A. VOL. XCVII., 1920.

"The Process of Diffusion through a Rubber Membrane." H. A. Daynes, M.Sc.

"The Theory of the Katharometer." H. A. Daynes, M.Sc. (with an introductory note by G. A. Shakespear).

Calcometer.

"*The Caleometer,*" *by* PROF. LEONARD HILL, *F.R.S., and* MISS HARGOOD-ASH.

THE Caleometer designed by the late O. W. Griffith and Leonard Hill is an instrument which measures the heat loss of a wire coil maintained at a constant temperature.

The arrangement of the instrument is shown diagrammatically in Fig. 1. *A B C D* form the arms of a Wheatstone bridge. The arm *AD* contains the coil which is made of a resistance wire having a large temperature coefficient. The bridge is balanced when the coil is at a temperature of 34°C. and the galvanometer *G* then shows no deflection. If the coil changes temperature the balance is upset, the galvanometer needle is deflected, and makes contact with *H* by means of one of the two platinum pointers *E* and *F*. This closes either the circuit *QHELMPS* or *QHFNMPS*, in the former case, which occurs when the temperature of the resistance coil has fallen, the soft iron plunger, attached to the movable contact *K* by a string passing over a pulley wheel, moves into the part of the coil *LM* and the resistance *R* in the battery circuit is reduced, and the current in the bridge increased, in the latter case which occurs when the temperature of the resistance coil rises the plunger moves into the part of the coil *NM* and the resistance at *R* is increased, the current in the bridge being decreased. In either case the temperature of the coil is brought back to 34° C. In practice to prevent sticking of the contacts it is found advisable to have *H* revolving.

The ammeter *W* in series with the resistance coil shows the amount of current required to flow through the coil to keep it at 34°C., and enables the heat loss of the wire to be calculated. The instrument may be made self recording, when the position of the ammeter needle is marked, at regular short intervals of time, on a piece of paper drawn at a known speed beneath it. The records from two coils may be obtained in this way for the same period, the coils being in different atmospheric conditions.

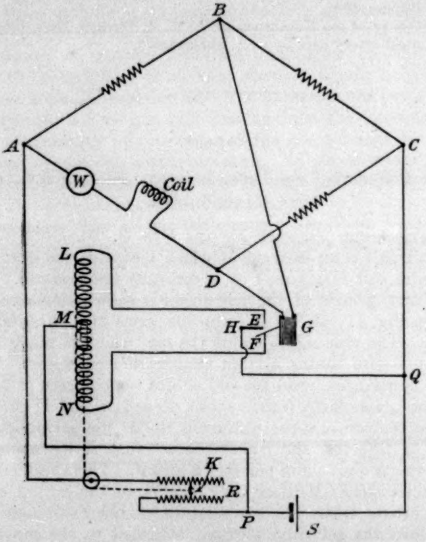

Fig. 1.—The Caleometer. Diagram of Connections.

The heat loss from the coil is affected by air currents, also, owing to its low temperature, by changes of temperature. The Caleometer, therefore, enables the meteorologist or hygienist to record daily variations of the rate of cooling of a surface at body temperature produced by atmospheric conditions. The records taken out of doors thus obtained throw light on the causation of those feelings which lead us to define the weather as pleasant, unpleasant, bracing, muggy, &c. Records taken indoors enable one to measure the cooling power of the atmosphere on a surface at body temperature and guide one to the securing of adequate ventilation. The Caleometer acts as a recording kata-thermometer the use of which for measuring the adequacy of ventilation has become established. The resistance coil is then placed within the bulb of a kata-thermometer. The instrument is made by the Cambridge and Paul Instrument Co., Newton Avenue Works, New Southgate, London, N.

Liquid Depth Gauge.

"*Liquid Depth Gauge, Distant Reading Type,*" *by* EDGAR A. GRIFFITHS.

IT is a well-known fact that the cooling power of a liquid is vastly greater than that of a gas, and this was taken advantage of by the author in the design of a gauge for measuring the depth of petrol in the tanks of aeroplanes. The gauges usually employed for this purpose are of the float type, sometimes involving transmission to a dial by the use of a string. In practice such gauges are not very satisfactory, particularly so if the tank is under air pressure. In the present instrument the gauge has no moving parts in the tank.

It consists essentially of a thin wire of platinum electrically heated to a temperature excess of about 20 deg. above the surrounding air. The wire is insulated and suitably protected by a tube projecting to the full depth of the tank. The portion of the wire immersed in the liquid is cooled down to practically the same temperature as the liquid, while the part above the surface is at the excess temperature.

Since the temperature coefficient of the resistance of platinum is of the order of 0·4 per cent. per degree, it follows that the average temperature and hence the resistance of the wire, will depend upon its depth of immersion in the liquid. Now the most convenient method of measuring changes of electrical resistance is by means of the Wheatstone bridge, and in this gauge the wire forms one arm of the bridge as shown in the diagram (Fig. 1). The influence of changes in the temperatures of the liquid and the atmosphere above are eliminated by arranging alongside a similar wire, totally sealed off from the liquid and electrically connected in the other arm of the bridge.

The changes of the resistance of the partially immersed wire, with variations of liquid level, is indicated by the deflection of the galvanometer pointer. The sensitivity of the method is such that a very small elevation of temperature of the wire suffices, and in practice a robust form of moving coil instrument is employed as indicator.

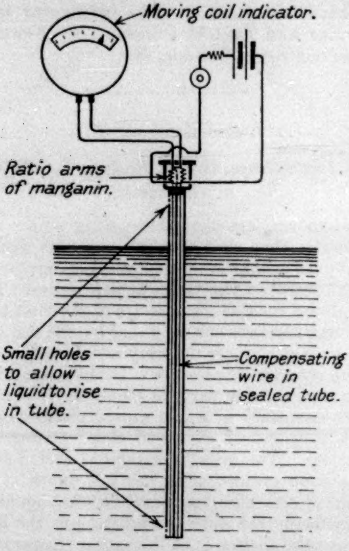
Moving coil indicator.
Ratio arms of manganin.
Small holes to allow liquid to rise in tube.
Compensating wire in sealed tube.

FIG. 1.—LIQUID DEPTH GAUGE.

Since the sensitivity of any bridge arrangement is a function of the current, it is necessary to keep this constant. The customary procedure is to have an adjustable series resistance in the battery circuit, and by means of a throw over switch, replace the wire by a dummy coil totally immersed. The current in the circuit is then adjusted to give full-scale deflection, the procedure being identical with that employed with direct reading resistance thermometers.

An alternative way would be to employ the same indicator with a shunt as ammeter when it is desired to adjust the battery current to a predetermined value.

The necessity for such periodical adjustments is a great disadvantage for aircraft purposes, when the attention of the pilot is fully occupied with more vital controls, and, consequently, the apparatus was modified to avoid this necessity. In this arrangement an iron wire ballast resistance is inserted in the battery circuit. The iron wires are sealed in a bulb similar to an electric lamp, with an atmosphere of hydrogen.

Such a resistance will maintain a fairly steady current even if the voltage of the battery changes by as much as 20 per cent., so frequent adjustment of the current is not necessary. This property of iron wire is, of course, well known and iron wire resistors will be found in most commercial types of Nernst lamps to compensate for the negative temperature coefficient of resistance of the rare earth mixture, of which the glower is composed.

As the two ratio arms of the bridge are of manganin of negligible temperature coefficient, calculation shows that a constant battery current gives a constant sensitivity for the instrument for temperature changes within ordinary limits.

XII. *On Tracing Rays through an Optical System.* (*Fourth Paper.*) *By* T. Smith, *M.A.*, *F.Inst.P.* (*From the National Physical Laboratory.*)

Received October 21, 1920.

ABSTRACT.

The Paper, which is mainly mathematical, is a further development of the system described by the author in the previous Papers of the same title presented to the Society; formulæ for skew rays are put into a shape as far as possible similar to those applying to rays in one plane.

The formulæ most generally used for tracing through an optical system rays which do not lie in a plane through the axis are not arranged along lines parallel to those followed in tracing rays contained by such a plane, and hence they do not readily exhibit the character and extent of the differences to be expected between rays of these two classes. The formulæ given in this Paper are the result of an attempt to bring the treatment of the more general rays into closer conformity with that adopted in the simpler case.

To secure the desired features in the formulæ it is proposed to obtain, the equations for the simpler case should be derivable at once by substituting zero for some quantity on which the skewness of the ray depends. The obvious variable to assume which may thus vanish is the distance between the ray and the axis. Let this be denoted by Δ. The distance from the origin at which this closest approximation is attained will evidently be of importance in the problem. Let it be denoted by l, reckoned positive when the origin is to the right of the position of nearest approach. The equation to the ray may then be written in the form

$$\frac{x+l}{\cos\psi}=\frac{y-\Delta\sin\theta}{\sin\psi\cos\theta}=\frac{z+\Delta\cos\theta}{\sin\psi\sin\theta}, \quad . \quad . \quad . \quad . \quad (1)$$

so that Δ may be calculated from

$$\Delta=y\sin\theta-z\cos\theta=\frac{Ny-Mz}{(M^2+N^2)^{\frac{1}{2}}}. \quad . \quad . \quad . \quad (2)$$

and l from

$$\left.\begin{aligned} l&=(y^2+z^2-\Delta^2)^{\frac{1}{2}}\cot\psi-x=\left(\frac{y^2+z^2-\Delta^2}{M^2+N^2}\right)^{\frac{1}{2}}L-x \\ &=\frac{L(Lx+My+Nx)-x}{M^2+N^2} \end{aligned}\right\}, \quad . \quad (3)$$

where x, y, z are the co-ordinates of any point on the ray, ψ is the inclination of the ray to the axis, and L, M, N are its direction cosines. The problem for solution need not involve the actual direction and position of the ray, but on the contrary does not concern the value of θ. It is thus only necessary to know l, Δ and ψ to enable a solution to be reached. The origin of co-ordinates may be taken at the vertex of the refracting surface under consideration, and the problem is to find from these quantities and the data of the system the powers of the various surfaces for the given ray and the length of its path between successive surfaces. It will suffice to consider a single surface and to show how l', Δ', and ψ', or rather $\sin\psi'$, after refraction, may be obtained from the known values of l, Δ and $\sin\psi$, before refraction.

The determination of these quantities will be most readily followed by reference to the accompanying figure.

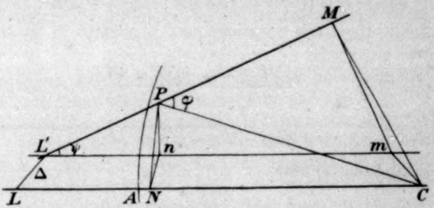

FIG. 1.

LC is the axis of the system, coinciding with the axis of x, and meeting the vertex of a refracting surface at A and its centre at C. $L'\,P\,M$ is an incident ray, $L\,L'$ is the shortest distance between the ray and axis, and is perpendicular to both. P is the point of refraction, and M is the foot of the perpendicular from C to the ray. Let the rectangle $L'\,L\,C\,m$ be completed, and m joined to M. Then mM is perpendicular to the ray and to mC, and it at once follows that if the radius of the surface is r,

$$r^2\sin^2\varphi=(l+r)^2\sin^2\psi+\Delta^2. \quad . \quad . \quad . \quad . \qquad (4)$$

The evaluation of $\sin\varphi'$ and K at once follows from

$$\mu'\sin\varphi'=\mu\sin\varphi,$$

and

$$rK=\mu'\cos\varphi'-\mu\cos\varphi,$$

equations common to other algebraic methods of calculation. Before refraction can be considered the x co-ordinate of P is required. Let $P\,N\,n$ be the plane through P normal to the axis. Then

$$\begin{aligned} L'M &= LC \cos \psi, \\ &= L'P + PM, \end{aligned}$$

or

$$(l+x) \sec \psi = (l+r) \cos \psi - r \cos \varphi. \quad . \quad . \quad . \quad (5)$$

The analytical equations of refraction may be written in the form

$$\mu' \cos \psi' = \mu \cos \psi + (r-x)K \quad . \quad . \quad . \quad . \quad (6)$$

$$\mu' \sin \psi' \cos \theta' = \mu \sin \psi \cos \theta - \{\Delta \sin \theta + (l+x) \tan \psi \cos \theta\} K,$$

$$\mu' \sin \psi' \sin \theta' = \mu \sin \psi \sin \theta - \{-\Delta \cos \theta + (l+x) \tan \psi \sin \theta\} K,$$

and from the two latter it at once follows that

$$\mu'^2 \sin^2 \psi' = \{\mu - (l+x) \sec \psi K\}^2 \sin^2 \psi + \Delta^2 K^2. \quad . \quad . \quad (7)$$

Again, these two equations may be written in the familiar form

$$\frac{\mu' M' - \mu M}{y} = \frac{\mu' N' - \mu N}{z},$$

or

$$\mu'(N'y - M'z) = \mu(Ny - Mz),$$

showing from (2) that

$$\mu \sin \psi \Delta$$

is invariant on refraction, and indeed is constant for all refractions in a coaxial system. Thus, Δ' is determined from

$$\mu'^2 \sin^2 \psi' \Delta'^2 = \mu^2 \sin^2 \psi \Delta^2 = C^2, \quad . \quad . \quad . \quad . \quad (8)$$

this form being adopted since Δ always appears in the equations squared, as is to be expected since a change in its sign has no influence on the problem considered. It now remains to find l'. This is derived at once from the consideration of the two values of y^2+z^2 obtained from the co-ordinates of the incident and refracted rays. The equation is

$$(l'+x)^2 \tan^2 \psi' = (l+x)^2 \tan^2 \psi + \Delta^2 - \Delta'^2, \quad . \quad . \quad . \quad (9)$$

where $(l'+x) \tan \psi'$ usually has the same sign as $(l+x) \tan \psi$. Any ambiguity in sign may be avoided by using the alternative equation

$$\mu' \left\{ \frac{x + l' \sin^2 \psi'}{\cos \psi'} + x \cos \varphi' \right\} = \mu \left\{ \frac{x + l \sin^2 \psi}{\cos \psi} + x \cos \varphi \right\}.$$

The value of l for the next refraction is, of course, found by increasing the l' after refraction at the present surface by t, the axial separation of the surfaces. The ray separation of the surfaces is obtainable from a number of equations, of which

$$d \cos \psi = t - x + x',$$

where x' denotes the x of the following surface, will be as convenient as any.

The equations given above afford a complete solution of the problem. One additional equation—viz., (8)—is necessarily introduced on account of the additional degree of freedom possessed by the ray, and in place of the simple angular relation $\psi' = \psi + \varphi' - \varphi$ ordinarily used two other equations have had to be substituted. This change, however, is no drawback, and the two equations will take no longer to apply to a ray than the single equation for the simpler case. The outstanding difference in the treatment required is the extraction of three square roots—those for $\cos \varphi$, $\cos \varphi'$ and $l' + x$. These present no difficulty where a calculating machine is employed, and in such cases the new equations are not likely to be found inferior to those given previously.

Although the preceding equations form a complete solution to the problem they are not in the best form for calculation where it is desired to apply the same equations to all surfaces whether their curvatures be large or small. This fault, however, is readily remedied. For equation (4) may be rewritten as

$$\sin^2 \varphi' = (1 + lR)^2 \sin^2 \psi + \Delta^2 R^2, \quad . \quad . \quad . \quad . \quad (10)$$

and equation (5) may be combined with (4) to yield

$$\frac{l + x}{\cos \psi} = \frac{2l + (l^2 + \Delta^2)R}{\cos \varphi + (1 + lR) \cos \psi}. \quad . \quad . \quad . \quad . \quad (11)$$

The remaining equations, apart from (6), which is an additional equation, and may be written

$$\mu'(\cos \psi' - \cos \varphi') = \mu(\cos \psi - \cos \varphi) - xK,$$

require no modification. It is to be noted that l and $\sin \psi$ are necessarily always finite, and no special cases arise for separate consideration. It may notwithstanding this be convenient to regard $l \sin \psi$ as a separate variable in place of l, with a view to increasing the accuracy obtainable with a given number of figures. Let this product be denoted by λ. The modified equations then become

$$\left.\begin{aligned}\lambda&=(y^2+z^2-\Delta^2)^{\frac{1}{2}}\cos\psi-x\sin\psi\\ \text{or}\qquad \lambda\sin\psi&=L(Lx+Ny+Nz)-x\end{aligned}\right\}, \quad . \quad . \quad . \quad (3)$$

$$\sin{}^2\varphi=(\sin\psi+\lambda R)^2+\Delta^2R^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{x+\lambda\sin\psi}{\cos\psi}=\frac{2\lambda\sin\psi+(\lambda^2+\Delta^2)R}{\cos\psi+\cos\varphi}=r(\cos\psi-\cos\varphi) \quad . \quad (5)$$

$$\mu'^2\sin^2\psi'=\left(\mu\sin\psi-\frac{\lambda+x\sin\psi}{\cos\psi}K\right)^2+\Delta^2K^2 \quad . \quad . \quad (7)$$

$$\left(\frac{\lambda'+x\sin\psi'}{\cos\psi'}\right)^2=\left(\frac{\lambda+x\sin\psi}{\cos\psi}\right)^2+\Delta^2-\Delta'^2, \quad . \quad . \quad (9)$$

and the λ for the new surface is obtained by adding $t\sin\psi'$ to this value of λ'. A check on the accuracy of the calculation is afforded by the relation

$$\sin{}^2\varphi'=(\sin\psi'+\lambda'R)^2+\Delta'^2R^2,$$

and others may be constructed if desired. For example, it readily follows from (7) and (8) that

$$\begin{aligned}\mu'\sin\psi'\left(\mu'\sin\psi'+\frac{\lambda'+x\sin\psi'}{\cos\psi'}K\right)&=\{(\mu\mu'\sin\psi\sin\psi')^2-C^2K^2\}^{\frac{1}{2}}\\ &=\mu\sin\psi\left(\mu\sin\psi-\frac{\lambda+x\sin\psi}{\cos\psi}K\right),\end{aligned}$$

and equation (6) may have the form

$$\mu'\left(\frac{x+\lambda'\sin\psi'}{\cos\psi'}+x\cos\varphi'\right)=\mu\left(\frac{x+\lambda\sin\psi}{\cos\psi}+x\cos\varphi\right).$$

It may be noted that the extraction of square roots may, if desired, be avoided, since the equations are in a form suitable for rapid approximation by the method of continued fractions.

XIII.—*On the Refractometry of Prisms.*

By J. GUILD, *A.R.C.Sc., F.Inst.P., F.R.A.S.* (*From the National Physical Laboratory.*)

RECEIVED NOVEMBER 26, 1920.

ABSTRACT.

In this Paper a generalised formula for the refraction of light through a prism is obtained, and the particular cases pertaining to practical methods of refractometry deduced from it. The sensitivity of various methods for various prism angles and refractive indices is shown in a series of curves, as is also the liability to error due to errors in auxiliary constants.

IN a previous Paper* the writer developed various formulæ in connection with the refraction of light through prisms. As the interest of that Paper centred round the Pulfrich refractometer, the treatment dealt only with the case of grazing incidence at the first surface of the system.

In view of their importance in connection with the design of instruments for refractive index determinations it may be worth while drawing attention to the equivalent relationships in other cases with a view to determining the comparative accuracy of different methods of refractometry, and the actual accuracy likely to be obtained with a given instrumental equipment.

Let *AB*, Fig. 1, be a ray of monochromatic light incident on the first of two plane surfaces which separate three regions in which the refractive indices for the light concerned are μ_1, μ_2 and μ_3.

We need only consider the case in which the incident ray and the normals to the surfaces all lie in the plane of the paper.

In developing prism formulæ it is desirable, as in lens formulæ, to follow a definite convention with regard to the signs of angles. Otherwise confusion and ambiguity frequently arise. The convention which the author himself finds most convenient—although it has certain disadvantages—is to regard all angles as positive when measured counterclockwise from their appropriate reference directions. For angles of incidence and refraction the reference directions are the normals to the respective surfaces: for the angles between surfaces, the normal to the surface first encountered by the ray is the reference direction for the normal to the second surface.

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* Proc. Phys. Soc., Vol. XXX., Part III., p. 157.

Figure 1 is drawn in such a way that all the angles with which we are concerned are positive on the above convention. The resulting formulæ will differ in the signs of some of the quantities from those deduced in the textbooks where only the magnitude of the angles are taken into account.

Let the angle of incidence $A\hat{B}N_1=i$. The refracted ray BC in the second medium makes an angle $N'_1\hat{B}C,=r$, with the normal at B, and finally enters the third medium, making an angle of emergence $N'_2\hat{C}E,=e$, with the normal to the second surface at C. If θ is the angle between the surfaces, it follows from the figure that $B\hat{C}N_2=r-\theta$, and that $C\hat{D}O=\theta+e-i$. But $C\hat{D}O$ is the angle by which the ray has been deviated by the two refractions $=\delta$, say.

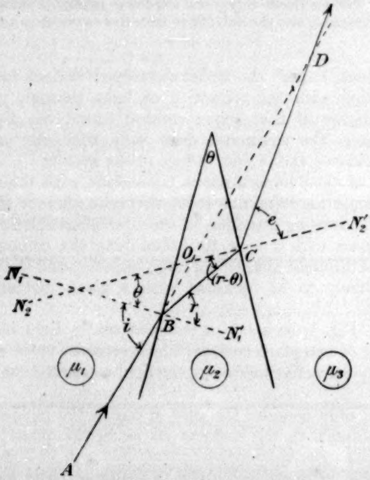

FIG 1.

$\therefore$ the deviation, δ, $=\theta+e-i$.

$$\text{Sin}\,^2\theta=\{\sin r\cos(r-\theta)-\cos r\sin(r-\theta)\}^2$$
$$=\sin^2 r+\sin^2(r-\theta)-2\sin r\sin(r-\theta)\cos\theta.$$

From the law of refraction

$$\mu_1\sin i=\mu_2\sin r;\quad \mu_2\sin(r-\theta)=\mu_3\sin e.$$

Therefore

$$\mu_2^2\sin^2\theta=\mu_1^2\sin^2 i+\mu_3^2\sin^2 e-2\mu_1\mu_3\sin i\sin e\cos\theta \qquad (1)$$

Equation (1) is the starting point from which all special cases may be deduced.

PRISM IN AIR.

Refractive index from deviation and angle of incidence or emergence.—In the case of a prism surrounded by air $\mu_1=\mu_3=1$ (approx.) and equation (1) becomes

$$\mu_2^2\sin^2\theta=\sin^2 i+\sin^2 e-2\sin i\sin e\cos\theta \quad .\ .\ .\ .\ (2)$$

which is the general equation for this case.

To determine μ_2, the index of the prism, it is necessary to know both i and e in addition to the angle, θ, of the prism. Alternatively either i or e and the deviation, δ, which the light suffers in passing through the prism will give μ_2 since $\delta=\theta+e-i$.

Angle of incidence for a given deviation.—Before departing from the general case we may treat the converse problem which occasionally arises in practice, viz., given a prism of a certain index and refracting angle to choose the angle of incidence which will give a specified deviation.

From equation (2)

$$\mu_2^2\sin^2\theta=\tfrac{1}{2}\{2-(\cos 2e+\cos 2i)\}-\cos\theta\{\cos(e-i)-\cos(e+i)\}$$
$$=\tfrac{1}{2}[2-2\{\cos(e+i)\cos(e-i)\}]-\cos\theta\cos(e-i)$$
$$+\cos\theta\cos(e+i)$$
$$=1-\cos(e+i)\{\cos(e-i)-\cos\theta\}-\cos\theta\cos(e-i).$$

But $\quad e+i=\delta+2i-\theta$ and $e-i=\delta-\theta$;

$$\therefore\quad \mu_2^2\sin^2\theta-1+\cos\theta\cos(\delta-\theta)$$
$$=-\cos(\delta-\theta+2i)\{\cos(\delta-\theta)-\cos\theta\},$$

or

$$\cos(\delta-\theta+2i)=\frac{1-\mu_2^2\sin^2\theta-\cos\theta\cos(\delta-\theta)}{\cos(\delta-\theta)-\cos\theta}.$$

Since δ is always—ve, provided $\mu_2>1$, we may put $\delta=-\bar{\delta}$, where $\bar{\delta}$ is the numerical value of the deviation, and

$$\cos(2i-\theta-\bar{\delta})=\frac{1-\mu_2^2\sin^2\theta-\cos\theta\cos(\theta+\bar{\delta})}{\cos(\theta+\bar{\delta})-\cos\theta}, \quad . \quad . \quad . \quad (3)$$

from which, since θ and μ_2 are supposed known, the value of i corresponding to the required value of $\bar{\delta}$ can be calculated.

Minimum Deviation. It is comparatively rarely that the general case is utilised in refractometric work. It simplifies experimental arrangements to make one of the variables i, e, or δ conform to some easily attained condition so that only one quantity remains to be measured. The most familiar condition is that the deviation δ should be a minimum. This happens when the angles of incidence and emergence are equal.* In this case $i=-e$ and equation (2) reduces to

$$\mu_2^2\sin^2\theta=2\sin^2 i(1+\cos\theta),$$

or,

$$4\mu_2^2\sin^2\theta/2\cos^2\theta/2-4\sin^2 i\cos^2\theta/2,$$

whence

$$\mu_2=\frac{\sin i}{\sin\theta/2}.$$

The deviation

$$\delta=\theta+e-i=\theta-2i, \text{ or } i=\tfrac{1}{2}(\theta-\delta).$$

$$\therefore \quad \mu_2=\frac{\sin\frac{1}{2}(\theta-\delta)}{\sin\theta/2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

GRAZING INCIDENCE.

Another convenient experimental arrangement is that in which $i=90$ deg. In this case the ray is refracted at the so-called critical angle, and equation (2) reduces to

$$\mu_2^2=1+\left(\frac{\cos\theta-\sin e}{\sin\theta}\right)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

NORMAL EMERGENCE.

In the Abbé Autocollimating spectrometer the light is reflected normally from the second face of a 30 deg. prism. This is clearly equivalent to putting $e=o$ in equation (2), which then becomes

$$\mu_2=\sin i/\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The reader may have observed that the signs of δ in equation (4) and of $\sin e$ in equation (5) differ from the usually given formulæ. This is the only disadvantage in the convention with respect to signs followed in obtaining the general equation. The fault, however, really lies in the ordinary formulæ in which the angles are assumed to have magnitude only. No confusion can arise in the case of minimum deviation for δ is invariably negative, and so the $+$ sign obtained by neglecting signs of angles always holds. In the case of grazing incidence, however, the formula is usually developed from a figure in which e is $-ve$, and, consequently, a $+$ sign is obtained by neglecting this fact. In practice e may quite frequently be positive and the $-$ sign is required. There are thus two formulæ to be remembered, and confusion easily arises in particular cases. By following one sign convention which holds for every case such ambiguity is prevented.

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* See any elementary textbook.

PRISM WITH ONE SURFACE IN CONTACT WITH ANOTHER REFRACTIVE SUBSTANCE.

In this case, which applies to most methods of commercial refractometry, $\mu_1 \neq 1$, and equation (1) becomes

$$\mu_2^2 \sin^2\theta = \sin^2 e + \mu_1^2 \sin^2 i - 2\mu_1 \sin i \sin e \cos\theta.$$

The only case of practical importance is that of grazing incidence, or $i = 90$ deg., in which case the equation reduces to

$$\mu_1 = \pm \sin\theta \sqrt{\mu_2^2 - \sin^2 e} + \sin e \cos\theta.$$

The index μ_1 is $+ve$, and greater than unity, whereas the product $\sin e \cos\theta$ is necessarily less than unity, so that the positive sign before the square root must always be taken.

Thus $$\mu_1 = \sin\theta \sqrt{\mu_2^2 - \sin^2 e} + \sin e \cos\theta. \quad . \quad . \quad . \quad (7)$$

If μ_2 and θ are known, μ_1 can be determined from equation (7), the only measurement required being e, the angle of emergence which the critical ray makes with the normal to the second face of the prism.

SENSITIVITY OF VARIOUS METHODS.

The foregoing equations, which connect the various essential quantities in the most usual methods of refractometry, do not give any information concerning the sensitivities of the various methods, that is to say, the change in the measured quantity per unit change in the refractive index, nor do they indicate the extent to which the results are affected by errors in the values of the auxiliary constants, such as the angle of the prism, or, in the case to which equation (7) refers, the refractive index, μ_2, of the auxiliary prism.

Taking, first, the most familiar method, the quantity measured is δ, the minimum deviation. The sensitivity S may be measured by the number of seconds which δ varies when the refractive index of the prism changes by one unit in the fifth decimal place.

Thus
$$S=\frac{\partial\delta}{\partial\mu_2}\times\frac{57\cdot3\times60\times60}{10^5}=2\cdot06\frac{\partial\delta}{\partial\mu_2},$$

$$\mu_2=\frac{\sin\frac{1}{2}(\theta-\delta)}{\sin\theta/2},$$

whence
$$\frac{\partial\mu_2}{\partial\delta}=-\frac{1}{2}\frac{\cos\frac{1}{2}(\theta-\delta)}{\sin\theta/2},$$

or,
$$S=-\frac{4\cdot12\sin\theta/2}{\cos\frac{1}{2}(\theta-\delta)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Thus, the sensitivity is a function of the deviation and of the angle of the prism. We can calculate its value for any values of μ_2 and θ, first finding the appropriate value of δ from equation (4).

For the method of grazing incidence, the similar formula as shown in the previous Paper,* is

$$S=-\frac{2\cdot06\mu_2\sin\theta}{\cos e\sqrt{\mu_2^2-1}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For the method of autocollimation by normal reflection from the second face of the prism

$$\mu_2=\frac{\sin i}{\sin\theta}.$$

The quantity measured is i, the angle of incidence for which the beam is exactly returned on its path, so the sensitivity

$$=2\cdot06\frac{\partial i}{\partial\mu_2},$$

or
$$S=2\cdot06\frac{\sin\theta}{\cos i}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The method is usually employed with a prism of 30 deg. refracting angle; by comparing equations (10) and (8) we see that the sensitivity with the autocollimating method using such a prism (θ=30 deg.) is always just half the sensitivity of the minimum deviation method using a 60 deg. prism.

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* *Loc. cit.*, page 179, eqn. (7).

For a given material (μ_2=constant) it is useful to observe how the sensitivity depends on the angle to which the prism is cut. In Fig. 2 this is plotted for two values of the refractive index (1·6 and 1·75) for the minimum deviation method.

The sensitivity becomes infinite for the maximum angle which the prism may have and still transmit light. In this case both the angles of incidence and emergence are grazing, and the deviation $=\theta-180$ deg. If $i=-e=90$ deg. and $\delta=\theta-180$ deg. equation (4) reduces to

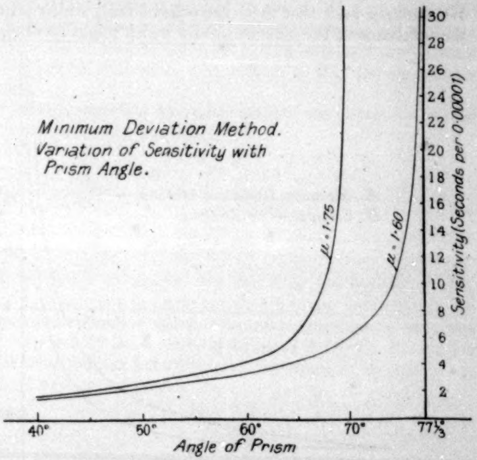

FIG. 2.

$$\mu_2=\frac{1}{\sin\theta/2},$$

which gives the largest refractive index that can be determined with a prism of angle θ or alternatively the largest angle which a prism of index μ_2 may have and still transmit light. Thus, if $\mu_2=1·6$ the largest possible angle is 77⅓ deg., while for the usual 60 deg. prism the largest possible index is 2.

The curves for the grazing incidence method are similar in character. They are given in the previous Paper* and need not be repeated here.

From Fig. 2 it is evident that for $\mu_2=1{\cdot}6$, which is about the average of the values encountered in practical work of high precision, an angle of 75 deg. would give more than five times the sensitivity of the usual 60 deg. prism. Sensitivity is, however, only one factor in the final accuracy of the results. We shall see later that there are other factors which prevent the utilisation of the high sensitivity which might be obtained by suitable choice of prism angle, and the balance of advantages does in fact lie with the 60 deg. prism.

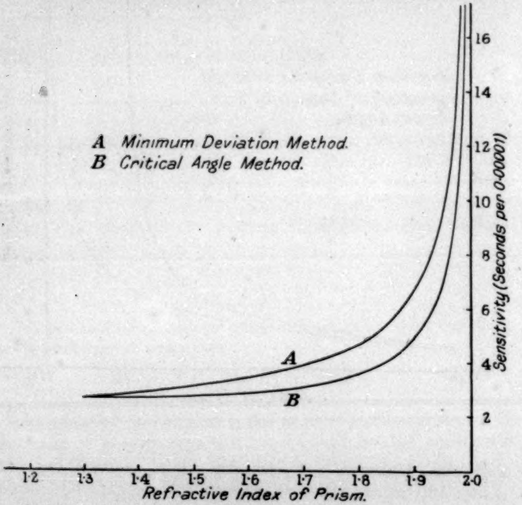

FIG. 3.

Taking this prism as basis we may compare the sensitivity of the minimum deviation and grazing incidence methods throughout a considerable range of refractive index, since these are the two methods of greatest importance in precise work.

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* *Loc. cit.*, Fig. 10.

These are plotted in Fig. 3, from which we can find the sensitivity by either method for any index. The minimum deviation method has clearly a slight advantage over the other for all ordinary materials.

EFFECTS OF ERRORS IN AUXILIARY CONSTANTS.

In the preceding formulæ the angle, θ, of the prism occurs. It is usually necessary to determine this experimentally, and its value is, therefore, subject to error. The assumption of an erroneous value for θ in calculating the index will lead to a corresponding error in the result. This is clearly proportional to the partial derivative with respect to θ of the expressions for μ_2.

For the minimum deviation method we obtain by partial differentiation of equation (4)

$$\frac{\partial\mu_2}{\partial\theta}=\frac{\frac{1}{2}\left(\cos\frac{\theta-\delta}{2}\sin\frac{\theta}{2}-\sin\frac{\theta-\delta}{2}\cos\frac{\theta}{2}\right)}{\sin^2\theta/2}$$

$$=\frac{1}{2}\frac{\sin\delta/2}{\sin^2\theta/2}.$$

It is better to express the result as the inverse of this, namely, the error in angle which will cause unit error in the calculated refractive index. We may conveniently term this the *tolerance in prism angle* and denote it by T_θ, introducing the factor 2·06 as before to give the result in seconds of arc per 0·00001 error in index.

Thus $$T_\theta=4{\cdot}12\,\frac{\sin^2\theta/2}{\sin\delta/2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

For the grazing incidence method, by differentiation with respect to θ of equation (5)

$$2\mu_2\frac{\partial\mu_2}{\partial\theta}=2\left(\frac{\cos\theta-\sin e}{\sin\theta}\right)\left\{-\frac{\sin\theta}{\sin\theta}-\frac{(\cos\theta-\sin e)}{\sin^2\theta}\cos\theta\right\}$$

$$=-2\left(\frac{\cos\theta-\sin e}{\sin\theta}\right)\left\{1+\cos\theta\,\frac{\cos\theta-\sin e}{\sin^2\theta}\right\}$$

$$=-2\,\{\sqrt{\mu_2^2-1}+(\mu_2^2-1)\cot\theta\}.$$

$$\therefore\quad T_\theta=2{\cdot}06\,\frac{\partial\theta}{\partial\mu_2}=\frac{-2{\cdot}06\mu_2}{\sqrt{\mu_2^2-1}+(\mu_2^2-1)\cot\theta}. \quad . \quad . \quad (12)$$

In Fig. 4 T_θ is plotted for these two methods for different prism angles, the refractive index being taken as 1·6. Except near the limiting prism angle, for which, of course, the two methods tend to become identical, the minimum deviation method has a considerable advantage.

In Fig. 5, the tolerance with a 60 deg. prism is plotted for various values of the index.

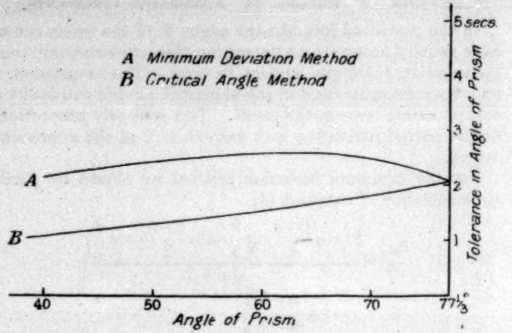

FIG. 4.

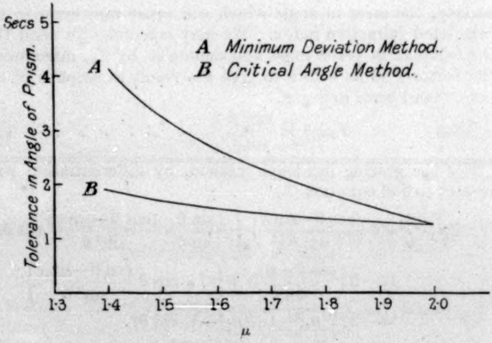

FIG. 5.

With either method we observe that the prism angle has to be known to an even greater accuracy than the deviation or angle of emergence of the refracted light. It is this that gives the 60 deg. prism the balance of advantage. For standardisation work a prism polished on all three sides can be used and each angle employed in turn, the mean being taken as 60 deg. exactly.* Where this cannot be done, on account of anisotropy, or in using a prism with only two polished faces, there is still an advantage in using a 60 deg. angle, as this can readily be determined by special methods to a high degree of accuracy. The corresponding functions for sensitivity and tolerance in angle for the method to which equation (7) refers, in which the substance under test is placed in optical contact with the first face of a prism of known properties, are fully discussed in the previous Paper, as is also the effect of error in the assumed optical constants of the standard prism. This case need not, therefore, be discussed here.

The author hopes that these two Papers, taken together will be of considerable utility to refractometrists, particularly those who have to attempt the most precise determinations, by giving in easily accessible form the means of judging at once the most useful method to employ for any particular purpose and by indicating readily the final accuracy which is possible of attainment with any particular instrumental equipment.

DISCUSSION.

Mr. T. **Smith** agreed that information such as that in the Paper would help to dispel the illusions concerning the accuracy of their observations which most workers on refractive index measurement seemed to possess. He had had occasion to analyse results supposed to be the last word in accuracy, but had found quite serious errors in many of them.

He would like to see the general adoption of some system of angular notation such as that used in the Paper. The only case which presented difficulty was when applying it to a system in which there were reflecting as well as refracting surfaces. He had found it convenient in such cases to keep to the angle convention, but to change the signs of the refractive indices after reflection.

Dr. D. **Owen** said the author had been speaking of tolerances of a few seconds. As a rule one felt satisfied when one got angles correct to a minute. What was the accuracy in angle which the author considered should be attained in precise measurements ?

Mr. **Guild** said that accuracy in angle measurements was largely a matter of choosing proper methods and apparatus. For the cases most frequently met there was no great difficulty in measuring to a second provided the surfaces of the prism were such that this accuracy had any meaning. He was at present experimenting with a very cheap and simple arrangement with which he hoped to reach a tenth of a second in cases which were fit for such precision.

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* Assuming no pyramidal error. In the case of a 60 deg. prism, if an edge of the prism is inclined to the opposite face by P seconds instead of being quite parallel to it, as in a true prism, the sum of the three angles of the prism is 180 deg.$+0{\cdot}279\ P^2\times 10^{-5}$ seconds (*vide* Guild, Trans. Opt. Soc., Vol. XXII., No. 3, 1920–21).

XIV. *A Note on the Hot Wire Inclinometer.* By R. H. Humphry, *M.Sc.* (*Birm.*) (*The Sir John Cass Technical Institute.*)

Received September 25, 1920.

ABSTRACT.

Two fine platinum wires were stretched parallel to each other in a hole in a copper block, and were heated electrically in a Wheatstone bridge arrangement. The changes caused by rotation were investigated with hydrogen, air and carbon dioxide surrounding the wires. It was found that both for the same current, and for the same temperature of the wires, the inclinometer filled with carbon dioxide was much more sensitive than one filled with air. The shape of the curves obtained suggests that the temperature gradient in the region traversed by the wires is nearly uniform, the difference of temperature of the layers of gas in which the wires happen to be being sensibly proportional to the vertical distance between them.

The original purpose of the experiments which are described below was two-fold. It was part of an attempt to elucidate the nature of the convection in the neighbourhood of hot wires, and also to determine if the principle could be applied to the construction of an inclinometer for use in inaccessible places where a spirit level could not be used. In view of the recently published work of J. S. G. Thomas, some of the results obtained may be of use as indicating how such an inclinometer as he describes may be made still more effective.

The apparatus consisted of two thin platinum wires, about $2{\cdot}5 \times 10^{-3}$ cm. thick, and 4 cm. long, stretched parallel to each other about 2·5 mm. apart, and symmetrically placed parallel to the axis in a hole in a copper block, the hole being about 6 mm. in diameter. The two wires formed two arms of a Wheatstone bridge, which could be balanced by moving a contact on a bridge wire. By a suitable arrangement different gases could be introduced into the cylindrical chamber containing the wires. The block could be rotated through any desired angle about a horizontal axis coinciding with that of the chamber, the zero position being that in which the wires lay in a horizontal plane.

The results for air at atmospheric pressure are shown in Fig. 1 for different bridge currents (corresponding to wire temperatures of from 60°C. to 150°C.).

For hydrogen, the effects for the same current are much smaller, while for carbon dioxide they are much greater. This is shown in the sensitivity curves of Fig. 2, and is to be expected, for the temperature of the wires would be much less in hydrogen, and greater in carbon dioxide for a given current.

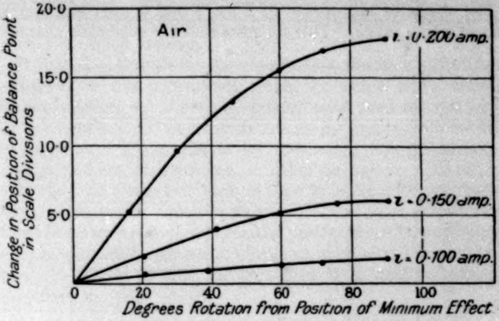

FIG. 1.—CURVES SHOWING RELATION BETWEEN CHANGE IN POSITION OF BALANCE POINT AND ANGLE OF ROTATION FOR AIR.

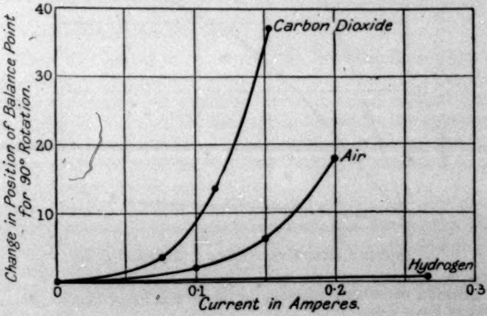

FIG. 2.—CURVES SHOWING RELATION BETWEEN CHANGE IN POSITION OF BALANCE POINT FOR 90 DEG. ROTATION AND CURRENT IN MAIN CIRCUIT FOR HYDROGEN, AIR AND CARBON DIOXIDE.

For the same temperature of the wires, the effects are also greater for carbon dioxide, and less for hydrogen as shown in

Fig. 3. This also is to be expected, for convection depends on difference of temperature, and the difference of temperature in different parts of the gas will be greater, the lower the thermal conductivity of the gas.

The shape of the curves in Figs. 1 and 3 approximates to that of a sine curve. This suggests that the gas in the chamber is warmer at the top than at the bottom, the gradient, at least in the region traversed by the wires, being nearly uniform, so that the difference of temperature of the two layers in which the wires find themselves is sensibly proportional to the vertical distance between them. There was no sharply defined

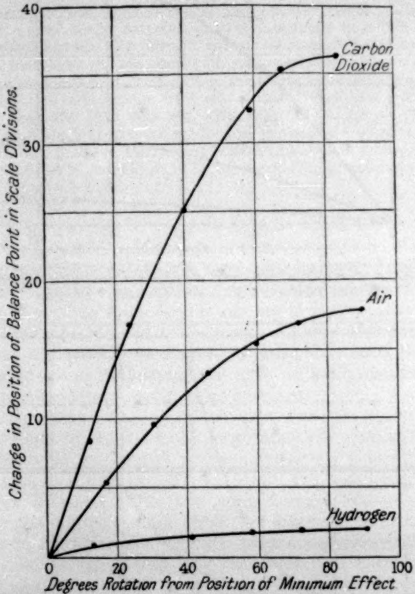

FIG. 3.—CURVES SHOWING RELATION BETWEEN CHANGE IN POSITION OF BALANCE POINT AND ANGLE OF ROTATION FOR SAME TEMPERATURE OF WIRE IN EACH GAS.

maximum as there would have been if a narrow upward current arose from the lower wire. Probably the upward stream had lost its sharpness, by conduction, by the time when it reached the upper wire when the latter was vertically over the lower wire. The case is different from that occurring in Mr. Thomas's experiments where the distance between the wires was much smaller.

It would appear from the results here described that as an inclinometer the instrument would be much more sensitive for a given current if carbon dioxide were used instead of air. In fact (as Dr. G. A. Shakespear points out), it may be modified to serve as a katharometer for measuring the percentage of carbon dioxide, or hydrogen, or other gas mixed with air.

It may be remarked that except in special circumstances, the instrument is unnecessarily elaborate, and in practice a spirit level would in most cases be preferable. It would moreover be subject to the same disadvantages as the latter instrument in failing to give the true level if it were subjected to acceleration.

These experiments were carried out in the Physics Department of the University of Birmingham in August, 1919, under the direction of Dr. G. A. Shakespear.

Note added December 29, 1920.

Since the above was written an investigation into the temperature distribution round a heated wire has been made at the Sir John Cass Technical Institute by a method due to Mr. J. Guild, to whom the author is indebted for the suggestion. It consists in placing the hot-wire system in one of the interfering beams of a Michelson interferometer arranged to give contour fringes with sodium light.

This method is being applied to the elucidation of problems in convection and conduction in gases, and the results which have already been obtained with a hot-wire inclinometer go to confirm what has been said above.

It is hoped to be able to publish these and other results at a later date.

DISCUSSION.

Dr. J. S. G. THOMAS said that the conditions under which the author's experiments were carried out made it difficult, if not impossible, to compare the results with those described in his own Paper on the subject. In his experiments the free convection loss was of much greater magnitude than the conductive loss, whereas in the present Paper the dimensions of the apparatus were such that probably conduction losses preponderated. The wires were disposed too near the walls of the containing chamber to permit convection losses to be of any great magnitude. The statement that for the same current the effects in air are less than those in carbon dioxide but greater than those in hydrogen is exactly what would be anticipated if conduction losses predominated. This result could not, however, be anticipated if convection losses predominated. Thus he had shown that, contrary to the author's contention, the temperature of a wire in carbon dioxide was not markedly different from what it was in air, provided the free convection effect was predominant. As King had shown, convection losses involved, not only the conductivity data, but likewise the specific gravity and specific heat of the medium. On the basis of King's formula, he had made a calculation of the thermal losses by convection, assuming the velocity of the convection current to be small, and had found the respective thermal losses on rotation in the case of hydrogen, air and carbon dioxide, assuming the wires initially at the same temperature, to be approximately in the ratio 12 : 100 : 200, which was not far different from the ratio of corresponding ordinates in Fig. 3. The wires in the author's experiments were so far apart that in all probability the lower wire did not experience any cooling effect when the second wire was rotated into a position at higher level. This effect itself would increase the sensitivity of the device considerably, and it was desirable to ascertain whether the sensitivity in carbon dioxide were greater than that in air when the disposition of the wires was such as to afford the maximum sensitivity in each case. It would be interesting to know whether experiments were carried out with larger currents than those shown in Fig. 2. His air results indicated that in air, with increasing current, the sensitivity increased as shown in Fig. 2, but ultimately reached a maximum value and thereafter decreased slightly. The experiments described in the previous Paper covered the range of temperature employed in the present experiments and extended to considerably higher temperatures. It would be useful to know the temperatures of the wires used in obtaining the results shown in Fig. 3, and also particulars as regards the purity or otherwise of the gases employed. In this connection, as was well known, contamination of hydrogen with minute quantities of impurities would have a marked effect upon the density of the gas, and hence of the convection loss.

Mr. J. GUILD said it was of interest to him that so many workers seemed to have taken up the somewhat obvious suggestion he had made, in discussing Mr. Thomas's Paper, of investigating the temperature distribution in the neighbourhood of a hot body by means of the interferometer. He would like to give a word of caution in connection with the method, which was that the contour fringes observed would only be true isothermals if the wire were infinitely long in the direction of vision. In ordinary conditions a slight correction for end effects would have to be made in obtaining quantitative data from the fringe system.

Mr. F. E. SMITH asked if experiments had been made at different pressures.

Mr. HUMPHRY, in reply, said that it was probably true that conduction losses from the wires in his experiments were of the same magnitude as the convection losses. But the change which had been investigated must necessarily have been caused by a change in convection, since, if there were no convection at all, there would be no change in the heat losses as the wires were rotated round one another. He did not know that the ordinates in Fig. 3 were in such close agreement with those calculated from King's equation for small velocity convection currents. The region of mutual effect between the wires could easily be seen from the interferometer experiments.

The effects of rotation were not investigated with higher currents than those shown in Fig. 2, nor with temperatures higher than that in Fig. 3, except for hydrogen. The temperature of the wires in Fig. 3 was about 130°C., and with the wires surrounded by hydrogen, a higher current than 0·2 ampere was necessary to maintain this temperature. At the time when the experiments were carried out it was not thought advisable to use higher currents with air and carbon dioxide.

The gases employed were obtained from cylinders of compressed air and hydrogen, and a cylinder of liquid carbon dioxide. The purity of the hydrogen was determined by means of a katharometer, and care was taken to ensure the purity of the gas in the chamber.

Mr. Guild had said that the bands seen in the interferometer were only isothermals if the wire were infinitely long. This had been considered in connection with the experimental work which was being carried out.

In reply to Mr. Smith, experiments at different pressures had not been made with the inclinometer.

XV.—*On the Magnetic Susceptibility of Certain Natural and Artificial Oxides. By* Prof. E. F. Herroun, F.I.C., *and* Prof. E. Wilson, M.I.C.E., M.I.E.E.

Received October 6, 1920.

SUMMARY.

(1) The susceptibility of ferric oxide, as occurring in nature, varies through a wide range, but in the case of artificial preparations the range of variation may be much greater.

(2) The highest susceptibility exhibited by artificial ferric oxide occurs when either ferrous or magnetic oxide is oxidised at a high temperature in the anhydrous condition. From hydrated magnetic oxide the susceptibility, though considerable, is much less.

(3) The lowest susceptibility is obtained from native hæmatites, or naturally oxidised chalybite, or by the ignition or precipitation of ferric salts.

(4) It appears that the passage through the stage of magnetic oxide impresses more pronounced magnetic properties upon the resulting ferric oxide.

(5) The actual values of susceptibility have been obtained in the case of ferrites, one of the most conspicuous being that of copper.

(6) Heating feebly magnetic ferric oxide with a basic oxide, *e.g.*, lime or magnesia, increases susceptibility (confirming List and others).

(7) When higher susceptibility has been produced by heating ferric oxide with a basic oxide such as that of silver or mercury, subsequent removal of the metal leaves the ferric oxide in a magnetic condition.

(8) The chemical compounds (aluminates) formed when ferric oxide is replaced by aluminic oxide show no definite increase in susceptibility, which is in marked contrast with the ferrites.

INTRODUCTORY.

Arising out of certain work on the magnetic susceptibility of Rock Specimens* and several varieties of Magnetite,† an endeavour has been made in the present Paper to explain certain peculiarities which were noticed in the course of those investigations. Though in general ferrous compounds, *e.g.*, Chalybite, show a higher susceptibility than the ferric oxides derived from them, certain specimens of specular iron ore were found to have a susceptibility higher than that of chalybite itself, while specimens of Red Hæmatite gave values smaller than either of the above. The view that substances having a constitution of the type of Magnetite ($FeO . Fe_2O_3$) exhibit a specially high susceptibility, appeared to merit further examination, and accordingly a large number of natural minerals and artifically prepared oxides were tested. Although several early workers at this subject prepared oxides of the above type, no determinations of susceptibility were then possible, and even later workers, who have investigated the transformation temperatures of such compounds, have not given absolute values of susceptibility. The relative importance of the FeO and Fe_2O_3 groups needed investigation, and this has been attempted in the present research. In this connection the authors regret that they have been unable to procure a specimen of Hercynite ($FeO . Al_2O_3$) sufficiently pure to test the influence of the FeO group in comparison, say, with minerals of the type of Franklinite ($ZnO . Fe_2O_3$). The number of minerals in which iron occurs in the form of FeO is very limited, Chrome Iron Ore and Pleonaste being the only examples so far tested apart from Magnetite.

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* "Phil. Trans." Roy. Soc., Series A., Vol. CCXIX. (Appendix 1919), pp. 83-87. Also "Proc." Roy. Soc., Series A., Vol. XCVI., 1920, p. 429.

† Proc. Phys. Soc., Vol. XXXI., Pt. V., 1919, p. 299.

TABLE I.

Natural Minerals.

Name.	Composition.	Density	H in C.G.S. units.	K_m in 10^{-3} C.G.S.
1.†Magnetite crystal ...	Fe_3O_4 or $FeO . Fe_2O_3$	5·06	22·5	615·0
2.†Magnetite massive .	Fe_3O_4	4·86	32	300·0
3. Franklinite	$(Zn, Mn) O . Fe_2O_3$	5·00	315	0·455
4. „	„	5·01	160	0·64
5. Pleonaste	$(Mg, Fe) O . Al_2O_3$	3·80	315	0·0243
6. Automolite or Gahnite	$Zn O . Al_2O_3$	3·65	318	0·013
7. Spinel	$MgO . Al_2O_3$	3·54	765	0·00062
8. Corundum Crystal ...	Al_2O_3	3·98	756	−0·00034
9. Sapphire (blue)	„	3·68	656	+0·00119
10. Sapphire (Siam)	„	3·995	776	0·00143
11. Ruby (Mysore)	„	3·96	790	0·00047
12. Red Hæmatite . (I.)	Fe_2O_3	4·80	305	0·025
13. „ (II.)	„	4·91	315	0·0368
14. Specular „ (I.)	„	4·89	222	0·205
15. „ (II.)	„	4·75	222	0·107
16. „ (III.)	„	4·90	113	0·517
17. Brown Hæmatite ...	$2Fe_2O_33H_2O$	3·70	315	0·0316
18. „ powdered and heated	Fe_2O_3	—	315	0·0175
19. Chrome Iron Ore (I.)	$(Fe Mg) O . (Cr, Al)_2O_3$	3·88	315	0·0565
20. „ (II.)	„	4·15	315	0·079

† Maximum values of K_m found for cut bars, by ballistic method.

In view of the fact that nearly all the tests were made with powders, it was necessary to adopt the mass-susceptibility as a standard. In this respect, therefore, the present results differ from those given in the previous communications referred to, in which the volume-susceptibility was used.

It should be mentioned that, with the exception of natural Magnetite (1 and 2 of Table I.), all the determinations were made with a magnetic balance, either of the form described in the Proceedings of the Royal Society, A., Vol. 96, 1920, p. 429, or of the portable type described in the Proceedings of the Physical Society, Vol. XXXI., Pt. V., 1919, p. 338.

NATURAL MINERALS.

The minerals comprised in Table I. can be conveniently divided into three groups. Nos. 1-7 and 19-20 are of the type represented by the formula $R''O . X'''_2O_3$. Nos. 8-11 are representatives of naturally crystallised alumina. Nos. 12-18 are examples of native ferric oxide in various forms.

As regards the first group, the first four contain ferric oxide combined with ferrous oxide or zinc oxide, whilst in the next three ferric oxide is replaced by alumina, the basic oxide being iron magnesium or zinc. As might be expected where the whole of the iron is substituted by elements like magnesium and aluminium, as in spinel (No. 7), the susceptibility falls to an almost insignificant figure. Where iron is present in the basic (FeO) or in the acidic (Fe_2O_3) condition, the susceptibility is higher in the latter case. For example, Franklinite (3, 4) gave susceptibilities $0{\cdot}455 \times 10^{-3}$ and $0{\cdot}64 \times 10^{-3}$, whereas Pleonaste (5) and Chrome Iron (19, 20) gave values so low as $0{\cdot}0243 \times 10^{-3}$, and about $0{\cdot}07 \times 10^{-3}$ respectively.

It might be thought from the importance of the ferric oxide in compound minerals as affecting susceptibility that this substance by itself would exhibit a high value. That this is not generally the case is shown by the low values given by red and brown Hæmatites (12, 13, 17). The specular ores (14, 15, 16) are all considerably higher, reaching the value of $0{\cdot}517 \times 10^{-3}$ (16). This shows that in native minerals the susceptibility of ferric oxide varies within wide limits, and this matter will be more fully dealt with later. In the case of these minerals in the solid it is noticeable that the higher susceptibility is associated with almost black crystalline form, and the red and brown Hæmatites are distinctly lower in susceptibility. When artificially prepared in the form of powders, considerable variations in colour are obtained, but give no indication as to the magnitude of the susceptibility. It is, of course, well known that the colour depends upon fineness of division and mode of preparation.

The close analogy between alumina and ferric oxide and the similarity of their mineral compounds induced the authors to examine the susceptibility of crystalline alumina in the form of Corundum, Sapphire and Ruby (8, 9, 10, 11), as well as the aluminates 5, 6, 7. The results show that the susceptibilities are very low, and, in the case of Corundum crystal (8), a small negative value has been found. The authors wish cordially to acknowledge the kindness of Dr. W. T. Gordon in supplying them with specially selected specimens.

IRON OXIDES AND FERRITES.

It is frequently stated that ferric oxide, in comparison with ferrous oxide, is practically a non-magnetic substance. The relatively high susceptibility of certain minerals in which there is no ferrous oxide suggests that this is not necessarily the case. For the purposes of comparison ferrous oxide and artificially-prepared magnetic oxide have been examined. The susceptibility of specimens of ferrous oxide, obtained by heating ferrous oxalate in the absence of air, gave a value of roughly one-half that obtained for artificially-prepared magnetic oxide. This result is given with some reserve, as it is impossible to guarantee the entire absence of magnetic oxide. The K_m value for precipitated magnetic oxide varied from 0·072 to 0·043 C.G.S. units, in a field $H=20{\cdot}4$, according to the method of preparation. As regards ferric oxide, the lowest value was obtained from powdered brown hæmatite, dehydrated by heating to bright redness: $17{\cdot}5\times10^{-6}$. In ascending order of magnitude the next example was obtained from basic ferric acetate by gradual heating, $K_m=63\times10^{-6}$. Ferric oxide, obtained from sodium ferrite by washing with hot water, gave the largely increased value 213×10^{-6}. This latter figure is greatly surpassed in the case of ferric oxide produced by the ignition of pyrophoric ferrous oxide. Rapid oxidation by full exposure to the air and subsequent heating on platinum foil gave a ferric oxide whose susceptibility was $4{\cdot}9\times10^{-3}$, and by slower oxidation the figure obtained was $28{\cdot}1\times10^{-3}$. A high susceptibility was exhibited by ferric oxide obtained by heating dried artificial magnetic oxide $K_m=26\times10^{-3}$. When the hydrated magnetic oxide was oxidised on platinum foil, a lower but still high value was obtained.

An important but neglected Paper "On Iron Rust Possessing Magnetic Properties," by Professor A. Liversidge, M.A., F.R.S.,* deals with this interesting subject, and he shows that ferric oxide which has been slowly produced on iron by long exposure to the air is comparatively a highly magnetic substance, even in cases in which chemical analysis showed a complete absence of ferrous oxide. In this Paper reference is made to earlier work on this subject, notably that of J. Robbins (*Chemical News*, Vol. I., 1859, p. 11), who obtained a peroxide of iron by igniting magnetic oxide, which still possessed strongly magnetic properties although proved to be free from ferrous oxide. Malaguti† described a magnetic variety of ferric oxide. A sample of iron rust was finely powdered and examined for ferrous oxide, no trace being found. Its susceptibility was $4{\cdot}4 \times 10^{-3}$, and this confirms the statement above referred to. After heating to redness on platinum foil the value of K_m was reduced to $3{\cdot}0 \times 10^{-3}$. C.G.S.

As a comparison with the oxide formed in the rusting of iron, an outer layer of chalybite, which by its colour was in a highly oxidised state, caused by weathering, was reduced to powder, which had a value of K_m equal to $34{\cdot}7 \times 10^{-6}$. The original chalybite in the deeper layers was then tested, and found to have a susceptibility of 103×10^{-6}. In the conversion of the carbonate into ferric oxide by the action of air and water at atmospheric temperature the susceptibility is reduced to a value almost identical with the values found in cases of brown and red hæmatites.

On heating the true chalybite in powder freely exposed to the air, the resulting oxide (free from ferrous compounds) was found to have a susceptibility of $13{\cdot}8 \times 10^{-3}$, which is indicative of the effect of the conversion of ferrous compounds at a high temperature. The specimen of chalybite, used as above, contained, as is usually the case, a small percentage of manganese and magnesium carbonates. In conformity with other experiments described in this paper it is possible that the high value of the susceptibility is in part due to the influence of the oxides of these metals.

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* Transactions of the Australasian Association. Hobart Meeting, 1892.

† "Ann. de Chim. et de Phys." Series (3) 69, 214.

It should be mentioned that the ferric oxides examined by the authors were chemically tested for ferrous oxide with a negative result.

Moissan* stated that magnetic oxide of iron exists in two modifications, one having a density of 4·86, obtained by reducing ferric oxide at 500° C., and the other a density of 5·18, obtained at higher temperatures, and Berthelot† showed that the form of lower density can be transformed into the one of higher density by heating strongly in nitrogen. He further drew a distinction between the two forms in that the one of lower density is soluble in nitric acid, and easily oxidised by heating in air on platinum foil, while the denser form resists both actions. The authors found that, although artificial magnetic oxide was easily converted into ferric oxide by heating in air, the finely powdered natural crystal (Traversella) resisted all attempts to oxidise it completely; even fusion with potassium nitrate produced only slight oxidation. In the case of magnetite from Altenfjord the oxidation was much more pronounced.

Early workers on this subject, notably Elliot (1862), Karl List‡ (1878), Percy, Ebelmen, Daubrée, Reich and others, obtained ferrites, such as franklinite, many of which were variously described as "feebly" or "strongly" magnetic. More recently Bemmelen and Klobbie§ and S. Hilpert‖ have examined a large number of ferrites with special reference to their transformation temperatures, but apparently no determinations of the actual susceptibilities were made.

Largely from this standpoint, it appeared desirable to prepare other ferrites, in which the ferrous oxide was substituted by some other oxide. Owing to the close similarity of cobalt to iron, one of the first to be tested was cobalt ferrite. The method employed in the preparation of this and other ferrites was the precipitation of the hydrated oxides from solutions in molecular proportions of the soluble salts of the metals concerned, either by sodium or ammonium hydroxide according to their relative suitability. The resulting precipitate, after washing, was collected and dried at from 95° to 100°C., and the susceptibility in the hydrated state was determined. The proportion of water actually found was

* "Ann. de Chimie" (5), 21, 233.
† "Ann de Chimie" (5), 23, 118.
‡ "Deut. Chem. Gas. Ber.," 1878, p. 1512.
§ "Journ. f. prakt. Chem." (2), 46, 496 (1892).
‖ "Deut. Chem. Ges. Ber." XLII. (1909), B.2., p. 2248.

not always that indicated by theory for a true hydroxide; nor is it possible to say how the water is shared between the oxides in a given case. The hydrates were rendered anhydrous by strongly heating on platinum foil, and the susceptibility of the resulting ferrite was then found. This method was the one employed by List, who obtained a number of ferrites, and mentions that many of them are magnetic. Hydrated cobaltous and ferric oxides had a susceptibility of 107×10^{-6}. After heating to about 900°C. the susceptibility was found to have increased about a hundred-fold. Similar results were obtained in the case of nickel.

Manganese presents a difficulty in the endeavour to form a ferrite, $MnO . Fe_2O_3$, as it appears possible that manganous oxide may reduce ferric oxide at a high temperature. A mixture of manganous and ferric sulphates, in molecular proportions, was precipitated by sodium hydroxide, and the precipitate washed and dried with the least possible air exposure, as manganous hydroxide is well known to absorb oxygen readily. Before heating the mixture gave a value of 98×10^{-6} for K_m, and after ignition on platinum foil it had increased to $26{,}900 \times 10^{-6}$, but, as this enormous increase may in part be due to the formation of true magnetite, the result must be accepted with some reservation. The following equation might express the change: $3\ (MnO . Fe_2O_3) = Mn_3O_4 + 2Fe_3O_4$. Chemical analysis of the product would not show the distribution of oxygen, as on solution in an acid, *e.g.*, hydrochloric, the higher oxide of manganese, would convert the ferrous into a ferric salt, and no ferrous compound would be indicated, although it might have existed as such in the solid.

Remarkable figures were obtained with copper ferrite. Precipitated from hot solutions, but not boiling, the susceptibility was 102×10^{-6}. From boiling solutions the value was found to be 486×10^{-6}. Both of these after ignition gave the very high figure $33{,}000 \times 10^{-6}$. The susceptibility of copper ferrite therefore considerably exceeds cobalt or nickel ferrites, or, indeed, any similarly constituted compound excepting magnetite itself. Hilpert (*loc. cit.*) has found that a lead-copper ferrite, if rapidly cooled from 900 to 1,000°C., is as strongly magnetic as copper ferrite, but if slowly cooled (five to six hours) it is weakly magnetic. The authors have examined copper ferrite in this connection, and have found that slow cooling (four hours) from about 960°C. reduced the susceptibility to about one half. Subsequent heating to the same temperature, followed by rapid cooling, restored the susceptibility to nearly its original value. In this connection it is interesting to note that copper pyrites shows a considerable increase in susceptibility on being heated in the presence of air with the formation of the double oxides. The susceptibility of one specimen of powdered $CuFeS_2$ increased from $6{\cdot}4 \times 10^{-6}$ (its initial value) to $4{,}480 \times 10^{-6}$.

Zinc ferrite is interesting as being an artificial representation of franklinite, and, whereas the susceptibility was 77×10^{-6} before heat treatment, it was 258×10^{-6} after treatment. This final value is about a half of that given by the specimens of franklinite actually tested (*see* Table I.). Cadmium behaves similarly.

The increase is more marked in the case of magnesium, the actual figures being 79×10^{-6} and $3{,}100 \times 10^{-6}$ before and after heating. Calcium ferrite, prepared by strongly heating a mixture of slaked lime ($Ca(OH)_2$) and ferric oxide in a crucible, was found to have a susceptibility of $8{\cdot}1 \times 10^{-3}$. Subsequent boiling with water and drying in air and re-heating reduced this figure to 3×10^{-3}.

Silver was chosen because on heating its oxide is resolved into the metal and oxygen, and consequently no ferrite, but a mixture of ferric oxide and metallic silver would result. As in the other cases, it showed an increase in susceptibility, the figures being $93{\cdot}6 \times 10^{-6}$ and $6{,}700 \times 10^{-6}$. Metallic silver was finally removed by treatment with hot nitric acid, and the susceptibility of the resulting ferric oxide was found to be $8{,}380 \times 10^{-6}$—a not unexpected result if the silver merely acted as an inert body. Evidently during the change the ferric oxide in contact with the silver oxide had acquired a marked increase in susceptibility, which it retained after the reduction of the silver oxide and removal of the silver itself.

The figures obtained in the case of mercury ($K_m = 65{\cdot}8 \times 10^{-6}$ and 915×10^{-6}), though not so marked as in the case of silver, show that a similar effect is produced when the mercury is entirely volatilised in the process of heating.

Up to this point basic oxides have been employed in conjunction with ferric oxide. A variation was now made by using aluminia and ferric oxide, the susceptibility being $25{\cdot}6 \times 10^{-6}$ before, and $1{,}080 \times 10^{-6}$ after heat treatment. Chromic and ferric oxides show a similar result, though less marked ($K_m = 40{\cdot}5$ and 418×10^{-6}). The result in the last two examples indicates that a definitely basic oxide is not essential to the production of enhanced susceptibility. It appears, therefore, that whenever ferric oxide is heated in intimate contact with other oxides, more especially the basic ones, *e.g.*, lime, combination may occur leading to a considerable increase in the susceptibility.

Alumina and Artificial Aluminates.

In Table I. it is shown that corundum crystal (8) has a small negative susceptibility. The hydrated alumina has also a negative susceptibility, almost of identical value $-0{\cdot}337\times10^{-6}$. After ignition the susceptibility was still negative, but smaller in value—$0{\cdot}098\times10^{-6}$. Special care had to be taken in the preparation of this substance, as the presence of a minute trace of iron would cause its susceptibility to become positive in value. Additional figures were obtained from varieties of aluminates. Cobalt (38 to 43×10^{-6}) and copper ($6{\cdot}2$ to $9{\cdot}2\times10^{-6}$) show a small increase after heat treatment, and manganese a small diminution (76 to $71{\cdot}2\times10^{-6}$). The result in the case of copper is specially

Table II.

Certain Hydroxides and Oxides.

Substance and Formula.	H in C.G.S. units.	K_m in 10^{-6} C.G.S. units.
Nickel hydroxide $Ni(OH)_2$	315	48·3
Nickel Oxide NiO	305	53·7
Cobaltous oxide CoO	318	74·5
Cobalto-cobaltic oxide Co_3O_4	318	39 to 43·6
Cobaltic oxide Co_2O_3	318	34·3
Manganous oxide MnO	315	81·0
"Red oxide" Mn_3O_4	430	68·0
Manganic oxide Mn_2O_3	318	67·0
Manganese dioxide MnO_2	288	66·2
Chromic hydroxide $Cr_2(OH)_6 . H_2O$	300	45·5
Chromic oxide Cr_2O_3	300	29·4
Cuprous oxide Cu_2O	670	1·2
Hydrated cupric oxide $Cu(OH)_2 . CuO$	305	5·0
Cupric oxide CuO	305	3·8
Zinc hydroxide $Zn(OH)_2$	724	−0·487
Zinc oxide ZnO	717	−0·305
Hydrated alumina $Al_2O_32H_2O$	650	−0·337
Alumina Al_2O_3	670	−0·098

interesting, as it shows the vast difference in behaviour between a copper aluminate and a copper ferrite. Copper chromite, $CuO.Cr_2O_3$, also has a low susceptibility, $26{\cdot}7\times10^{-6}$.

Certain Hydroxides and Oxides.

In the course of the investigation a number of pure oxides had been prepared and their susceptibilities measured, and these are given in Table II. It is interesting to note that, in the case of cobalt and manganese, the susceptibilities of the different oxides show a steady decrease with increase of oxygen, and, unlike iron, there is no increase in the susceptibility associated with Co_3O_4 or Mn_3O_4 corresponding with Fe_3O_4.

The susceptibility of pure copper is well known to be negative, but its oxides are all positive, though small in magnitude.

The figures obtained in the cases of chromium and zinc call for no special comment, but are given because these substances have been employed in the earlier stages of the work. It is mentioned in Watts' "Dictionary of Chemistry" that Wohler many years ago described a magnetic oxide of chromium. Recently Soné and Ishiwara * have investigated this substance, to which they assign the formula Cr_5O_9.

For further information on the subject of pure oxides standard books of reference, such as Landolt and Börnstein, should be consulted, where the results obtained by various investigators will be found. It is noteworthy, however, that in the 1912 edition of the above work no values are given for the oxides of iron, cobalt or nickel and the results quoted for other oxides show considerable variation as measured by different workers.

DISCUSSION.

Prof. **Bragg** showed a model of the molecule of magnetite as deduced from its X-ray spectra, and pointed out various conclusions which could be drawn from it. For instance, it was impossible to deduce the properties of the magnetite from those of Fe_2O_3 and FeO, nor could any of its properties be ascribed to one or the other, because the crystalline form of the Fe_2O_3 and FeO in the magnetite molecule differed from their crystalline form when separate. It was possible by X-ray spectroscopy to say what crystals were present in any powder. No doubt magnetic effects were associated with the crystal structure; but experiments on pyrrotine showed no difference between the magnetised and unmagnetised state.

Dr. **Rayner** asked if differences in aggregate form rather than in crystal form might not account for the different magnetic properties of some substances of the same composition.

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* "Science Abstracts," Vol. XVIII., No 926.

Prof. **Wilson** said he was disappointed by Prof. Bragg's remark about pyrrotine as it partially answered the question he wished to ask. Would the X-ray spectroscope be likely to show any difference in the two ferric oxides of such widely different susceptibilities mentioned in the Paper ?

Prof. **Bragg** : If there is any difference in crystal structure it would show it.

Mr. **Guild** said the case of pyrrotine was different from that of the two ferric oxides. The experiment with pyrrotine showed that there was no difference in the molecular structure of a magnetisable substance when magnetised and unmagnetised. This one would expect since the phenomena of magnetisation are accounted for by the orientation of the molecules, and not by any alteration of their properties. The case of the two ferric-oxides in which Prof. Wilson was interested was quite different. Here we had magnetisable and non-magnetisable modifications of the substance. In this case one would expect to find a difference in molecular structure revealed by the X-ray spectrometer.

Dr. J. S. G. **Thomas** asked if the differences might not be due to differences in the orientation of the magnetic couplings between the electronic orbits. Did X-ray analysis afford any evidence of such differences of orientation ?

Prof. **Bragg** : That is the problem we are all working at. In the case of the diamond there is no doubt that the carbon atoms point in opposite directions.

Prof. **Herroun** said that in pyrrotine the susceptibility in one direction was 70 times that in another. Would the X-ray spectra give any clue to this ?

XVI. *A New Registering Microphotometer.* *By* DR. W. J. H. MOLL, *University of Utrecht.*

RECEIVED JANUARY 1, 1921.

(COMMUNICATED BY MR. R. S. WHIPPLE.)

ABSTRACT.

A diminished image of a slit, on which the filament of a half-watt lamp is focussed, is projected by means of a microscope objective on the photographic plate or other object of which the absorption is to be measured. A second similar objective focusses an image of the slit, magnified up to its original size, on a second slit behind which is mounted a sensitive thermopile of the author's own design connected to an improved D'Arsonval galvanometer. By suitable gearing to an electromotor the photographic plate is given a slow motion at right angles to the beam of light, and at the same time a drum of photographic paper, on which the spot from the galvanometer is focussed, is caused to rotate at a suitable rate. A record is thus obtained of the density of the plate at different parts encountered by the beam of light. The arrangement is dead beat and so quick in response that intensity curves of close spectrum lines, Zeeman triplets, &c., are accurately recorded.

PROFESSOR HARTMANN gave the name microphotometer to an instrument which he had designed to investigate the density of very small areas of a photographic plate. He compared each area with a "photographic wedge," *i.e.*, a plate of continuously decreasing density. The arrangement of his apparatus will be clear from the accompanying diagram, Fig. 1, on which the photographic plate and the wedge are indicated by P and K respectively; they receive their light by way of the mirrors S and S' from a plate of ground glass, illuminated by the small lamp, L. W is a Lummer and Brodhun double prism upon the diagonal plane of which the images of small parts of P and K are thrown by the objectives O and O'. The observer, looking through the eye-piece A, which is focussed on the centre of the diagonal plane, obtains an image of the small area of the plate P to be examined, surrounded by the image of a certain part of the wedge W. By moving the wedge, the observer may find a position such that the contrast between both images has disappeared. By reading this position and repeating the observation for other parts of the photographic plate, this latter may be investigated all over the required surface.

Hartmann's microphotometer, in its original form, was designed solely with a view to searching for areas of equal density; as such it was only a supplementary instrument in photographic photometry. It has also been applied, however, to a problem of no less importance and of wider bearing, viz., the registering of the structure of complicated photographic pictures (*e.g.*, absorption and emission spectra).

An investigation of this kind implies, however, a long series of measurements by visual photometry. This is a tedious piece of work, and very trying to the observer's eye; notwithstanding the utmost care the precision of measurement will gradually diminish. Moreover, the fact that the observer has to compare areas of some extension renders it nearly impossible to detect delicate details, especially in objects of linear structure.

These considerations led Koch to the construction of an apparatus working automatically and provided with a slit; in this self-registering microphotometer Koch rejects the principle of photometry by contrast, the density being determined by directly measuring the transmitted light with the aid of a photo-electric cell. The principle of Koch's apparatus is shown in the diagram, Fig. 2. The light of a Nernst-glower N_1 is concentrated upon the photographic plate P by means of a condenser C. The objective O throws an enlarged image of the plate on the slit S_1, which cuts off all the light except that from a narrow strip of the plate P. The transmitted light falls on the cathode of a photo-electric cell F, which is connected with the negative pole of a storage-battery B. The

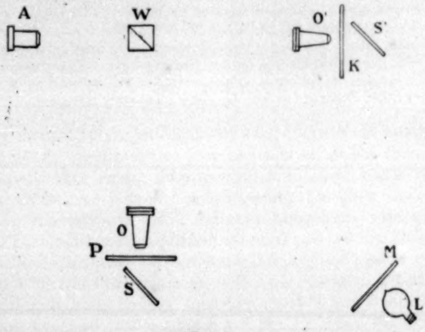


FIG. 1.

electric charge of the anode is allowed to flow to earth through a high resistance; the potential of the anode is measured by a string-electrometer *E* of the usual type. Evidently the deflections of the electrometer will more or less accurately indicate the degree of darkness of the narrow portion of the plate under observation. These deflections can be photographically recorded on a sensitive plate *R* by means of a second Nernst-lamp N_2, and a couple of lenses L_1 and L_2. *P* and *R* are simultaneously shifted by one and the same mechanism, *P*'s motion being slow and horizontal, whereas *R* moves rather quickly in a vertical direction. In this way a curved line is recorded on the plate *R*, its abscissæ indicating different places on the plate *P*, and its ordinates giving the degree of density at these places.

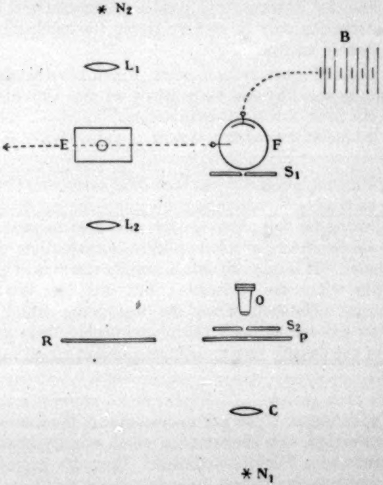


FIG. 2.

Koch himself has pointed out the difficulties encountered in the effort to obtain reliable results with this very delicate apparatus. Even if extreme care has been taken in regard to the electrostatical protection of the electrometer, the cells and their connections, unexpected movements of the string are to be expected; the latter, moreover, is extremely sensitive to vibrations of the ground.

But the most serious objection to the method is the slowness of reaction of the photo-electric cell, which inevitably lags behind the variations of the incident light. Now abrupt changes of density seldom occur on photographic plates, and therefore, though the curve secured with a slow cell may seem to give a reasonable record of the phenomenon, this record does not represent the true distribution of density. On account of the lagging of the cell's indications a fully symmetrical spectral line, for instance, will yield an asymmetrical curve. This objection can only be met by giving the mechanism the slowest possible motion.

In all kinds of spectographic work, which have lately been carried on in the Physical Laboratory of the University of Utrecht, the need of a trustworthy microphotometer was felt. This has led me to the construction of an apparatus of my own design.

The new microphotometer has been built mainly on the same principle as Koch's, but an important improvement is effected by substituting for the photo-electric cell used in connection with an electrometer, a thermopile in combination with a galvanometer. It is now, of course, mainly the rays of greater wavelengths which are operative; but this does not make any essential difference, since the weakening which they undergo on passage through the photographic plate gives a measure of the density just as definite as when visual rays are employed.

The new arrangement, on the other hand, presents a number of great advantages. The galvanometer and the thermopile, of my own design, are instruments which exactly answer to the demands of a microphotometer. They are rapid, very sensitive, and so indifferent to external disturbances that, while in use, they do not need any special supervision.

I now proceed to the description of the new microphotometer, the principle of which will be clear from Fig. 3. In this diagram T represents a thermopile and S_1 a narrow slit. L is a lantern containing a small incandescent lamp and fitted with a condenser C, which projects the glowing spiral of the lamp upon the slit S_2. O_2 and O_1 are two microscope objectives; the former throws a greatly reduced image of the slit S_2

upon the photographic plate *P*, and the latter forms an enlarged image of this plate on the slit S_1. The plate *P* is given a slow shifting motion by a micrometer screw. The thermopile is directly connected with the galvanometer, whose mirror reflects a small spot of light on to a registering drum *R*. The way in which the motion of *P* is connected with that of *R* will be explained later on.

Fig. 4 gives a sketch of the whole instrument, one-third of the actual size. The base of the instrument consists of two

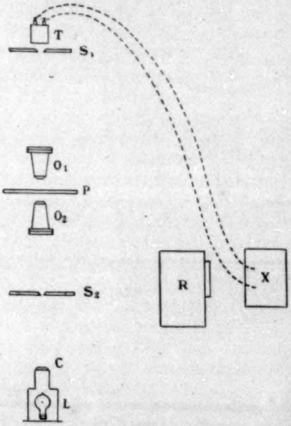


FIG. 3.

parts, mounted together on a marble slab *Z*: firstly, a heavy cast-bronze plate *U*, bearing a table *D*, and, secondly, an iron rail *V*, of triangular cross-section, along which the three bearers *I*, *II* and *V* may be shifted. *I* carries the small lantern *L*, *II* the slit S_2, and *V* the thermopile *T*. The table *D* carries the bearers *III* and *IV* of the objectives O_2 and O_1.

The axle *A* is the principal part of the motive machinery; that part of it which goes through the metal block *B* carries a micrometer screw with a pitch of 1 mm., which causes this block to slide between the two guiding rails F_1 and F_2. Both the micrometer screw and its nut should be constructed with the utmost care. The plate-holder consists of two parts *M* and *N*. *N* is fixed to the block *B*, and upon it the photographic

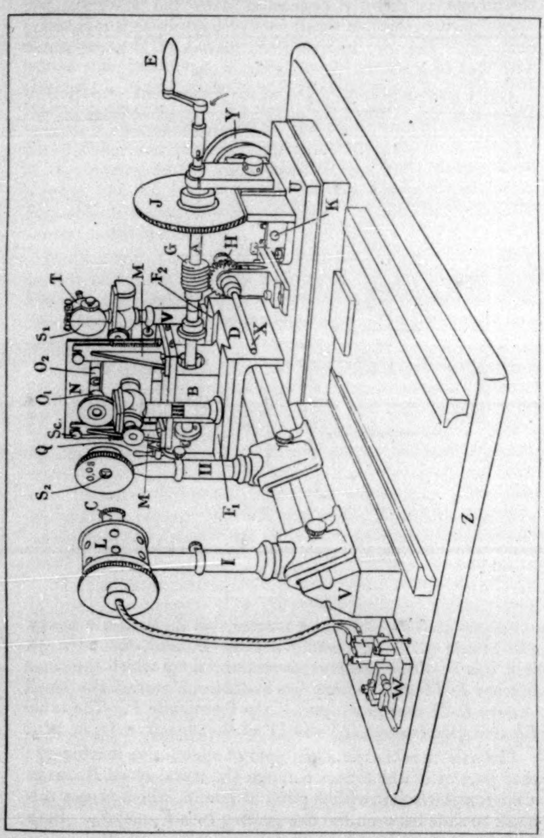

E
Y
J
U
K
H
G
F2
M
T
V
S1
O2
N
O1
B
D
X
Sc.
Q
II
M
F1
S2
0,05
C
L
I
V
Z
W

FIG. 4.

plate is rigidly clamped. The part *N* may be moved up and down a little by the handle *Q* and clamped to *M* by means of the screws *Sc*, so as to allow different parts of one and the same spectral line to be examined.

The axle *A* bears also a triple wormscrew *G*, which causes a wormwheel *H* to rotate at a 10-fold slower rate. The steel axle *X* of this wheel can be clamped to the driving mechanism of the registering drum (not shown in the sketch). This drum, whose circumference measures 40 cm., can be made to rotate either with the same speed as the wormwheel *H* or four times more slowly. These speeds correspond to a shift of the photographic plate of 40 and 10 mm. respectively for one revolution of the drum.

The axle *A*, finally, bears a second wormwheel *J* with 200 teeth, engaged by the wormscrew *K*, which by means of a triple pulley *Y* and a cord is driven by a moderate-speed electric motor (not shown in the sketch). By means of an adjustable resistance and a commutator the speed of rotation of the motor may be varied within wide limits and the sense of rotation reversed. Moreover, an arrangement has been devised to confine the motion of the block *B* between two limits; with *B* in either of the two extreme positions, the rotation of the axle ceases without the micrometer screw being injured. The screw *K* may be released from the wormwheel *J* and the axle turned by a handle *E*, in order to bring the desired part of the photographic plate between the two objectives. By doing this the registering drum, of course, is also turned, but it can be replaced in its original position by a special device.

The optical arrangement of the microphotometer has already, in its principal lines, been explained in Fig. 3. For further particulars we may refer to Fig. 4. The thermopile *T*, protected by a glass window of 6 mm. diameter, is mounted in a heavy brass cylinder. The jaws of the slit S_1 are formed by two half-circles of white celluloid, providing a surface on which careful focussing is possible. The width of the slit can be adjusted by a screw with divided head. The lantern *L* with condenser *C* holds a small incandescent lamp with special fitting, which secures a steady position of the lamp. A key *W* of special design ensures a quite reliable electric contact. The two objectives are adjustable in the direction of the optical axis; the thermopile and slit S_1 may be shifted sideways.

The microphotometer is adjusted as follows: Remove the slit S_2 and bring the objective O_2 close to the photographic plate; a relatively large area of this plate is then illuminated. Now produce a sharply defined image of the plate on the jaws of the slit S_1; this leads to the correct position of the objective O_1. If a line-spectrum is to be examined, care must be taken, by turning the slit S_1 a little, that it runs parallel to the projected lines. Replace the slit S_2, and let a sharp image of the spiral of the lamp be formed upon it. Next move the objective O_2, and turn the slit S_2 so as to obtain a sharp image of it on the slit S_1, running parallel to the latter. Finally, shift S_1 into coincidence with the image of S_2, and open the slit to the required width.

In contrast to S_1, the slit S_2 has fixed jaws; it can, however, be replaced by others, so that the observer may have at disposal four different widths, *viz.*, 0·05, 0·1, 0·25 or 0·4 mm.

One might suppose that the slit S_2 and even the condenser C might as well be omitted, so as to project an image of the spiral directly on the photographic plate. It is true that this simpler arrangement would yield even greater deflections of the galvanometer. But omitting the slit S_2 would entail serious diffuse scattering of the light in the sensitive film, so as to injuriously affect the delicate details of the phenomenon under consideration. In Koch's microphotometer this danger is not met by use of the slit, which he places *behind* the photographic plate (S_2 in Fig. 2); this slit only improves the sharpness of definition by partly shutting off the "false light." In my instrument the slit S_2 not only cuts off the false light totally, but also reduces the unavoidable scattering by the photographic plate to a minimum.

As to the choice of a suitable place for the instrument, no special conditions are necessary. It is important that the galvanometer should be free from mechanical vibration; but this can be secured merely by mounting it on an ordinary bracket attached to a solid wall. The registering apparatus need not be placed in the dark, if care is taken that no direct light can fall on the cylindrical lens in front of the drum.

In conclusion I should like to show some of the results obtained with, the new microphotometer, which may give an idea of its efficiency.

Fig. 5 is the reproduction of a photogram yielded by a Zeeman sextet; the photographic plate was kindly provided by Professor P. Zeeman. The sextet, obtained with a spark

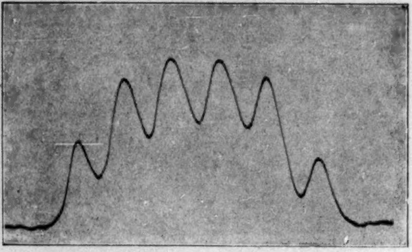


FIG. 5.—PHOTOGRAM OF ZEEMAN SEXTET.

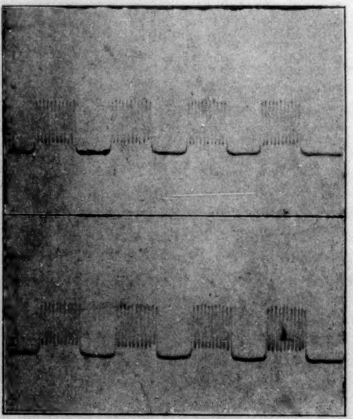


FIG. 6.—PHOTOGRAM OF ABBEY TEST-PLATE.

To face p. 215.]

between Na-electrodes, had a total area of about 1 mm.^{2} Fig. 6 represents the photogram yielded by an "Abbe test-plate." This is a glass plate with a layer of silver in which 4 series of 11 lines each have been engraved; each line has a width of about 0·02 mm., and the space between two adjacent lines is also about 0·02 mm; the lines have been irregularly engraved on purpose. The photogram shows very clearly the differences between the successive lines. The object has been recorded twice in succession; the two curves are sensibly identical.

DISCUSSION.

Mr. R. S. Whipple thought that the dead-beatness and quickness of the apparatus were unequalled. There was a beautiful feature of the records to which Dr. Moll had not referred; they showed the actual texture of the photographic plate as ripples on the base line. He believed the author was engaged in another research which would throw light on some important problems in connection with the blackening of photographic plates. It would be difficult for most of those present to imagine that the record of the Zeeman effect shown had been taken inside half a minute. It was a triumph of experimental skill.

Prof. Rankine said he had had the privilege of being about during the last few days when Dr. Moll and Prof. Ornstein had been erecting the apparatus. He had suggested that the instrument might be used to settle the question of the existence of the canals on Mars. In this connection Dr. Moll had told him that the eye saw many things which did not exist and missed many things which did. An example which might be of interest to radiologists present was the case of an X-ray photo of a square hole in an opaque screen. It was frequently asserted that the intensity was greater at the edges of such a square than at the centre. The thermopile showed that this was wholly an illusion.

Mr. Guild said he could only express the greatest admiration for the instrument they saw before them. He was particularly interested in the galvanometer. They had recently been installing radiometric apparatus in the Optics Department of the National Physical Laboratory, and were using a Paschen galvanometer. They were troubled so much with the zero changes due to stray magnetic fields from the adjacent electric railways and from sources within the laboratory that the advantages due to the sensitivity of the galvanometer were largely nullified. He looked forward with very great interest to the advent of Dr. Moll's thermopiles and galvanometers on the British market.

Dr. J. S. Anderson said he had had the pleasure of hearing Koch describing his microphotometer. He had thought the chief objection to Koch's instrument was the variability of the photo electric cell. The modern forms were much less variable and probably the objection did not now hold to the same degree. Dr. Moll was, however, to be congratulated on so successfully employing the much more easily manipulated thermopile for the purpose.

Mr. C. R. Darling thought the instrument would be readily adapted to form a recording radiation pyrometer.

Mr. F. E. Smith said he would like to know how Dr. Moll had made such an excellent thermopile. Through the kindness of Mr. Whipple he had had the pleasure of working with one of them, and there was no question of its excellence, but he had not been able to see how it was made.

Dr. **Moll**, in reply, said the secret of his thermopiles consisted in making the junctions which received the radiation of very low heat capacity. With most thermopiles the bulk of the heat capacity was due to the solder at the junctions. His method of avoiding this was as follows: He took two thick plates of the metals to be used, constantin and manganin, and soldered them end to end, scraping off the excess of solder. The metals already joined in this way were then rolled out to the desired thinness, and cut into strips. In this way strips only 2μ thick were obtainable with no excess of solder at the junctions. The other junctions were made of large heat capacity to give constancy of temperature of the "cold" junctions.

As regards the galvanometer, the principal modification which he had introduced was to have the coil held by two strips of bronze in tension instead of suspending it by the upper strip in the usual way. This gave greatly improved constancy of zero.

XVII. *A Balance Method of Using the Quadrant Electrometer for the Measurement of Power.* *By* HUBERT PARRY.

RECEIVED JANUARY 1, 1921.

(COMMUNICATED BY R. S. WHIPPLE, ESQ.)

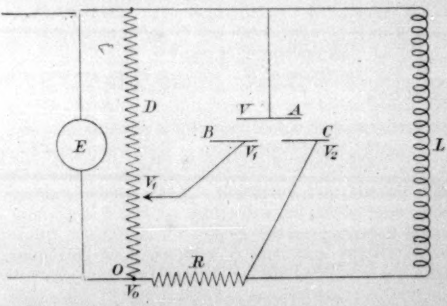
ABSTRACT.

A new method of using the quadrant electrometer for the measurement of electrical power is described. It involves the use of a potential divider across the supply circuit, and a standard non-inductive resistance in series with the load. An ammeter and a voltmeter are employed to measure the supply voltage and the load current.

An example of its application to determine the power-factor of a condenser is quoted.

INTRODUCTION.

ANYONE who has used a quadrant electrometer as a deflectional wattmeter will know the difficulties in getting this instrument to obey the "Heterostatic Law" over fairly large angular deflections, and in keeping it so from day to day; it requires very great accuracy in workmanship so that the instrument shall be free from electrical control with the vane or needle in any position between the quadrants. It was to get over these difficulties that I conceived the method described in this Paper.



Method.—The electrometer is used as a balance indicator, as shown in the figure, where *A* represents the needle, *B* and *C* the quadrants, *R* a noninductive resistance in series with the load *L*, *D* a calibrated potential divider coupled between the potential point O on the supply side of *R* and the far side of the load, to which point the needle A is coupled. E is a voltmeter.

It is, of course, necessary that the instrument shall be free from electrical control: that is, that the zero shall not change at any voltage over the working range, when the voltage is applied to the needle and the quadrants shorted; this condition is easily got by slightly tilting the instrument.

Now, if a voltage be applied to the needle A, and we have a voltage on quadrant C proportional to the current, we can apply with the potential divider D a voltage in phase with the needle voltage to quadrant B, so that the average torque in the instrument is zero. As the instrument is used it will obey the "Heterostatic Law" and, should the power-factor of the circuit be unity, the voltages on B and C will be equal, hence the product of the voltages on the needle A and quadrant B will be proportional to the watts expended in R and L. If the power-factor is not unity, then the average voltage on B will be less than on C, but a voltage can be applied to B such that the average torque is zero.

Theory.—

Let v be the instantaneous voltage on the needle A,
,, v_1 ,, ,, ,, ,, quadrant B,
,, v_2 ,, ,, ,, ,, quadrant C,
,, v_0 ,, zero potential at point 0,
and ,, R ,, resistance of R: where v, v_1, v_2 are periodic functions of time.

The theoretical law of the electrometer is given as

$$\text{Torque} = K\{v - \tfrac{1}{2}(v_1 + v_2)\}(v_1 - v_2)$$

where K is the instrument constant. As the needle is always in the same position this law will hold good.

Hence the instantaneous torque (t)

$$= K\{v - \tfrac{1}{2}(v_1 + v_2)\}(v_1 - v_2),$$
$$= K[vv_1 - vv_2 + \tfrac{1}{2}(v_2^2 - v_1^2)].$$

Then by integrating over a complete period and taking the average we have the average torque (T)

$$= K[VV_1 - VV_2\cos\varphi + \tfrac{1}{2}(V_2^2 - V_1^2)] = 0,$$

where V, V_1, V_2 are square root of the mean square values and $\cos\varphi$ the power-factor of the circuit.

If we divide the above expression by R and transpose terms, we have

$$\frac{VV_2}{R}\cos\varphi = VI\cos\varphi = \frac{VV_1}{R} + \frac{1}{2R}(V_2^2 - V_1^2) =$$

power expended in the resistance R and load L, where I is the R.M.S. current in the circuit.

As an example, and to show the fair agreement with results got with a standard Duddell wattmeter, a 1 mfd. condenser was tested at 90 volts, 40 cycles, for losses at air temperature.

The Duddell instrument gave :—

Watts, 0·0039 ; power-factor, 0·0012, at 92 volts, 40 cycles.

The method described gave :—

(1) Watts, 0·0040 ; power-factor, 0·0019 at 90 volts, 40 ∿, 11°C.

(2) Watts, 0·0030 ; power-factor, 0·0015 at 90 volts, 40 ∿, 11°C.

Below are given the detailed figures :—

V.	Potential divider ratio.	V_1	V_2	R	Freq.	Milli-amps.	Temp.
(1) 90·1	$36{\cdot}7\times10^{-5}$	0·033	$2{\cdot}26_8$	100	40·25	$22{\cdot}6_8$	11°C.
(2) 90·75	$35{\cdot}3\times10^{-5}$	0·032	$2{\cdot}28_2$	100	40·25	$22{\cdot}8_2$	11°C.

The total resistance of the potential divider is 100,000 ohms, and the division can be altered by unit steps, keeping the total resistance constant. The first decimal place is by estimation.

This is quite good when one considers the low power-factor, and that the instrumental losses are of the same order as those to be measured. In the above test the voltage V was measured by a reflecting electrostatic voltmeter where the sensitivity is such that 8 cm. of scale represent 1 volt. V_1 is calculated from potential divider data and V; V_2 from the current and resistance R, or measured with an A.C. potentiometer.

CONCLUSION.

It will be seen that the voltmeter is the only instrument needing a scale ; these instruments can be made with strong control and will keep their accuracy from day to day. The potential divider and series resistance need no special comment, except to say that the potential division must be true as regards phase as well as magnitude. The advantages are that no wattmeter scale is needed, and therefore no calibration, and the electrometer does not need to be so accurately made as for the deflectional method. The voltmeter and potential divider would still be needed for calibration if the electrometer was used as a deflectional instrument.

The disadvantages are, slowness of working, and the need to have a very steady supply for accurate work.

I have used the method for over a year with quite good results, and as far as I am aware it is new. I have published it in the hope that it may be of interest and use to others.

DISCUSSION.

Mr. C. C. **Paterson** said he had worked for 10 to 15 years with electrometers for power measurements. He would like to see the method tried out more fully under varying conditions. With small power-factors one always had the result as the difference of two large terms when using the electrometer.

Another point was that the two voltages V_1 and V_2 were not in phase. Of course, V_2 was small but it would surprise him to find that it was legitimate to add them as the author had done.

Dr. E. H. **Rayner** said they had not yet tried the method at the National Physical Laboratory, but were certainly going to. He wondered if the difficulty mentioned in connection with small power-factors could not be overcome by connecting the needle to the mid-point of a large dividing resistance. There was always a danger, however, with small power-factors that the dividing resistance was not true as regards phase on account of the distribution of capacity.

Dr. A. **Russell** congratulated the author. The method was novel to him. The old method was quite good for high voltages, but for low voltages he thought the author's method would be superior.

Dr. D. **Owen** said: The author's Paper, of modest dimensions, contains an idea which promises to add decidedly to the value of the quadrant electrometer as used for the measurement of electrical power. The method, being a null one, possesses several advantages. In the first place, it makes it possible to utilise to the full the possession of an electrometer of the highest sensibility attainable in virtue of mechanical design. Further, as the constant of the electrometer disappears from the expression for the power, the usual calibration experiment with non-inductive load becomes unnecessary.

In the form proposed, however, the method has one decided disadvantage, namely, that at low power-factors the power required was obtained as the difference of two separately determined quantities, each of which might be of much greater magnitude. This is so, indeed, in the very test chosen by the author for illustration, and the effect is evident in the considerable discrepancy between two successive tests on the same condenser.

Dr. Rayner has expressed the possibility of this source of error being overcome by connecting the needle to the mid-point of the non-inductive resistance across the mains. This had occurred to me, and on trying it I found it to be the case. The adoption of this modification would not only eliminate the source of error occurring at low power-factors, but would simplify the expression for the power in the general case. Denoting by V/n the voltage drawn from the potential-divider (denoted by V_1 by the author) the expression for the power supplied to the load becomes

$$W=\frac{V^2}{R}\cdot\frac{n-1}{n^2}$$

All that is required in a test now is to adjust the tapping-point on the potential-divider until the electrometer needle is at zero, and at that instant to read off V on the voltmeter. n may be observed at leisure, and the only remaining factor in the calculation of the power is the standard non-inductive resistance R.

Mr. **Parry** communicated the following reply :—

The theory of the method is based on the conception of the electrometer acting as two instruments in opposition—the one having elements A and B working at unity power factor with potentials V, V_1 and V_0; the other with elements A and C working at the power-factor of the load and potentials V, V_2 and V_0. If Mr. Patterson will look at it from this viewpoint I think he will find the latter part of his remarks answered.

As pointed out by Dr. Owen and Dr. Rayner, the difficulties at low power factor are eliminated by making the needle potential $\frac{V}{2}$; it had not occurred to me to do this, though I knew of this property in the deflectional methods.

Dr. Owen's formula can be expressed in a slightly more convenient form by taking $V_1 = KV$; then

$$W = \frac{V^2}{R} (K - K^2).$$

K can generally be read directly on a potential divider.

The method has checked out very well with results got by dynamometer wattmeters working over a power factor range of 1 to 0·3. I shall be very pleased to hear the N.P.L. opinion after they have tried the method.

In conclusion, I should like to thank Dr. Owen for the large trouble he has taken over my small Paper.

XVIII. *Application of the Ionisation Spectrometer to the Determination of the Structure of Minute Crystals. By Prof. Sir* W. H. BRAGG, *F.R.S.*

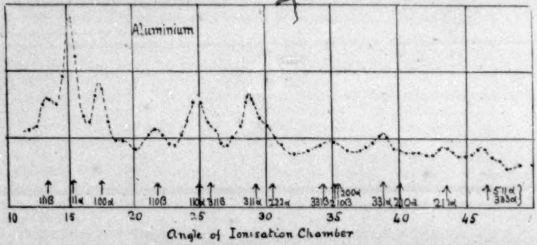
RECEIVED MARCH 31, 1921.

ABSTRACT.

It is shown that crystals in the form of powder can be examined by the ionisation method. The powder is pasted on a flat surface and placed on the spectrometer table in the position ordinarily occupied by the face of a single crystal. A bulb current of 1 milliampere is sufficient to give satisfactory records.

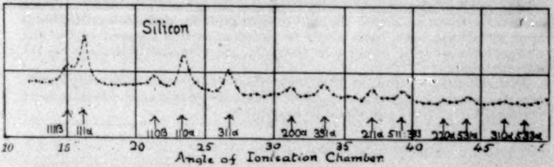
IN the first determinations of crystal structure by the help of X-rays large single crystals were employed; and the reflections were observed by means of the ionisation effects of the rays. It is not convenient, however, to handle in this way crystals weighing less than a few milligrams. The great majority of substances do not yield crystals even as large as that.

Debye and Hull have independently developed a method of dealing with crystals in a state of minute subdivision. A monochromatic beam of X-rays is passed through a small quantity of the powder; some of the small crystals are oriented so as to yield each and every one of the reflections proper to the particular wave length and the particular crystal. The results are recorded photographically; and it is usual to give long exposures with heavy discharges in the X-ray bulb. This is necessary because only a small fraction of the crystalline matter is at any one moment so oriented as to reflect.

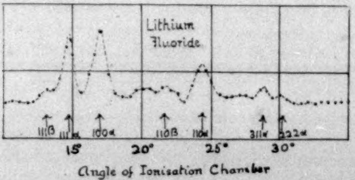


A few instances given below show that crystals in the form of powder can be examined satisfactorily by the ionisation method. The powder is pasted on to a flat surface and placed on the table of the spectrometer in the position ordinarily occupied by the face of a single crystal. The slit of the spectrometer may be wide enough to take in reflections over half a degree. The intensity in different parts of the spectrum can be examined in detail. Each reading occupies a few seconds and the main part of the spectrum can be run through in half an hour. A bulb current of one milliampere is sufficient.

The method has some advantages of its own. It measures relative intensities without further calculation, and avoids the difficulty of interpreting the blackening of a photographic plate.



The aluminium spectrum shown in the figure displays the reflections characteristic of a face-centred cubic lattice, and of the simple piling of atoms into a close packed structure. The corresponding photographic spectrum is given by Hull in the "Physical Review," X., 1917. The rays used in finding spectra here given were those of the K series of rhodium. The silicon spectrum is characteristic of the diamond lattice; its most obvious difference from that of aluminium is the absence of the 100 line. The spectrum of lithium fluoride is notable for the peculiarity in the relative magnitude of 100α and 111α. If the lithium atom had been quite negligible compared to the fluorine atom the spectrum would have been



similar to that of aluminium, with its large 111 reflection. But the lithium atom is not negligible, though small; and so the 111 spectrum is considerably reduced. In rock salt it is known that the 111 is quite small; in that case the metal atom is larger in comparison to the halogen atom.

DISCUSSION.

Mr. J. GUILD said that while listening to this Paper, with the preceding Paper by Dr. Moll on improved thermo-electric measurements still in his mind, he had been wondering whether the modern improvements in the methods of X-ray production and of radiometry had possibly brought us within reach of direct measurement of the energy of X-radiation. Using a sensitive thermopile with lead foil receiving plates, and a heavy discharge from a Coolidge tube, there might be sufficient energy stopped by the foil, although most of it would pass through, to give an indication on the galvanometer.

Prof. ORNSTEIN thought that insufficient energy would be stopped by the thermopile to produce any effect unless the lead was so thick that all the sensitivity of the pile would be destroyed.

XIX. *The Effect of Viscosity on Orifice Flows.* By W. N. Bond, *B.Sc.* (*Engineering*), *M.Sc.*(*Lond.*), *A.R.C.S.*, *A.Inst.P.*

RECEIVED NOVEMBER 16, 1921.

ABSTRACT.

Determinations were made of the coefficient of discharge through an orifice 0·1469 cm. in diameter of solutions of glycerine and water, varying in kinematic viscosity from 0·01 to 7. The results are plotted in a manner which combines both purely viscous and purely turbulent flows in one graph. It is shown that the effect of slight viscosity is to increase the coefficient of discharge.

A LARGE number of experiments have been carried out by different workers on the flow through orifices when the effect of viscosity can be neglected. When viscosity has to be taken into account, the value of the coefficient of discharge C, given by $C=Q/(A\sqrt{2gh})$, should be the same for all cases having the same value of $vl\rho/\mu$,

where Q=observed volume passing per second.
h=the difference of " head " between the two sides of the orifice.
A=the area of the orifice.
$v=Q/A$=the average velocity at the orifice.
l=the diameter of the orifice.
ρ=the density of the liquid.
μ=the viscosity of the liquid.

The kinematic viscosity μ/ρ may be denoted by ν.

The following experiments were carried out for J. L. Hodgson, Esq., B.Sc., A.M.I.C.E., of Messrs. Geo. Kent, Ltd.; the object of the investigation being to obtain the values of C for lower values of $vl\rho/\mu$ than were previously available, in the case of a circular sharp-edged orifice in a thin plate, having liquid on both sides of it.

It was known from previous experiments that as viscosity becomes more important the value of C is at first increased; and it is necessary that for yet more viscous liquids C should decrease to zero. Since the completion of these experiments the author has found that G. F. Davidson* had previously attempted to obtain similar information in the case of a round edged orifice. For his experiments a thick engine oil was used, the viscosity being varied at will by changing the temperature.

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* Proc. Roy. Soc., A., Vol. LXXXIX., 1913-14, pp. 91-99

The values obtained for the coefficient of discharge were not constant for any one value of $vl\rho/\mu$, and he traced the cause of the discrepancy to the fact that the oil he was using had not a definite value of ν independent of the rate of shear. The value of ν was found to decrease for an increase in the rate of shear. Similarly an increase in the value of ν when the rate of shear is made small has been found by Kurt Molin,* using Searle's viscosity apparatus.

In the present experiments, at Mr. J. L. Hodgson's suggestion, mixtures of glycerine and water were used, the value of ν being thus varied from about 0·01 to 7.

The viscosity of each mixture used was determined by the capillary tube method, the capillary tube being placed in a water bath. The values obtained were consistent and agreed in the cases of water and pure glycerine with the values usually accepted.

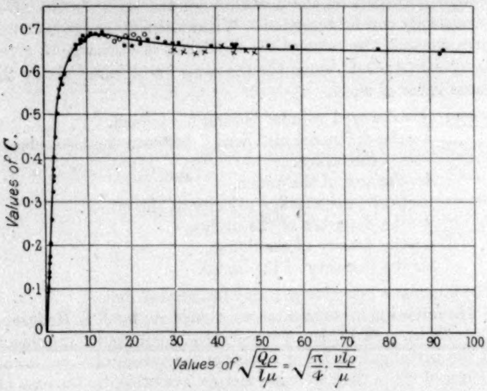

FIG. 1.

The density of each mixture was determined by means of a hydrometer.

The orifice was made by clamping a piece of thin hard-rolled sheet German silver between two gunmetal plates and drilling through the three together. After unclamping, the orifice plate was polished on both sides to ensure the edge being right-angled. This plate was then soldered to a small gunmetal ring, which in its turn was fastened to one end of a wide glass tube with plasticine. The wide glass tube, which was about 76 cm. long by 2·85 cm. diameter, was clamped vertically with the orifice at the lower end. This end was arranged to dip into a large beaker containing some of the liquid being used. A scale divided in millimetres was pasted along the length of the tube. The area of cross-section of the tube at different points was obtained by closing the lower end, pouring in successive known volumes of water and noting the corresponding rise in level.

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* Kurt Molin, Proc. Camb. Phil. Soc., Vol. XX., Pt. I., pp. 23-34, 1920.

The diameter of the orifice was measured by means of a microscope fitted with an eyepiece scale micrometer. Check measurements were obtained by other means. The mean of the determinations was 0·1469 cm. diameter. The thickness of the orifice plate was obtained by means of a micrometer screw gauge, which gave a value of approximately 0·0075 cm.

In the flow experiments the vertical tube was filled with liquid, the orifice being meanwhile closed by one finger. An air-tight cover was placed on the top of the tube to prevent flow taking place till desired. The beaker was put in position and any air bubbles removed from the orifice plate. Time was allowed to let any bubbles in the liquid rise to the surface. On removing the top cover the time was noted when the falling liquid surface in the tube passed suitable divisions on the scale. The level in the beaker was read when desired, and a correction for its rise applied to the value of the "head." The temperature of the liquid was observed and a viscosity test carried out at the same temperature.

From the determinations of the time of passing successive divisions on the tube scale, the rate of flow could be deduced, since the area of cross-section of the tube had been obtained by the preliminary experiment. This very convenient method of experimenting was suggested by Mr. F. Gray.

For the timing in the case of the less viscous mixtures a chronoscope reading to 1/50 sec. was used; and it was found that successive readings could be taken satisfactorily at intervals of as little as six seconds, the observing and writing being carried out by two observers.

In all the more viscous mixtures, great care had to be taken to remove air bubbles. Also in the case of the most viscous liquids there was an appreciable quantity of liquid left clinging to the inside of the glass tube. The error this would cause was made small by the use of the small size of orifice adopted.

The space between the orifice and the bottom of the beaker was varied in the case when $\nu=0{\cdot}35$ from 2·2 cm. to 4 cm., and no measurable change in C occurred. For most of the experiments the length of this free downstream was 2·2 cm. which is about 15 times the orifice diameter.

At the end of this set of experiments the orifice plate was fitted into a pipe line and the value of C obtained for larger water flows.

In plotting the results, instead of plotting the values of C against those of $v l \rho/\mu$ or its equivalent $4 Q\rho/\pi l \mu$, it was found better to plot against $\sqrt{Q\rho/l\mu}$ in order to obtain a more convenient representation of both viscous and turbulent portions of the curve on a single graph. This method of plotting has also the advantage that it causes both the purely viscous and the purely turbulent portions of the graph to become straight lines.

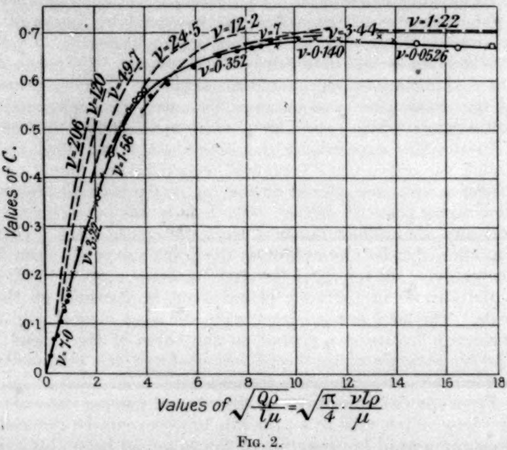

FIG. 2.

Fig. 1 shows the curve obtained by plotting the results on this basis. In Fig. 2 the portion corresponding to viscous flow is plotted on a larger scale. The curves given by C. F. Davidson[*] in Fig. 2 of his Paper have been reduced to the same basis and are indicated by the broken curves in Fig. 2. The values of the kinematic viscosity $\nu=\mu/\rho$ are given by each curve. The figures above refer to Davidson's experiments; those below to the present results. The points obtained with different values of ν are distinguished by plotting successive sets with dots, crosses, circles, and so on. In Fig. 2 the viscous flow determinations (shown in detail in Fig. 2) are for simplicity indicated entirely by dots.

It will be noted that the extreme values of ν used are in ratio of about 1 to 700 (cf. Davidson's 1 to 1,000). Moreover, the results obtained with the more viscous mixtures lie extremely closely along a straight line through the origin. This indicates that true viscous flow is obtained; and further that under the conditions of these more viscous experiments the viscosity of the mixtures is to a close degree independent of the rate of shear. However, when less viscous mixtures were used, there are indications of successive experiments with different amounts of dilution showing slight disagreement; and it is probable that for the less viscous mixtures (and consequently greater rates of flow) the value of ν is no longer independent of the rate of shear. Thus though the left-hand end of the curve is probably determined with fair accuracy, the shape of the apex may be slightly in error, due to variations in ν similar to those observed by Davidson. Any error due to this cause is probably confined to the portion of the curve near the apex, since the linear law agreement obtained for purely viscous flows renders error there improbable; and for purely turbulent flows any variation in ν with rate of shear can have no effect on the resultant curve.

The fact that viscosity, when its effect is small, produces an increase in the coefficient of discharge (perhaps due to its causing an increase in the coefficient of contraction) is in agreement with previous experiments carried out at Messrs. Geo. Kent's, and with the results obtained by Davidson in the case of water. The results obtained by Osborne Reynolds[†] and subsequent experimenters for the pressure gradient in the case of flow in pipes, show a similar increase in the discharge caused by slight viscosity. Another set of experiments showing a similar result is the determination at the National Physical

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[*] Proc. Roy. Soc., A., Vol. LXXXIX., 1913-14, p. 94.

[†] Osborne Reynolds, Phil. Trans. Roy. Soc., 1883.

Laboratory of the force exerted on a cable by a transverse air current.*

In conclusion the author would like to thank Messrs. Geo. Kent, Ltd., for the facilities that have made these experiments possible; Mr. J. L. Hodgson, at whose instance the experiments were made, for his kindness in suggesting such an interesting problem for investigation; and Mr. F. Gray, conjointly with whom the tests were made, for his kindly co-operation throughout the progress of the work.

DISCUSSION.

Mr. A. S. E. **Ackermann** said he had made experiments of a similar character in which the substance tested was clay. The clay was packed in a box with an orifice of from 3 to 20 millimeters diameter in the bottom. The space above the clay contained air, and the pressure was raised by means of a pump.

The speaker then drew curves on the board to show his main results. With any particular orifice a certain initial pressure is required before any clay is forced out. As the pressure increases the rate of the flow becomes more rapid, and finally becomes very rapid indeed. The critical pressures and the orifice diameters were connected by the law $pd^{0{\cdot}883}=5408$ for a clay with 29 per cent. water. Similar results are obtained in forcing a disc through clay. The speaker mentioned that these and other results were given in a Paper by him to the Society of Engineers, which was at present in the Press.

Mr. **Bond** said the phenomena described by Mr. Ackermann were probably due to plasticity, rather than viscosity.

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* Report of the Advisory Committee on Aeronautics, 1912-14.

XX. *Viscosity of Water at Low Rates of Shear.* *By* ALBERT GRIFFITHS, *D.Sc., and* CONSTANCE H. GRIFFITHS, *M.Sc.*

RECEIVED JANUARY 27, 1921.

ABSTRACT.

This Paper describes the determination of the coefficient of viscosity of water by a method in which water is forced along glass capillary tubes of about 1·5 to 2·0 mm. bore at rates of flow varying from 1 litre in two years to 1 litre in 24 years. The liquid fills a closed tubular circuit which for purposes of description may be said to be rectangular in shape. Two of the tubes of the rectangle are horizontal, and two vertical. The circulation is caused by a difference of density between the contents of the two vertical tubes. The difference is obtained by having a weak solution of uranine in one vertical tube, and pure water in the other.

In Apparatus I. the two vertical tubes are at a distance apart, and an allowance has to be made for the difference of temperature. In Apparatus II. a modification of the rectangular circuit is employed, the vertical tubes are close together, and no such correction is necessary.

Applying the temperature correction to the results recorded in this Paper, the value of the co-efficient of viscosity at 18°C. is 0·010624 ± 0·000229 (assuming observations of equal weight) and 0·010569 gm./cm. sec. from Hosking's formula. Thus there is no experimental evidence that at the extremely low rates of shear (0·0017 to 0·0233 radians/sec.) the viscosity of air-free water in glass capillary tubes differs from its value at normal rates of shear (5,000 to 10,000 radians/sec.).

§1. Introduction. §2. Apparatus I.
§3. Apparatus II. §4. Results.

§1. INTRODUCTION.

THIS Paper describes the determination of the coefficient of viscosity of water by a method in which water is forced along glass capillary tubes of about 1·5 to 2 mm. bore at rates of flow varying from 1 litre in two years to 1 litre in 24 years.

The liquid fills a closed tubular circuit which for purposes of description may be said to be rectangular in shape.

Two of the tubes of the rectangle are horizontal and two vertical. The circulation is caused by a difference of density between the contents of the two vertical tubes. The difference of density is obtained by having a weak solution of uranine in one vertical tube, and pure water in the other.

In 1912[*] the authors studied the flow of distilled water along a capillary tube, the maximum rate of shear at the boundary of the tube being about one radian per second. In the experiments to be described the rate of shear at the boundary of the tube was reduced to about one six-hundreth of a radian per second.

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[*] Proc. Phys. Soc., Vol. XXIV., Part V., Aug. 15, 1912.

In 1912 the water was freed from air at the commencement of an experiment, but was exposed to air during the progress of the experiment, freedom from fungoid growths being ensured by the addition of a trace of copper salt. No evidence of abnormality was discovered in the coefficient of viscosity.

In the present experiment air-free water completely fills a closed circuit, and as there can be no absorption of air during the course of an experiment, it is not necessary to add a trace of copper salt to keep down the fungoid growths.

§2. Apparatus I.

(A) General Description of Apparatus.
(B) Theory of Method.
(C) Numerical Details of Apparatus.
(D) Difference of Density between Water and a Dilute Solution of Uranine.
(E) Details of Procedure.

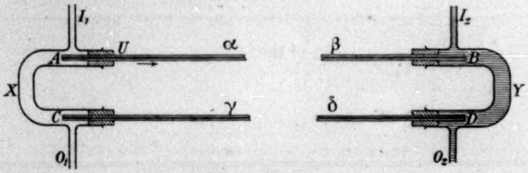

Fig. 1.

(A) *General Description of Apparatus.*—The apparatus is attached to a rectangular beam pierced at a distance equal to one-quarter its length from each end by two transverse arms at the ends of which work vertical screws supported on upright blocks. By rotating these screws the apparatus can be adjusted with considerable delicacy and the upper face of the beam made truly horizontal. Two straight capillary tubes, *AB*, *CD*, are fastened with hasps to the upper and lower faces of the beam so as to be parallel to one another, and, as indicated in Fig. 1, are connected so as to form a closed circuit by two U-tubes, *X* and *Y*, the joints being made with rubber stoppers. The circuit is first filled with air-free distilled water, and the water in Y is then replaced by a dilute aqueous solution of uranine, the small driving force in the circuit being due, as already stated, to the difference in density between the solution and water. The solution of uranine is so dilute that to the degree of accuracy of the present experiments its viscosity may be assumed to be that of water. It may be noted that the present apparatus differs from that described in 1912 in that there is no surface of separation between air and water, and what is more important, there is no line of contact of air, water, and glass.

In the tube, AB, is inserted a short column, U, of dilute uranine solution which acts as an index whose linear movement measures the flow.

(B) *Theory of Apparatus I.*—

Let h=the vertical distance in centimetres between the centres of the capillary tubes, AB and CD.

δ=the difference between the densities of the liquids in X and Y in gms./cc.

g=the acceleration due to gravity in cms./sec.^{2}.

Then the driving head in dynes per square centimetre=$hg\delta$.

Let l_1=the length of the tube, AB, in centimetres.

$\frac{1}{r_1^4}$=the mean value of the fourth power of the reciprocal of the radius of the tube, AB.

Let l_2, $\frac{1}{r_2^4}$, l_3, $\frac{1}{r_3^4}$, l_4 and $\frac{1}{r_4^4}$ indicate corresponding values connected with the tubes, CD, X and Y.

Let η=coefficient of viscosity.

Q=rate of flow of liquid in cubic centimetres per second.

Then it can easily be seen that the head in dynes per square centimetre required to overcome the viscous resistance equals

$$\frac{8\eta Q}{\pi}\left(\frac{l_1}{r_1^4}+\frac{l_2}{r_2^4}+\frac{l_3}{r_3^4}+\frac{l_4}{r_4^4}\right).$$

Equating the two expressions for the head we obtain

$$hg\delta=\frac{8\eta Q}{\pi}\left(\frac{l_1}{r_1^4}+\frac{l_2}{r_2^4}+\frac{l_3}{r_3^4}+\frac{l_4}{r_4^4}\right).$$

The apparatus is of such dimensions that the last two terms within the brackets may be neglected.

Thus

$$\eta=\frac{\pi h g \delta}{8Q\left(\frac{l_1}{r_1{}^4}+\frac{l_2}{r_2{}^4}\right)}.$$

It may be mentioned that for ease of description the rectangular circuit is considered to be vertical, and in the experiments of this Paper this was the case. But, the tubes, *AB* and *CD*, retaining their horizontality, the circuit can be placed at any angle with the vertical. Let a be the angle the circuit makes with the vertical. Then h varies as $\cos a$, and may have any value between zero and the distance separating the tubes, *AB* and *CD*. The same apparatus may thus be used over a wide range of rates of flow.

(C) *Numerical Details of Apparatus Used in Method I.*—

$$l_1=129{\cdot}2 \text{ cms.}$$

$$\frac{l_1}{r_1{}^4}=4{\cdot}233\times10^{+6}\text{cms.}^{-3}$$

$$l_2=129{\cdot}2 \text{ cms.}$$

$$\frac{l_2}{r_2{}^4}=5{\cdot}449\times10^{+6}\text{cms.}^{-3}.$$

$$h=5{\cdot}811 \text{ cms.}$$

(D) *Difference of Density between Water and a Dilute Solution of Uranine.*—The method employed to determine the difference of density was a slight modification of that employed by F. Kohlrausch.* Instead of using a silk fibre for the suspension, a platinum wire was employed, and this wire, where it entered the liquid, passed through a short glass tube, dipping into the liquid. Into this tube was poured a small quantity of pure paraffin. This arrangement avoided a platinum-air-water contact.

In the experiment for the determination of the viscosity the uranine solution in Y is not at exactly the same temperature as the water in X, and it is necessary to apply a correction for the difference of temperature.

Let e=the difference between the density of the uranine solution and that of the water. Let θ=the excess of the temperature of Y above that of X. Then, assuming that the coefficient of expansion of the dilute solution of uranine is not appreciably different from that of pure water, to the degree of accuracy necessary in the experiments, in the neighbourhood 18·5-19·0°C., we may write

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* F. Kohlrausch and Hallwachs, Wied. Ann., LIII., 1894, p. 1.

$$\delta = e - 0{\cdot}0002\theta.$$

It may be mentioned that the uranine used was the ordinary commercial product.

The graph in Fig. 2 gives the relation between density and concentration with one sample of commercial uranine. It suggests that there is a linear relation between the density and the concentration.

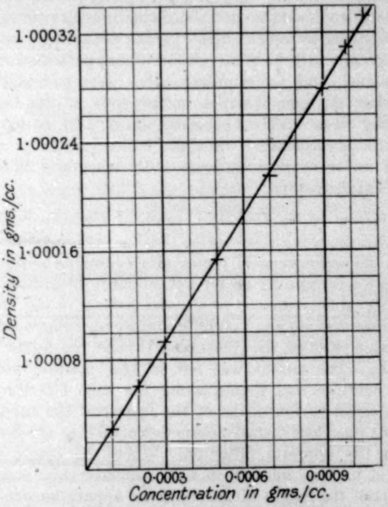

Fig. 2.—Variation of Density of Dilute Aqueous Solutions of Uranine with Concentration.

(E) *Details of Procedure in Method I.*—The apparatus is not provided with taps, and the procedure in filling the apparatus is troublesome. In the first place, the authors designed an apparatus easy to fill and requiring taps of special construction; but in the conditions prevailing at the end of the war they could not obtain their requirements. The absence of taps has the advantage that there is no need to consider the effect of any tap-lubricant on the viscosity of water.

It is necessary not only to fill Y with weak uranine solution, but also the neighbouring parts of the tubes AB and CD, otherwise the concentration of the solution in Y would vary with the flow. In addition, an index must be introduced in AB.

The inlet and outlet tubes (I_1, O_1, I_2, O_2) of the connecting tubes were fitted with stout caoutchouc pressure tubes, which were controlled by screw clips. (These details are not shown in the figure.) All the glass parts of the apparatus were carefully cleaned, and the capillary tubes were passed through indiarubber stoppers inserted in the ends of the U-shaped connecting ends. All rubber-glass joints were sealed externally with seccotine, and when this had dried the circuit was fastened to its supporting beam with hasps and filled with air-free distilled water.

Introduction of the Driving Column of Uranine Solution.—The apparatus was supported so that the two capillary tubes were in the same vertical plane, and I_1 and O_1, being closed, an aspirator containing an air-free solution of uranine in distilled water was connected to O_2, and 200 c.c. of this solution, whose concentration was 0·00097 grammes per cubic centimetre, were passed through the right-hand U-tube, the liquid emerging at I_2. The circuit was left in this position until the uranine solution had flowed along the tube CD through a distance equal to four-fifths of the length of the tube. The apparatus was then turned upside down, so that the flow took place in the opposite direction. When the length of the column of uranine solution in each capillary tube was about two-fifths of the length of the tube, the apparatus was placed on its side, the two capillary tubes being in the same horizontal plane.

The object of having columns of uranine solution in the right-hand side of each of the capillary tubes was to minimise any discontinuities at the ends B and D of the capillaries, and to ensure constancy in the value of the driving head. To make certain that the solution in Y was of the standard strength, an additional 250 c.c. were passed through after the apparatus had been allowed to rest for about 40 hours.

Introduction of the Recording Index into AB.—The apparatus was placed vertical, and, I_2 and O_2 being closed, the solution of uranine was passed in at I_1, emerging at O_1. There were two paths for the solution—one through the connecting tube X and the other through AB, Y, and DC; but on account of the greater resistance of the latter path only a short column of the solution entered the left-hand side of the capillary AB. Air-free distilled water was then passed through from I_1 to O_1 until all the uranine in the U-tube X and in left-hand end of the tube CD had been removed, only the short index in the left-hand end of AB remaining. This required a large volume of water, and during its passage the coloured column of uranine was driven further into AB. I_1 and O_1 were then closed with screw clips.

Setting up of the Apparatus.—After the apparatus had been properly filled and the index introduced, it was allowed to rest for a week in a horizontal position. The apparatus was then taken into a thermostat room and supported as described in §2A, the screw feet being adjusted until the upper surface of the supporting beam was horizontal. To maintain a uniform and steady temperature around the U-tubes, each end of the apparatus was partly covered with a cylindrical sheet of iron standing on the bench. The top of each cylinder was covered with a sheet of paper, and the U-tubes completely surrounded and insulated with paper and cotton wool. A Beckmann thermometer was inserted in each cylinder, and so arranged that the end of the mercury column could be easily read.

Method of Reading Index and of Obtaining Volume Flow.—The method employed is similar to that made use of by the authors in earlier work.* An observer took three pairs of readings of the head and tail of the index U in the calibrated tube AB.

The mean of each pair was calculated, and then the mean of the three pairs taken as the reading of the observer. A numerical example will explain:—

Observer	I.			II.		
Reading of tail of index.........	8·7	8·9	8·9	10·0	8·5	9·2
Reading of head of index.......	19·7	19·2	19·2	18·4	19·5	18·8
Mean of these two readings ...	14·2	14·05	14·05	14·2	14·0	14·0
Mean reading of each observer	14·10			14·07		

The age of the index was 13 days.

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* A. Griffiths, Proc. Phys. Soc., Vol. XXIV., Part V., August 15, 1912.

It will be noted that Observer II. had no definite points for the head and tail, but simply aimed at reading points where the intensity of the fluorescence was equal. As the index was symmetrical his method gave results consistent with themselves and with those of Observer I., who rather measured the points where the variation in the intensity of the fluorescence was most rapid. A scale drawn with Chinese white on black paper was placed behind *AB*, and the source of illumination was a wire-lamp fixed in a small portable box provided with a window of cobalt blue glass. These results gave the linear flow from which the volume flow was obtained by reference to the results of the calibration of this tube embodied in an appropriate correction curve.

§ 3. Apparatus II.

In the experiments performed with the first apparatus, the correction necessitated by the difference of temperature between the contents of tubes *X* and *Y* amounted to 20 per cent. of the head which would have existed in the absence of the variation in temperature. The need for this correction has been eliminated in a second apparatus in which the tubes *X* and *Y* are brought together, and both contained in the same insulating enclosure.

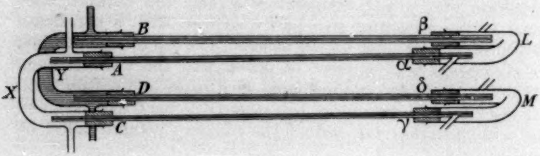


Fig. 3.

The form of the apparatus is the same as would be obtained by bending the capillary tubes *AB* and *CD* twice at right angles near their middles at α, β, γ, δ, so that the drying tubes *X* and *Y* and the portions of tubing $A\alpha$, $B\beta$ and $C\gamma$, $D\delta$ lie side by side, $A\alpha$, $C\gamma$ and $B\beta$, $D\delta$ being in parallel planes.

In reality the short portions of tubing $\alpha\beta$ and $\gamma\delta$ are replaced by drying tubes *L* and *M*, so that the apparatus actually consists of four parallel capillary tubes joined at their ends by four drying tubes to form one complete closed circuit (Fig. 3). It will be observed that as the drying tubes *L* and *M* are horizontal, and X and Y are vertical, the head produced is independent of the temperature of L and M, and it is substantially independent of the temperatures of X and Y provided that these are the same, for we may assume the water and uranine solution to have the same coefficient of expansion within the narrow range of temperature covered by the experiments (18·5°C.-19°C.).

As in the previous experiments the apparatus is mounted on a wooden beam planed so that it is accurately rectangular.

The principle of the method, and the theory of the experiment are precisely the same as before, except that, we have to consider the resistance of four capillary tubes instead of two.

If l_A, l_B, l_C and l_D are the lengths of the tubes $A\alpha$, $B\beta$, $C\gamma$ and $D\delta$ respectively, and

$$\frac{1}{r_A^4}, \quad \frac{1}{r_B^4}, \quad \frac{1}{R_C^4}, \quad \frac{1}{R_D^4}$$

the means of the reciprocals of the fourth powers of the radii, since θ is zero the expression for η reduces to

$$\frac{\pi hge}{8Q\left(\frac{l_A}{r_A^4}+\frac{l_B}{r_B^4}+\frac{l_C}{r_C^4}+\frac{l_D}{r_D^4}\right)}.$$

Numerical details of Apparatus II.—

$l_A=130$ cm. $\qquad \frac{l_A}{r_A^4}=1{\cdot}3252\times10^6$ cm^{-3}.

$l_B=130$ cm. $\qquad \frac{l_B}{r_B^4}=1{\cdot}8703\times10^6$ cm.$^{-3}$.

$l_C=130$ cm. $\qquad \frac{l_C}{r^4}=1{\cdot}7306\times10^6$ cm.$^{-3}$.

$l_D=130$ cm. $\qquad \frac{l_D}{r_D^4}=1{\cdot}5271\times10^6$ cm.$^{-3}$.

$h=5{\cdot}752$ cm.

The experiments were conducted just as before, the drying tube Y being filled with uranine, and an index U introduced into $A\alpha$ after filling the whole circuit with distilled evacuated water. In the course of three weeks the extreme variation of temperature at one point of the thermostat room was about 0·5°C. The extreme variation per day was about 0·2°C.

Apparatus	I.	II.			
Experiment	1	2	3	4	5
e (difference of density)	0·000406	0·000423	0·000423	0·0000423	0·0000423
θ in °C.	0·399	...	...	...	...
Total time of observation, in days	0·674	7·631	1·080	21·001	7·683
Total linear movement of index in cm.	24·73	283·41	39·33	75·81	27·85
Q in ccs./sec.	0·000007532	0·000013863	0·000013756	0·000001329	0·000001396
Mean temp. in °C.	18·67	18·89	18·83	19·02	18·67
η in gms./cm. sec.	0·010008	0·010403	0·010484	0·010850	0·010330
η calculated from Hosking's formula *	0·010394	0·010343	0·010357	0·010312	0·010394
Maximum rate of shear at boundary of index tube in radians/sec.	0·0233	0·0177	0·0176	0·00170	0·00179

* $\eta_t = \frac{0{\cdot}017928}{1+0{\cdot}03445t+0{\cdot}000235t^2}$ $(0 < t < 25°\mathrm{C})$, (Phil. Mag., Vol. XVIII., p. 262, 1909.)

§ 4. Results and Conclusion.

The value of Q was found by assuming a linear relation between the volume-flow v and the time t. Then v=a constant+Qt, the constant being introduced to allow for the possibility of an error at the origin. The final evaluation of Q was made by the method of least squares.

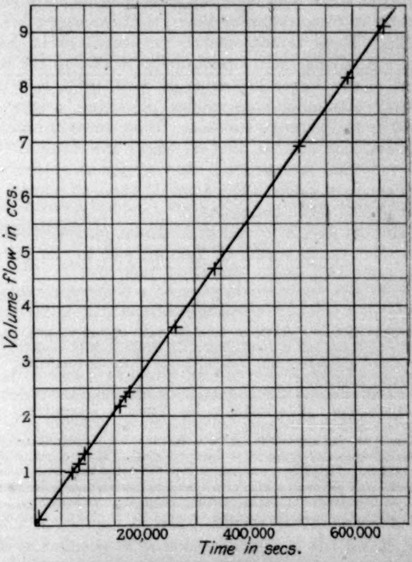


Fig. 4.—Apparatus II. Expt. 2.
Rate of shear=0·0177 rads./sec.

Fig. 4 shows the relation between the volume-flow and the time in the case of experiment (2)—*i.e.*, the first experiment with Apparatus II. In experiment 1 with apparatus 1 before applying the method of least squares it was necessary to allow for the variable difference of temperature between the two ends. The value of θ in the table is the average difference of temperature.

The discrepancy between Hosking's value of the coefficient of viscosity and the value determined in the experiment with the first apparatus is probably due to the difficulty in determining the difference of temperature between the two ends of the apparatus with sufficient precision. The fourth experiment was the first experiment to be performed at extremely low rates of shear, and in it the arrangements for obtaining uniformity of temperature at the ends of the apparatus were imperfect. These arrangements were improved in Experiment 5. Applying the temperature correction to the results recorded in the Paper the value of η at 18° C. is 0·010624 ±0·000229 (assuming observations of equal weight), and 0·010569 from Hosking's formula. Thus, there is no experimental evidence that at the extremely low rates of shear (0·0017 to 0·0233 radians/sec.), the viscosity of air-free water in glass capillary tubes differs from its value at normal rates of shear (5,000-10,000 radians/sec.).

The authors wish to record their thanks for much valuable help to their assistant Mr. G. Temple, and to the Birkbeck College instrument maker, Mr. H. G. Bell.

DISCUSSION.

Dr. **Vincent** said it would be useful if the same apparatus could actually be used for high and low rates of shear. It would then be possible to find out any minor differences that existed better than by comparing results with different apparatus.

Dr. **Rankine** suggested that there was not much point in giving the results to so many figures. With regard to Dr. Vincent's suggestion it would probably be sufficient if the same capillaries were employed in conjunction with different driving apparatus for the different rates of flow, since it was in the capillaries that any errors were likely to arise. He thought, however, that the evidence of the Paper was sufficient to show that there is no difference in excess of 1 or 2 per cent. This result may, of course, only hold for water.

Mr. W. N. **Bond** said that for viscosities as high as those used by Davidson and Kurt Molin, the viscosity changed by 50 to 100 per cent. at low rates of shear.

Dr. W. **Eccles** said the authors were to be congratulated on having surmounted very considerable experimental difficulties, particularly those due to temperature variations. Mr. Bond's remark suggests that with more viscous substances the range of variable viscosity is reached earlier, and this suggests that the author's next experiments should be with oils.

Dr. **Griffiths**, in reply to Dr. Vincent, said that in their experiments of eight years ago, in which they had been dealing with low rates of shear, though not so low as in the present experiments, they had for comparison made measurements in which the water was driven through the apparatus at ordinary rates. The difference in viscosity was less than ½ per cent. in that case. With the present apparatus there would be no difficulty in getting rates of flow a hundred times greater than those dealt with in the Paper.

XXI. *A Method of Measuring Frequencies.* *By* B. S. SMITH *and* G. F. PARTRIDGE, *B.Sc.*

ABSTRACT.

An heterodyne method of measuring frequency by comparison with a calibrated valve oscillator is described. The calibration is performed by means of two valve oscillators capable of giving frequencies of, say, 1,000—/sec. upwards. The frequency of the oscillators is raised alternately to give a beat note whose pitch is determined by comparison with a fork. Intermediate frequencies are found by interpolation on the calibration curve and a method of checking the capacities of the oscillator condensers is described.

For the measurement of acoustic frequencies the sounds are converted into alternating currents by means of a suitable transmitter.

IN experiments recently carried out by the authors it was found necessary to know, with a fair degree of accuracy, the frequency of sounds ranging from 2,000 vibrations per second to 10,000 vibrations per second. The usual methods of obtaining frequencies by comparison with standard forks are very uncertain at frequencies above 2,000 vibrations per second, since these depend on accurate tuning by ear of octaves, &c. The following heterodyne method has proved successful, and is available for acoustic or electric frequencies above 2,000 vibrations per second. It is also available for the determination of the frequency of damped oscillations, provided the damping is not too great.

The scheme is essentially an alternating current method and depends upon the conversion of the sound energy into electrical energy by means of a suitable form of sound receiver. In connection with this a valve oscillating circuit has been found a very constant form of alternating current generator, and a method is outlined for calibrating such a circuit for frequency by the same heterodyne method.

Reception of Sound.

For the reception of the sound an electrical receiver is required, which will not introduce other frequencies. A perfectly distortionless receiver is impossible, but it is usually sufficient to select a receiver whose resonant point is well above or below the frequency to be measured. The type of receiver also depends upon the sensitivity required and the conditions under which the sound is given out. If the sound is travelling through the air the sound waves can be made to act directly on the diaphragm of the receiver; if travelling through water, one of the well-known hydrophones can be used. In most cases either a telephone earpiece or a microphone transmitter connected through a transformer can be used. It is very necessary to ensure that the transformer does not introduce other frequencies.

Heterodyne Method.

The principle involved is the interference produced between two alternating currents when introduced into the same detector circuit. For the latter, either a valve or a crystal can be used (*see* Fig. 1, showing the circuit for a crystal detector). The function of the detector is to rectify both the alternating currents, so that the best note can be clearly heard, while the higher frequencies are practically eliminated.

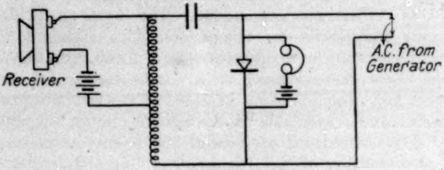


FIG. 1.

The alternating current coming from the sound receiver or from the circuit whose frequency is to be measured, is therefore introduced into the rectifying circuit together with the alternating current of about equal magnitude from the generator, the frequency of the latter being adjusted to a frequency greater than that of the sound under measurement by, say, 1,000 vibrations per second. This frequency is then carefully adjusted until the beat note heard is exactly in unison with a 1,000 fork. The frequency of the alternating current given by the generator is known from the calibration curve for the generating set, and hence the frequency of the sound in question.

Alternating Current Generator.

The oscillating valve circuit employed is that with variable capacity and fixed inductive coupling between the plate and grid circuits (*see* Fig. 2). The particular valve used, a Type Moyenne valve, French make, working with a filament voltage of 6 volts and a plate voltage of about 200 volts, enables all frequencies between 300 and 30,000 cycles per second to be obtained by variations of the capacity in the oscillating circuit from about 1·7 microfarads to zero.

Providing the coupling between the oscillating circuit and the output winding is sufficiently loose, no difficulty is experienced in maintaining a constant frequency, and for most purposes a 200-volt lighting circuit may be used in place of a high tension battery.

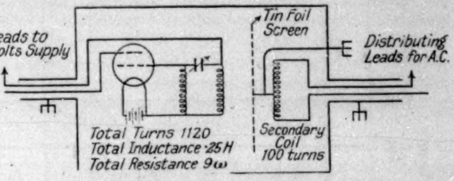


FIG. 2.—PORTABLE OSCILLATING SET.

Calibration of the Generator.

Since the frequency of the alternating current is adjusted by varying the capacity in the oscillating circuit, a curve connecting capacity and frequency is all that is necessary. This can be obtained as follows by the heterodyne method, a second alternating current generator being required for comparison. For clearness we will denote the set to be calibrated as *A* set and the comparison set as *B* set.

The alternating current from both sets is brought into a crystal detector connected through telephones. *A* is made to oscillate exactly at 1,000 cycles per second by comparison with a 1,000 fork, using the beat method without a rectifier. *B* is then raised in frequency, keeping *A* constant, until a beat of 1,000 is heard through the detector telephones, the beat note of 1,000 being adjusted exactly to unison with the 1,000 fork. The frequency of *B* is now 2,000 and is kept constant, while that of *A* is raised until the beat note disappears, then comes in again and rises to 1,000, to which it is accurately adjusted. The frequency of *A* is now 3,000. By repeating the process frequencies of 3,000, 5,000, 7,000, &c., cycles per second are obtained in the *A* set, and the required capacity noted in each case. Other series of frequencies commencing, say, at 1,500 and 2,000, are then obtained by the same process, the former requiring the use of a 500 fork.

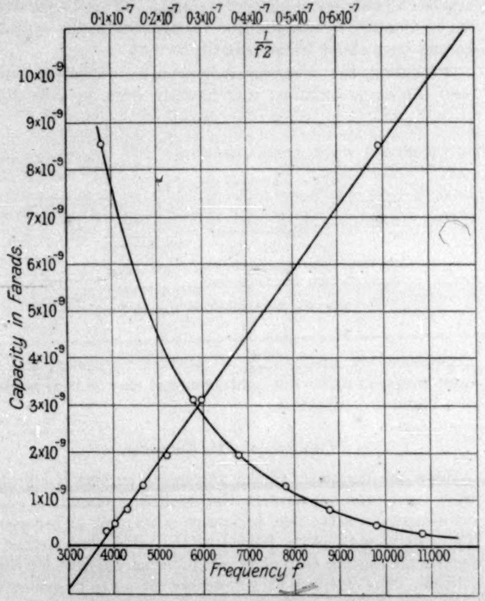


FIG. 3.—CALIBRATION CURVE FOR PORTABLE OSCILLATING SET.

The calibration curve giving frequency against capacity for the generating set described above is given in Fig. 3, and the graph connecting the reciprocal of the square of the frequency with capacity is practically a straight line.

Calibration of Condensers.

In order to use the curve for frequencies intermediate to those measured it is not essential to know the absolute value of the capacity, but it may be necessary to calibrate the condensers in terms of the vane condenser. For example, if plug condensers giving ½, 1, 2, 4, 8, &c., millimicrofarads and a vane condenser with a maximum capacity of 1½ millimicrofarads are used, the same frequency can be obtained usually with two or three different settings of the plug and vane condenser. In this way, starting at the highest frequency the values of the plug condenser can be found in terms of the vane condenser. The law of the vane condenser can be obtained by a bridge method, and the calibration curve of the generator, connecting frequency and capacity, may be made in terms of the vane condenser.

DISCUSSION.

Dr. VINCENT said he would like the authors to give more detail about the coupling of the middle portion of the apparatus with the end parts. The authors appeared to have reached the same conclusion as himself that a triple arrangement was most suitable for measuring frequency differences. Dr. Eccles and he had called attention to a somewhat similar apparatus which they had termed a differential wavemeter. The two outer parts generated the oscillation and the middle part resonates sufficiently to either of them that you could do anything you liked to the middle circuit without affecting the capacity, &c. With the old arrangements it was necessary for the observer to stand in a particular position and to abstain from making any movement whatever. He had only one point of criticism. He thought it was undesirable to get the electricity wholesale. A different supply should be used for the two circuits. It was important to use large batteries which could be left on for some time to give steady conditions. With such precautions he thought it would be possible to get an accuracy of 3 or 4 in 10 million.

Dr. D. OWEN asked to what purposes the authors had actually applied their method. Could it be used for calibrating a Galton whistle; would the sounds in this case be sufficiently intense for the method to be applicable? He presumed the authors had determined by it the limit of audibility. This was usually stated to be 40,000 vibrations per second, but he had found people who were deaf above 10,000. Could the method be used to study the overtones of a valve set?

Prof. RANKINE asked if the authors could actually produce a note of 10,000 frequency.

Prof. ECCLES said there was often some diffidence in confessing to deafness to high notes as the limit of audibility was a function of age.

Dr. RAYNER said he presumed the work had been done before the Abraham Block multi-vibrator was described.

Mr. PARTRIDGE, in reply to the discussion, said he had seen Dr. Vincent's work, but thought he was principally concerned with the measurement of inductances. As regards coupling, the middle circuit contained the telephone and crystal rectifier in series with two small secondary coils associated with the inductances of the outer circuits; there was no tuning. As regards the electric supply, their reason for using the 220-volt lighting circuit was because of its convenience on board ship. In reply to Dr. Owen, they had found the method very useful in determining the frequency of heavily damped steel diaphragms. It would be possible to use it with a Galton whistle if some means of picking up the sound at such high frequencies was available. In reply to Dr. Rayner, he was unaware of the device he had referred to.

(Mr. Partridge then demonstrated the production of a note of 10,000 frequency. This appeared quite loud to many of the audience, but was inaudible to others.)

XXII. *Note on Vacuum Tubes Used as Detectors of Electrical Oscillations.*

By L. HARTSHORN, *D.I.C., A.R.C.S., B.Sc.,* *and* E. S. KEEPING, *D.I.C., A.R.C.S., B.Sc.*

RECEIVED JANUARY 21, 1921.

ABSTRACT.

I. The Paper describes the development of a robust form of vacuum tube which was used as a detector of electrical oscillations in the "wireless" circuits carried by aeroplanes. Platinum electrodes are avoided being replaced by strips of tinfoil, to which contact may be made by the spring clips holding the tube in position.

II. It was found that when a discharge is passed through such a tube, the walls are affected in such a way that thenceforth it is much easier to get a discharge to pass. The change produced by the first discharge s annulled by heating the tube above 210°C. Further, if the walls are coated on the inside with a metallic film, this first discharge is unnecessary, and the tube is unaffected by heating, but when the walls are coated with an insulator it is if anything more difficult to pass a discharge. A silica tube behaves like one coated with metal. It seems possible that the change in the tube may be due to the formation of a layer of gas molecules on the walls by the first discharge. The explanation of the behaviour of the silica tube is a difficulty.

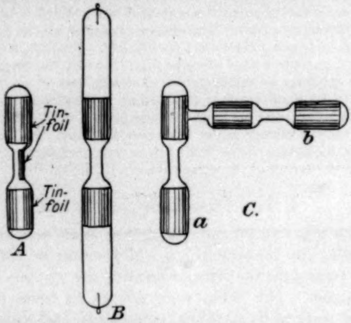
I. THE DESIGN OF THE TUBES.

THIS work was carried out during the War, and arose out of the demand for vacuum tubes which could be used as indicators of tuning for the circuits forming the wireless equipment of aeroplanes. The tubes were not to be more than 7 cm. long, they were to be as robust as possible, and since this form of detector is rather crude, as sensitive and as easily seen as possible.

Two tuned circuits were set up. The primary consisted of an inductance, a condenser and spark gap, and was energised by an induction coil. The secondary consisted of an inductance and variable air condenser, and was magnetically coupled to the primary. The vacuum tube was connected across the condenser of the secondary circuit, and of course should glow when resonance is established.

Preliminary Experiments.—As a preliminary several old vacuum tubes which happened to be available were tried. Some would glow under the above conditions, but others would not. The best results were obtained with a tube of krypton. It was then noticed that a tube which would not glow when supported in a wooden clamp would glow when held in the hand. This suggested wrapping the tubes round with tinfoil, which was earthed. A further improvement was obtained by connecting the tinfoil to one electrode instead of to earth.

A number of experimental tubes were made. These were of the form commonly used for spectroscopy. The diameters of the narrow portion varied from 1 mm. to 8 mm. They were all 7 cm. long, and had platinum-wire electrodes. They were connected to a vacuum pump, McLeod gauge, and drying tubes, and were in turn filled with air, carbon dioxide, and hydrogen at pressures varying from 3 mm. to ½ mm., but no glow could be obtained with or without the tinfoil.



A—Final form of tube detector.

B—The tube was made "soft" by a discharge passed either by way of the platinum wire electrodes or the tin foil.

C—"*a*" was made "soft" by a discharge in "*b*."

Experiments with Helium.—It was to be expected that helium and neon would give the best results. Accordingly helium was introduced into the apparatus, and the tubes again tested. In the process of filling, a discharge was passed through the tubes directly from the induction coil, in order to observe the progress of purification of the gas by examination of the spectrum.

After this treatment all the tubes would glow in the tuned secondary circuit. The results may be summarised thus :—

(1) The best pressure of gas was 15 to 20 mm.

(2) The best diameter of the narrow portion of the tube was 2 to 3 mm.

(3) None of the tubes would glow using the electrodes only. In all cases it was necessary to coat with tinfoil the middle as well as the two ends of the tube, and the best results were obtained by connecting the tinfoil to one electrode.

Experiments with Neon.—A new set of tubes was made and filled with neon. None of these would glow on the secondary circuit until a discharge had been passed through them from the induction coil. After this treatment they glowed brightly. The following points were noted :

(1) The wider tubes would glow without being coated with tinfoil, but the narrower ones would only glow when the foil was on, though it was not necessary to connect it to an electrode.

(2) Connecting the tinfoil to one electrode considerably improved the glow with the narrowest tubes, but made little difference in the other cases.

(3) The glow obtained when the tinfoil was used as one electrode was as good as that with both electrodes and the tinfoil.

(4) Using the tubes in this way the glow was not increased by raising the pressure above 10 mm., but at 6 mm. pressure the narrower tubes would not glow steadily, and the wider tubes were not as brilliant as at the higher pressures.

(5) At 11 mm. pressure a tube with a capillary portion of 2 mm. diameter was most satisfactory, but at 6 mm. pressure a diameter of 3 mm. was better.

(6) The glow was brighter than that given by the helium tubes.

In order to make the tubes as robust as possible, and in order to facilitate their construction, it was resolved to try to work without electrodes, using instead two pieces of tinfoil on the outside of the tube. This was found to be satisfactory, but it was found that the glow was more steady if the capillary was also coated with tinfoil connected to one of the end pieces, a gap of about 5 mm. being left between the two pieces of foil. The widest tube would not glow under these conditions, but the rest would ; indeed, better results were obtained in this way than by the other. Tubes with the narrow portion 1 to 2 mm. diameter were found to be most satisfactory. Using the tubes in this way a brilliant glow could be obtained at a pressure as low as 3 mm. A pressure of about 5 mm. seemed to give the best results.

Further tests showed that the platinum-wire electrodes were not at all necessary, and the form of tube finally adopted is shown at (*A*). The diameter of the narrow portion was 1 mm., and the gas pressure 5 mm. Before filling, all the tubes were strongly heated (a vacuum of something like 0·01 mm. being maintained) until they were on the point of softening. None of these tubes would glow on the secondary circuit after filling, until a discharge had been passed through them directly from the induction coil. Very few of the tubes would glow at once, even when connected to the induction coil; there was sometimes a lag of more than a minute, but after this treatment the tubes glowed without any lag on the secondary circuit. This last condition appeared to be fairly permanent, as tubes kept for two months glowed without any lag.

On trying air in tubes of this pattern it was found to glow under the same conditions as the neon, but the glow was not nearly as bright as that obtained with neon, and was unsatisfactory because there was often a considerable lag between the application of the voltage and the appearance of the glow, this lag gradually increasing with age.

II. PHENOMENA OCCURRING IN THE TUBES.

The above phenomena invite further investigation. These tubes when first made would not glow on the oscillatory circuit. For convenience such tubes were referred to as "hard." Even when directly connected to the induction coil they did not glow at once, but did so after a lag varying from a few seconds to a minute. When, however, this discharge had passed, the tubes would glow on the oscillatory circuit and without any lag. This condition was referred to as " soft " and seemed to be permanent.

Phenomenon Associated with the Walls of the Tube.—It was first necessary to find out whether the change in condition was due to change in the gas, or in the walls of the tube. For this purpose two tubes were made and connected by a piece of glass tubing 30 cm. or 40 cm. long. This arrangement was filled with neon at 4 mm. pressure. At first, of course, neither of the tubes would glow on the oscillatory circuit (both tubes were "hard"). A discharge from the induction coil was now passed through one tube and then it would glow on the oscillatory circuit (it was "soft"). If the softness is due to a change in the gas, then when mixing of the gases in the two tubes occurs by diffusion, we should expect the other tube to be soft also. However, such was not the case; the other tube remained "hard." Secondly, a tube of neon was rendered "soft" and the gas pumped out. Fresh neon was then admitted. The tube was still "soft." Hence the effect is associated with the walls of the tube.

Phenomenon not Confined to Neon.—It was found that the phenomenon was shown quite well by tubes containing air. This considerably simplified the experimental work, as the quantity of neon available was not large. The results hereafter mentioned refer always to tubes containing air unless the contrary is stated. It was found that a tube which had been rendered "soft" by a discharge through neon in it, remained so when the neon was replaced by air, even though in the meantime the tube had been standing for a day or two with full atmospheric pressure inside it, and in the same way a "hard" tube rendered "soft" in air, continued to be "soft" for neon.

Effect of Temperature.—In the preparation of the vacuum tubes, to be used as detectors, it was customary to heat the tubes before filling as strongly as possible without risking collapse, the pressure inside being kept as low as possible by means of the Gaede pump. In spite of this treatment further heating after sealing off a tube was found to spoil it, the glow obtained being a pale blue instead of the reddish neon glow. If, however, the tube was heated while still joined to the filling apparatus, after having been rendered "soft" by a discharge from the coil, the only effect was to render the tube "hard." This effect of heating was the same whether the tube contained air or neon, and whether the heating was conducted with the pressure inside very low or atmospheric. A tube not heated before filling was hard like the others, but this, of course, was only to be expected in view of the heating in process of manufacture.

It was thought worth while to investigate further this heating effect. A soft tube was heated to 100°C. in boiling water for some time, with apparently no effect. Similarly no change was observed when a tube was heated to 200°C. in a bath of oil. The tube was then heated in a small oven made of asbestos board and heated by passing an electric current through a coil of nickel wire. A tube, originally soft, after having been maintained at a temperature of 257°C. for about 20 minutes was found to be hard. A soft tube heated to a temperature of 205° for abour an hour was still soft, but when heated to 212° for about an hour, it was made hard. These results were confirmed several times. Extreme accuracy was not aimed at in the measurement of temperature, but we may safely say that the change takes place at about 210°C.

Effect of Nature of Walls of Tube.—(*a*) A little sodium was distilled into one of the tubes so as to form a film covering the walls but leaving the capillary clear. This tube was then filled with neon. It was found to glow at once on the oscillatory circuit.

(*b*) The walls of another tube were coated with a film of platinum. This was obtained by painting the walls with "liquid platinum" and then heating strongly. This tube (when filled with air) would also glow at once on the oscillatory circuit and it was not made "hard" by strongly heating, either when the atmosphere was in the tube, or the tube was evacuated.

Tubes whose walls were coated with various other substances were tested. In general it was found that when the covering was a conductor the tubes were "soft" and were unaffected by heating. When the covering was an insulator the tube was "hard." In the case of paraffin wax and sealing wax, the tube remained hard even after passing a discharge direct from the induction coil, but in the case of sulphur such a discharge made the tube "soft" exactly as for uncovered glass walls. When the sulphur was melted and run over the walls afresh the tube again became hard.

One tube was made of silica. This was found to be soft when first made, and it was found to be still soft even after making it red hot in the blowpipe flame.

The presence of a drop of mercury in a tube made it soft, and it was noticed that the discharge not only passed between the two pieces of tinfoil but also down to the mercury.

The first discharge, which was passed so as to make the tubes soft, was at first always passed by way of the tinfoil so that it passed through the walls of the tube. It was obviously of interest to find out whether it was necessary that it should pass thus or whether a discharge through electrodes had the same effect. Accordingly, a tube was made furnished with both electrodes and tinfoils, the two being some distance apart (*see* Fig. B). At first, of course, the tube would not glow on the oscillatory circuit, but it would do so after a discharge had been passed through the tube using the electrodes, and the presence or absence of the tinfoil whilst this discharge was passing made no difference. Thus a discharge through the electrodes has the same effect as a discharge which passes through the glass.

In order to find out whether the mere presence of the electrostatic field was sufficient to produce the change or whether it was necessary that a discharge should pass, the tube was evacuated as completely as possible, and then connected to the induction coil. In some cases the tubes were made soft, and in others not so. It was probable that in the former cases a discharge had passed, but the results were not conclusive.

As an alternative the atmosphere was admitted into the tubes, and they were then connected to the coil. In no case did this treatment make a tube soft.

It was further discovered that the effect of this discharge in rendering a tube soft was not only exerted on the glass immediately beneath the tinfoils, but also on that some distance away. Two tubes were joined by a cross-piece as at *C* (*see* Fig.). Tube *b* was made soft by a discharge. The room being darkened it was observed that the glow extended along the cross-piece as far as tube *a*. On testing *a* it was found to be soft.

These experiments seem to suggest that the effect of this discharge is due to the bombardment of the walls of the tube by the ions taking part in the discharge.

Measurements of the Discharging Voltages.—It was now resolved to measure the voltage required to make a tube glow, as it seemed possible that such measurements might give more information than merely knowing whether a tube was hard or soft. The induction coil was used as a transformer and fed by alternating current. The tubes were connected across the secondary circuit. By means of a rheostat in the primary circuit the voltage across the secondary circuit, *i.e.*, across the tubes, could be varied. The R.M.S. values of the voltages were measured by means of a multicellular electrostatic voltmeter.

The glow produced by this means was not nearly so brilliant as that produced by the previous arrangement. This is probably due to the difference of wave form. Probably in the first case the curve had sharp peaks whilst in the second it was approximately a sine curve. Further the phenomenon was not nearly so well marked in these experiments as might have been expected, *e.g.*, the average value of the voltage required to send a discharge through a " hard " tube containing air was 480, while for a soft tube it was about 390. In some cases a hard tube would glow at 450, while some soft tubes required about 410. Complications also arose from the fact that the discharge was not always of the same character. Sometimes a peculiar glow first appeared extending only half way down the tube, but as the voltage was gradually increased, the usual full glow more or less suddenly appeared. In some cases no glow whatever appeared until an unusually high voltage was reached, and then the ordinary full glow would come quite suddenly.

The neon tubes seemed to be the most consistent in their behaviour. They all first lit up with the faint glow only extending part of the way along the tube. For a soft tube, this glow appeared at 260–290 volts, whilst the full glow appeared at 360–400. For a hard tube, the first glow appeared at about 330 and the full glow at about 415.

With the air tubes it was exceedingly difficult to get consustent results. A new tube would generally light up at 460–500 volts. A tube which had just been made soft would generally glow at about 380, and if such a tube were heated and left for some time it would once more require 460–500 volts, but if it were tried *immediately* after heating, it seemed as though a lower value was required than if left for an interval. It seems possible that the various values obtained for air tubes corresponds to the varying lags obtained in previous experiments, though it should be mentioned that in making the measurements the voltage was increased very slowly to allow for the lag.

CONCLUSION.

In these tubes with external electrodes the discharge has to pass through the walls of the tube, hence it must pass as a " condenser " current, and the discharge must be alternating. This was strikingly shown by connecting a tube across a spark gap, excited from a high-tension direct-current generator. The tube only glowed when a spark passed across the gap, *i.e.*, when oscillations were occurring.

Thus we may regard the pieces of tinfoil on the tube as the outer plates of two condensers, the walls of the tube being the dielectric. The inner plates of these condensers must be the inner surface of the tube, and it seems possible that a

" soft " tube is one in which there is a conducting layer of gas molecules sticking to the walls, and that it is the function of the first discharge to form this layer. Heating might remove these molecules, and thus make the tube hard. This idea, however, leads to no simple explanation of the behaviour of the silica tube.

The recent publication of Papers* on the disappearance of gas in the electric discharge has drawn our attention to this phenomenon, and suggested that the effects observed in our experiments are intimately connected with it. It seems quite probable that the first discharge passed through our tubes was accompanied by the " disappearance " of some of the gas which became attached to the walls in some manner, and modified the glass-air surface so as to facilitate the discharge.

In conclusion we wish to thank the Rt. Hon. Lord Rayleigh for much valuable advice, and for the keen interest he showed in the work during its progress in his laboratory.

DISCUSSION.

Prof. G. W. O. **Howe** mentioned that comparisons of vacuum tube detectors both with internal and external electrodes were carried out at the Reichsanstalt. Helium at various pressures was employed. Certain pressures were used to obtain extreme sensitiveness, while others were used if extreme brightness was desired. In some of the tubes issued internal electrodes were fitted, not for actual use, but for the initial preparation of the tube by electrolytic deposition of potassium on the walls.

Dr. **Vincent** hoped the authors would not cease work on this interesting problem with the publication of the present Paper. They had dealt only with one aspect so far, but there were a number of interesting researches, particularly in connection with valve-maintained circuits, which would suggest themselves. Mr. Hartshorn had mentioned the advantage possessed by the method that it responded without lag during quick tuning. Another advantage was that there was no rectification of the current.

Mr. G. D. **West** asked if many of the authors' results in connection with the initial hardness of the tubes and the permanent softening effect of an initial discharge could not be accounted for by minute perforations of the walls by the discharge. The effects of subsequent heating appeared to fit in with this hypothesis.

Mr. W. R. **Cooper** asked whether the authors had looked up the work of Claude on neon tubes. These were large tubes for illumination, and it was found that a tube 6 metres long required less than 800 volts. Possibly some of the effects observed by the authors (*e.g.*, on heating the tubes) were due to the fact that the slightest impurity produced a marked effect. Claude found that it was not sufficient to use pure neon; he therefore purified the neon when in the tube itself by connecting the tube to a receiver containing carbon and immersed in liquid air.

Dr. **Borns** asked if a film of moisture might give rise to any of the effects.

A **Visitor** said that a common form of indicator in use for tuning purposes was an ordinary flash-lamp bulb. What were the conditions necessitating the use of vacuum tubes ? He noticed that when responding to spark discharge it was difficult to note the brightest point on account of the intermittence of the light. This flicker is absent with a flash lamp. The response of such bulbs was also quite quick enough for all ordinary conditions. Would a fluorescent screen in the tube increase its efficiency ? What was the effect of the frequency of the oscillations on the voltage necessary for discharge ?

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* " Phil. Mag.," November, 1920. This Paper gives references to the earlier work.

Mr. Hartshorn, in reply, said he was not familiar with all the early work which had been mentioned. They had assumed that the experts who had referred the problem to them were aware of all that had been done on it. He did not think perforations would explain the softening of the tubes. as this would not fit in with the behaviour of some of the compound tubes. As regards moisture, the greatest care was always taken to exclude this. As regards the use of vacuum tubes it was essential that the signals should be easily seen on an aeroplane, and flash lamp bulbs required more careful watching. As regards the intermittence, this was certainly a drawback, but was only present with spark discharges. It was not there with valve circuits. A fluorescent screen would make the tubes much too difficult to construct. Something very simple and robust was required. He did not know the effect of frequency on the discharge voltage. The figure he gave, 260 volts, was at ordinary supply frequency.

XXIII. *On the Coefficient of Diffusion of Certain Saturated Solutions.* *By* BASIL W. CLACK, *M.Sc.* (*Lond.*), *Lecturer in Physics at Birkbeck College.*

RECEIVED FEBRUARY 23, 1921.

ABSTRACT.

This Paper gives an account of experiments on the Diffusivity of saturated solutions of KCl, NaCl and KNO_3 at constant temperatures near 18°C., when the steady state of diffusion has been attained, employing a method similar to that previously used by the author (Proc. Phys. Soc., Vol. XXI., p. 863, 1908; Vol. XXIV., p. 40, 1911; Vol. XXVII., p. 56, 1914; Vol. XXIX., p. 49, 1916).

The solution under investigation is maintained at complete saturation by the presence of salt crystals in the diffusion vessel. The theory takes into account the change in volume of this salt as it dissolves, and an expression is obtained for the coefficient of diffusion at complete saturation, which depends on the rate of change in weight of the diffusion vessel with time. The experimental results are found to agree very closely with the values obtained by extrapolation from the results previously found for less concentrated solutions. By the present Paper the author has thus extended the range of concentration over which he has studied diffusion from very dilute solutions right up to complete saturation.

I. *Introduction.*

IN the study of the diffusion of salts through water, not only have investigators very rarely employed steady conditions, in which the concentration of the solution remains constant at each point of the diffusion tube all the time the experimental readings are being taken, but in the majority of instances the solutions under investigation have been comparatively dilute, and very few experiments have been undertaken in which relatively concentrated solutions are employed.

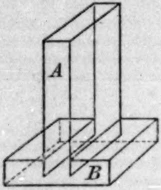
The author has already insisted on the importance of adopting the steady state in these researches (*see* Proc. Phys. Soc., Vol. XXI., p. 863, 1908; Vol. XXIV., p. 40, 1911; Vol. XXVII., p. 56, 1914; Vol. XXIX., p. 49, 1916), and in the present Paper a method is described in which this condition is employed in the study of saturated solutions.

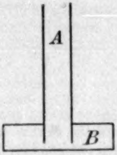
II. *Apparatus.*

The apparatus contains a diffusion tube, *A*, which has been made rectangular in cross-section in order to make the same apparatus applicable to another investigation which the author hopes to carry out in the future. The tube, which has a width of about 4 cm. and a breadth of 1 cm., and is

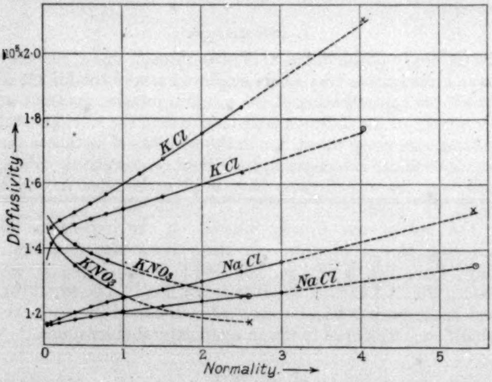
5 cm. high, is made of glass plates cemented together with zinc oxide and water glass cement, and is fitted at its lower end into a shallow glass box, *B*, about 4 cm. square and 1 cm. high, as shown diagrammatically in the figure.

The whole of this apparatus is filled with air-free saturated





solution, and the concentration of the solution at the lower end of the rectangular diffusion tube is maintained constantly at complete saturation by filling the shallow box with crystals of the salt under investigation. The diffusion cell thus filled



is suspended from one arm of a delicate balance by a fine platinum wire, gauge No. 33, and hangs a few centimetres below the surface in a large tank containing about 30 litres of distilled water maintained at a constant temperature of about 18°C. in a thermostat room.

After diffusion has been proceeding for a sufficient time, the concentration of the solution at the upper end of the diffusion tube may be taken as zero on account of the very large volume of water present in the tank. Thus the concentration of the solution in the diffusion tube varies from saturation at its lower end to zero at its upper end, and at every point remains constant with time—*i.e.*, the steady state has been established. When this condition has been obtained, the rate at which salt diffuses out of the cell has become constant, and observations of the weight of the cell every day enables this rate of diffusion to be determined. As the weight changes in these experiments by about 1 mg. in eight or nine minutes, it is only necessary to adjust the mass in the other pan of the balance to the nearest milligramme less than that of the cell, and wait until the pointer is seen to move to the zero of the micrometer scale fitted in the eyepiece of the observing microscope. The usual precautions already described are taken to avoid disturbance of the solution during these weighings.

As the diffusion cell has so small a volume, any variation of the density of the water in the bath, caused by slight variations in temperature, or by the addition of diffused salt to it, only produces a negligible effect on the weight observed, and hence it is not necessary in these experiments to employ a compensating cell suspended from the other arm of the balance.

III. *Theory*.

The theory of the method outlined above is slightly complicated by the change in volume of the salt crystals as they dissolve. It is known that on solution a decrease in volume usually occurs, so that water must enter the cell from the tank to make up for this change in volume, and the diffusion of the salt out of the cell takes place simultaneously with the entrance of water into it.

If v_0=velocity downwards with which the water enters the top of the diffusion tube, and if

L=total length of the diffusion tube in centimetres,
A=its area of cross-section in square centimetres,
N=concentration of saturated salt solution in gms./c.c.
D=density of saturated salt solution in gms./c.c.
c=mass of salt diffusing out of the cell/sec. when the steady state has been reached,
i=change in weight of cell/sec. when the steady state has been reached,

then it has been shown by the author (Proc. Phys. Soc., Vol. XXIX., pp. 51, 52, 1916) that

$$K=Li/\left\{AN\left[1-\frac{v_0N}{2c}+\frac{v_0(v_0-bc)N^2}{3c^2}\right](1-\delta)\right\},$$

where it has been assumed that the relationship between the density and the concentration of the solution, which is approximately a linear one, may be expressed with sufficient accuracy as $d=1+an$. Thus a is a constant whose value may be taken as equal to the mean slope of the true curve connecting d and n and b is written for $(1-a)$.

In previous work it was found unnecessary to include the third term in the square bracket, since (v_0-bc) was so small that this term was negligible, but in the present case rather different conditions result from the solution of the salt crystals in the reservoir, and this has the effect of slightly increasing the value of the term under discussion.

We have seen that as salt leaves the cell by diffusion, water enters it. Let $\delta=$the ratio of the mass of water entering the cell to the mass of salt leaving it in a given time when the steady state has been attained.

Then δ can be calculated as follows :—

Let $N=$concentration of saturated salt solution in gms/c.c.
$D=$density of saturated salt solution in gms/c.c.
$Z=$density of the salt crystals.
$V_1=$volume of solution in the cell of any given instant.
$v_1=$volume of salt in the cell of any given instant.
V_2, v_2 corresponding quantities at a later instant.

Then loss of salt in this time interval

$$=V_1N+v_1Z-V_2N-v_2Z$$
$$=(v_1-v_2)Z-(V_2-V_1)N.$$

Again, gain of water in the interval

$$=(V_2-V_1)(D-N)$$

Now

$$\delta=\frac{\text{gain of water}}{\text{loss of salt}}=\frac{(V_2-V_1)\,(D-N)}{(v_1-v_2)Z-(V_2-V_1)N}.$$

But $\quad V_1+v_1=V_2+v_2=$total volume of cell,

i.e., $\quad V_2-V_1=v_1-v_2$,

i.e., $\quad \delta=(D-N)/(Z-N)$.

From published tables of densities the value of δ can thus be determined. For the salts investigated in this Paper its value is in the neighbourhood of 0·5.

Again it is clear that in the steady state the mass of water entering the cell per second=(mass of salt leaving it per second) δ; that is

$$v_0 = c\delta.$$

Hence we have

$$K = Li/\{AN[1-\tfrac{1}{2}N\delta+\tfrac{1}{3}N^2\delta(\delta-b)](1-\delta)\}.$$

The difference between δ and b is, even in the present work, so small that the omission of the third term in the square bracket only produces an error in K which amounts in all cases to a fraction of 1 per cent.

IV. *Results.*

The results obtained by this method, using saturated solutions of the salts mentioned, are collected and tabulated below :—

Salt.	KCl.	NaCl.	KNO_3.
Temperature °C. ...	17·3	17·1	18·1
Normality	4·03	5·44	2·575
N.gms/cc.	0·300	0·318	0·260
D.gms/cc.	1·178	1·202	1·154
Z.gms/cc.	1·977	2·150	2·092
δ	0·5236	0·4827	0·4878
b	0·4059	0·3640	0·4080
L cms.	5·04	5·04	5·04
A sq. cms.	4·22	4·22	4·155
i gm/day	0·1687	0·1486	0·1115
$K\ X10^5$ C.G.S. ...	1·767	1·349	1·253

These results are indicated by circles in the graphs given below in which the curves already published in 1916 (loc. cit.) are extended so as to include the saturated solutions studied in the present research.

V. *Definite Concentration.*

The value of the Diffusivity obtained above is that for the solution in the diffusion tube, which varies in concentration from saturation to zero. This coefficient the author has called the "Mean Diffusivity" and has shown that the coefficient of Diffusion, K_n, for a solution of a definite concentration can be calculated theoretically from the value of the Mean Diffusivity by a formula (No. 6) given on p. 56 in the Paper referred to. In applying this calculation to the present case it is advisable as before to include the term containing n^2, and we obtain

$$
\begin{aligned}
K+\{1-\tfrac{1}{2}n\delta+\tfrac{1}{3}\,n^2\delta\,(\delta-b)\}\,/\,\{1-n\delta+n^2\delta\,(\delta-b)\}\,ndK/dn \\
=c/\{1-n\delta+n^2\delta\,(\delta-b)\}\,dl/dn \\
=K_n.
\end{aligned}
$$

The following results are obtained by this method for the coefficient of Diffusion of saturated solutions of the salts mentioned at the temperature indicated :—

Salt.	Temperature.	$K_n \times 10^5$.
KCl	17·3°C.	2·107 C.G.S.
NaCl	17·1	1·522
KNO_3	18·1	1·171

These results are also included in the graphs and are indicated by crosses. The curves associated with them show the value of the coefficient of diffusion at other definite concentrations calculated in the same way. It will be observed that both the Mean Diffusivity and the Diffusion Coefficient for the saturated solutions employed agree to within 1 per cent. with the corresponding values obtained by extrapolation from the results previously found for less concentrated solutions, and an inspection of the curves shows that the range of concentration over which the author has studied diffusion by this method has been approximately doubled by the work included in this Paper.

The research has been carried out at Birkbeck College, and I should like to record my continued indebtedness to Dr. A. Griffiths, in whose department the work was done, for his kind interest and assistance.

DISCUSSION.

Dr. A. GRIFFITHS dealt with the fundamental expression for diffusion. He explained the theoretical foundation of the diffusion-equation used by Mr. Clack and himself, and stated that this equation was an extension of Fick's Equation, or that Fick's Equation could be considered to be a limiting case of their equation. He said that Fick's Equation implicitly assumes that the liquid is at rest during a diffusion operation, whereas in fact the changes in concentration in all experiments known to himself inevitably produce movement of the liquid. An extension of Fick's hypothesis is, therefore, inevitable. He explained that the equation used by Mr. Clack and himself assumed that in addition to the diffusion as expressed by Fick, there is a movement of the diffusing salt equal in value to that of the water-component of the salt solution. He stated that the quantity defined by δ in earlier Papers by Mr. Clack and himself should have been defined as the ratio of the *mass* of water entering the diffusing cell to the mass of salt leaving the cell. The modification of the definition makes no change whatever when a linear relation may be assumed between density and concentration; and, in any case, makes a change smaller than errors of experiment. The change becomes smaller with the concentration, and is zero at infinite dilution.

The coefficients of diffusion for the weakest solutions of potassium and of sodium chloride were each 16 per cent. less than the theoretical values given by the Nernst equation for infinitely dilute solutions. The value of the coefficient for potassium nitrate was 11 per cent. less than the theoretical. These differences could not be considered very remarkable if it were remembered that Perrin's value (based on work with liquids) for the electronic charge was 11 per cent. less than the generally accepted value. The fundamental hypotheses for liquids probably require a close examination. One of his students had interpreted Mr. Clack's results by the equation $K=K_1r+K_2(1-r)$, where K is the coefficient obtained by Mr. Clack, K_1 is the coefficient of diffusion of the completely ionised salt, K_2 is the coefficient of diffusion of the non-ionised salt, and r is the ionisation-ratio. In the case of potassium nitrate this equation represents the results obtained by Mr. Clack wonderfully well over the whole range of concentration, and it represents the results well in the case of potassium chloride and of sodium chloride when the concentrations are not great.

Capt. C. W. HUME did not think the concentration at the top of the diffusion tube would be zero. The effect of this error would be equivalent to a constant increase in the length of the tube.

Mr. F. E. SMITH asked if experiments could not be made by measuring differences in electrical conductivity. Such methods, it appeared to him, would enable results to be obtained much more rapidly.

Mr. CLACK, in reply, said that earlier in his researches he had investigated the end-correction suggested by Mr. Hume and concluded that it was negligible. As regards the conductivity method, this had been tried, but he was at present experimenting with an optical method with which it should be possible to obtain results for all concentrations in one experiment.

XXIV. *Experiments on Thermal Transpiration Currents. By* GILBERT D. WEST, *D.Sc.* (*Lond.*).

RECEIVED JANUARY 4, 1921.

ABSTRACT.

This Paper is an attempt to demonstrate the actual existence of thermal transpiration currents. Theoretical considerations are first introduced to show that if a radial temperature gradient be maintained over a disc, so that the centre is the hottest part, thermal transpiration currents sweep radially inwards over the surface of the disc, and discharge themselves more or less radially outwards in the upper regions.

To detect these currents a narrow strip of foil is used which is placed perpendicular to the disc, and to one side of the hot region. When at a considerable perpendicullar distance from the disc, and when the gas pressure is sufficiently low to eliminate convection currents, the deflections of the strip of foil are always away from the hot region. When, however, the strip is placed very close to the disc, its deflections over a certain range of gas pressure are towards the hot region. These facts are explained by the tendency of the thermal transpiration currents to drag the strip with them.

The Paper emphasises one of the essential differences between thermal transpiration currents and convection currents—namely, that while the latter clearly depend on gravitation, the former do not. Further emphasis is laid on the differences between the conditions of molar and molecular equilibrium.

Introduction.

IN a previous Paper to the Physical Society* the author found it possible to explain the peculiar movements of heated strips of foil in rarefied gases, by means of thermal transpiration currents. Moreover, it was found possible to give a satisfactory explanation of the Crookes radiometer on a similar basis.† The actual existence of these currents was thus strongly favoured, and, in the present work, some of the phenomena to which they should give rise are discussed and conclusions drawn. These conclusions are then submitted to experimental tests.

General Considerations.

It is first necessary to realise that the properties of a gas vary according as we consider it from the molar or molecular point of view. To make matters clear, take the case of a vessel filled with gas and separated into two compartments, *A* and *B*, by a partition in which there is a small hole. If the diameter of the hole—although small compared with the dimensions of the vessel—be large compared with the mean free path of the molecules, the gas can be considered to behave as a continuous viscous fluid, and if, for instance, a difference of temperature be established between the two compartments, a flow of gas will take place from the hot to the cold compartment. In the equilibrium state, small disturbances due to convection currents may take place at the orifice, but, but for these, the pressures in the two vessels will be equal.

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* Proc. Phys. Soc., Vol. XXXII., p. 166, 1920.

† Proc. Phys. Soc., Vol. XXXII., p. 222, 1920.

Let us now consider the case when the hole is small compared with the mean free path of the molecules. It is clear that we cannot now regard the gas as a continuous fluid. On the contrary, everything must be considered from the molecular standpoint, and equilibrium will therefore only be reached when the numbers of molecules passing per second in opposite directions through the hole are the same. The consequent relation between the pressures in the two compartments is easily deduced if we divide the molecules up into the six conventional Joulean sets, and imagine that there are $1/6NV$ impacts per sec. per square cm. of wall, where N represents the number of molecules per cubic centimetre, and V the root mean square velocity. Clearly, equilibrium can only be reached when

$$\tfrac{1}{6}N_A V_A = \tfrac{1}{6}N_B V_B,$$

where the suffixes distinguish the molecules in the two compartments. Hence, if m be the mass of a molecule, the ratio of the pressures P_A and P_B will be given by

$$P_A/P_B = \tfrac{1}{3}N_A m V_A^2/\tfrac{1}{3}N_B m V_B^2 = V_A/V_B = \sqrt{T_A}/\sqrt{T_B},$$

where T_A and T_B represent the respective absolute temperatures.

The distribution of pressure is thus seen to be different according as the size of the hole is large or small compared with the mean free path of the molecules. It will be convenient to use the terms "molar" and "molecular" equilibrium to distinguish these two extreme cases. Intermediately, when the conditions are neither molecular nor molar, complex phenomena occur, and no very general cases of equilibrium under these conditions appear to have been dealt with satisfactorily. Sutherland,* however, has considered the case of a tube along which a uniform temperature gradient is maintained, and which connects hot and cold vessels containing gas. It is necessary even here to simplify the problem, and to restrict calculations to pressures at which convection currents are unimportant. When this reservation is made, it is found that there can be considered to exist two superposed systems of gas currents. The first, termed a thermal transpiration current, is from the cold to the hot vessel, is uniform over the whole section, and is similar to the flow that would take place in the simple case just discussed, were we to start with the hot and cold compartments *A* and *B* at the same pressure. It is clear that such a current exhibits the extraordinary power of flowing from a region of low pressure to a region of high pressure. This, however, is not surprising if we regard it as an attempt to establish molecular equilibrium.

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* Phil. Mag., 42, p. 373, and p. 476, 1896.

Superposed on this uniform thermal transpiration current, and discharging an equal amount of gas per second, is another current in the reverse direction, which follows the laws of viscous flow in a tube, and which thus flows from regions of high pressure to regions of low pressure, and which rises from a minimum value at the walls to a maximum at the axis. This current, of course, can be regarded as an attempt to establish molar equilibrium.

The result of the superposition of the two flows is to give a current from the cold to the hot vessel along the walls of the tube, together with a current from the hot to the cold vessel along the axis, whilst between the two there will be a surface of zero velocity.

At very high pressures, when molar conditions are approached, or at very low pressures, when molecular conditions are approached, the gas currents will be very feeble. On the other hand, at some intermediate pressure, when the conditions are neither molar nor molecular, the currents will be at a maximum. Starting from the lowest pressures, therefore, the velocity of the gas currents will rise in value with inrceasing pressure, reach a maximum when the mean free path is of the order of the diameter of the tube, and finally, at higher gas pressures, decline asymptotically to zero.*

It is to be noted that, whether the equilibrium is molar or molecular, or intermediate between the two, equilibrium is conditioned by the equality of the number of molecules

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* On the basis of a previous Paper (Proc. Phys. Soc., Vol. XXXI., p. 279, 1919) there is no difficulty in obtaining an expression for the distribution velocity of the gas over the cross-section of the tube. For any given point the velocity is proportional to dp/dx, as obtained from equation (2), p. 285. Note, however that 65·1 should be replaced by 71·9.

F

passing in opposite directions. The only difference is the way in which this may be accomplished. Under molar conditions it may be accomplished by oppositely directed currents, but, under molecular conditions, the gas has lost all resemblance to a continuous fluid, and such currents are no longer possible.

Let us now turn our attention to the variation of the excess pressure in vessel A with gas pressure. It has already been shown that at the lowest gas pressures

$$P_A/P_B=\sqrt{T_A}/\sqrt{T_B}.$$

If T_A and T_B are close we may drop suffixes and write

$$\text{Excess pressure in vessel } A=\delta p=\frac{\delta T}{2T}\cdot p.$$

It is thus seen that when molecular equilibrium is approached the excess pressure in A is proportional to the gas pressure. On the other hand, it can be shown* that when the equilibrium is nearly molar, the excess pressure in A is inversely proportional to the gas pressure. In fact, the function representing variation of excess pressure in A with gas pressure, rises, reaches a maximum, and again declines.

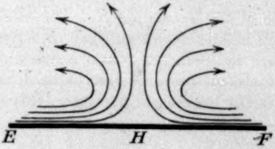

FIG. 1.—DISTRIBUTION OF GAS CURRENTS AROUND A DISC *EF* WITH CENTRAL HOT REGION *H*.

Thus both the gas currents and the excess pressures in the hot compartment follow the same type of variation. In a tube, moreover, both the maxima occur at the same gas pressure, but for more complicated cases this is not necessarily the case.

Let us now leave the case of the tube and try to extend our considerations to a case that lends itself to easy experimental verification, namely, that of a disc over whose surface a radial temperature gradient is maintained, so that the centre is the hottest part. We should here expect thermal transpiration currents to flow radially inwards over the surface, and to discharge themselves more or less radially outwards, as in Fig. 1.

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* Proc. Phys. Soc., Vol. XXXI., p. 279, 1919.

Let us further consider the effect of the introduction of a thin metal strip, *AB* (Fig. 2), perpendicular to the disc *EF*, and to one side of the hot region *H*. The mean temperature of region *C* will clearly be above that of region *D*, and this region of higher temperature must, according to principles previously enunciated, be associated with a higher pressure. If we start from the lowest gas pressures, this excess pressure will at first rise with increasing gas pressure, then reach a maximum, and finally decline asymptotically to zero. In fact, it might quite well be represented graphically by the curves *PQR* in Figs. 3 or 4. Moreover, such excess pressures will be capable of producing deflections of the strip away from the hot region *H*.

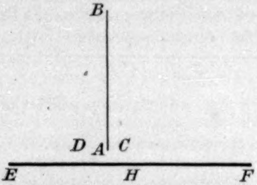

FIG. 2.—*EF* REPRESENTS A DISC WITH CENTRAL HOT REGION *H*, AND *AB* A STRIP OF THIN METAL LEAF.

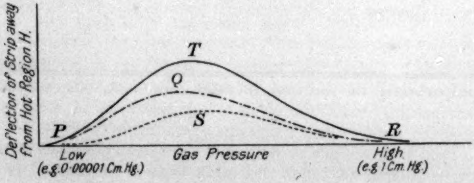

FIG. 3.—INDICATING DEFLECTIONS OF STRIP AT VARIOUS GAS PRESSURES—STRIP DISTANT FROM DISC.

The gas currents that sweep over the surface of the disc, however, are also capable of producing deflections of the strip, but the curve which will represent deflections of the strip due to this agency will depend on the perpendicular distance of the strip from the disc. If the strip is far away from the disc, the strip will merely have its deflections augmented by the outflowing currents, and we might well represent such augmentations by *PSR* (Fig. 3). If, on the other hand, the strip is close to the disc, the end *A* will tend to be driven towards the hot region by the inflowing gas currents, and we might now imagine such deflections represented by *PSR* (Fig. 4). It will be noted that we here suppose the maxima in the curves *PSR* (Figs. 3 and 4) to occur at higher pressures than in the curves *PQR*. This is admitted to be little more than a probable assumption, but if this assumption be made, the resultant deflections will be represented by *PTR* (Figs. 3 and 4), and it will be noted that, in one case, the deflections are always in the same direction, whereas, in the other, they change in direction with gas pressure.

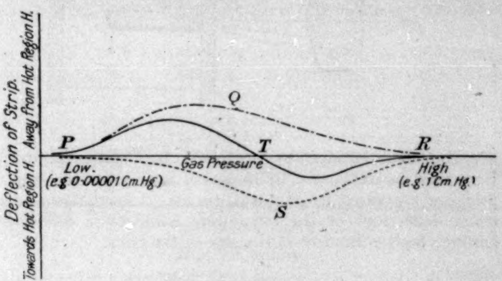

FIG. 4.—INDICATING DEFLECTIONS OF STRIP AT VARIOUS GAS PRESSURES —STRIP CLOSE TO DISC.

If now the thermal transpiration currents actually exist, and if they function in the way indicated, experiment should bear out these curves. In what follows, therefore, a set of experiments is described, and the results obtained are considered.

Apparatus.

The apparatus consisted of a cylindrical brass tube about 9 cm. in diameter, at the ends of which were cemented two glass plates, *V* and *W* (Fig. 5). To *V* was fixed a thin vertical disc *EF* of aluminium foil about 0·001 cm. thick, and, close to the centre of the disc, was fixed, in a horizontal plane, a small strip of aluminium leaf *AB* about 0·71 cm. long and 0·2 cm. broad. It was found that, owing to its slight curvature of section, *AB* would remain horizontal although *EF* was vertical. Radiation from a "Pointolite" lamp *P* was concentrated at *H* by means of a lens system *L*, and the box could be exhausted by means of a Gaede pump, and the pressure measured by McLeod gauges. Observations of the small deflections of the strip were made by a microscope, through a glass window in the side of the box.

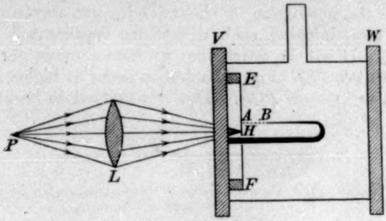

Fig. 5.—Diagram of Apparatus.

Results.

Results obtained with the strip *AB* at 0·29 cm. from the screen, and with the hot region in succession above and below the strip, are shown in Figs. 6 and 7. It will be seen that, below 1 cm. of mercury, the curve rises and falls in a way similar to *PTR* (Fig. 3), and that in each case the deflection is away from the hot region. Deflections upwards, however,

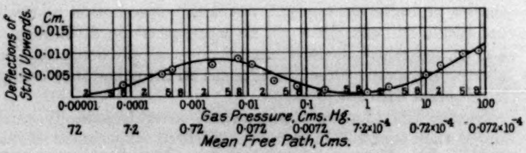

Fig. 6.—Showing Variation of Strip Deflections with Gas Pressure. (Hot region below strip. Distance of end of strip from screen 0·29 cm.)

are greater than deflections downwards, and this may be attributed to the slight concavity of the upper surface of the strip, and its consequent resistance to downward deflections.

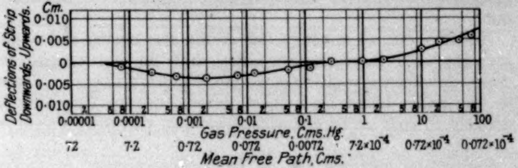

Fig. 7.—Showing Variation of Strip Deflections with Gas Pressure. (Hot region above strip. Distance of end of strip from screen 0·29 cm.)

In Figs. 8 and 9 deflections are shown with the strip much closer to the screen. At the lowest pressures, the deflections are in the same directions as before, but, with increase of pressure, the deflections are in each case reversed. The general shapes of the experimental curves, from the lowest pressures to 1 cm. mercury, are, in fact, strikingly similar to

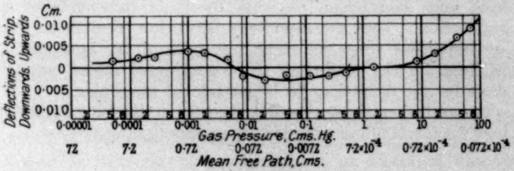

Fig. 8.—Showing Variation of Strip Deflections with Gas Pressure. (Hot region below strip. Distance of end of strip from screen 0·09 cm.)

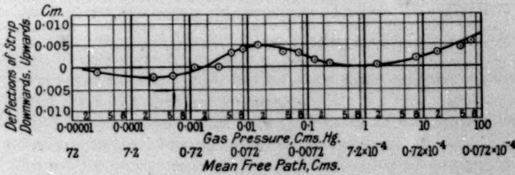

Fig. 9.—Showing Variation of Strip Deflections with Gas Pressure. (Hot region above strip. Distance of end of strip from screen 0·09 cm.)

that of the theoretical curve *PTR* of Fig. 4. We cannot, therefore, help feeling some confidence in the assumptions made in deducing it. In particular, a good case for the actual existence of thermal transpiration currents can be made out.

Above 1 cm. of mercury deflections also occur, and these increase with increasing pressure, but do not change in direction when the position of the hot region is changed. There is but little doubt that these deflections are due almost entirely to convection currents streaming vertically upwards over the disc.

Convection currents are, in fact, characteristic of molar equilibrium, and, with reduction of gas pressure, they quickly tend to disappear, whilst their place is taken by thermal transpiration currents. These thermal transpiration currents, although resembling convection currents in that they are associated with temperature inequalities, are differentiated from such convection currents by their independence of gravitation. They are essentially transition phenomena that are important when the mean free path of the molecules is too long for the conditions to be considered molar, yet too short for the conditions to be considered molecular.

In conclusion, the author has pleasure in thanking Professor Lees for the facilities afforded for prosecuting this research.

DISCUSSION.

Dr. D. OWEN said the author appeared to be working down to pressures of a ten-thousandth of a centimetre of mercury. How were these pressures measured ?

Dr. WEST.—With a McLeod gauge.

Dr. OWEN then asked if the author considered these to be reliable at such low pressures.

Dr. WEST replied that he always used two gauges and they gave similar results. The bulbs were from 200 c.c. to 300 c.c. capacity. He thought if care was taken to use perfectly dry air the gauges were quite reliable.

THE SIXTH GUTHRIE LECTURE.

Some Recent Applications of Interference Methods.

Delivered by PROF. A. A. MICHELSON, *University of Chicago.*

MARCH 11, 1921.

FOR some years it has been my hope to have an opportunity of putting into execution three investigations involving application of interference of light waves: first, a repetition of the measurement of earth tides; second, a redetermination of the velocity of light; third, an attempt to measure the diameter of a star.

Through the kindness of Dr. George E. Hale such an opportunity was presented in the form of an invitation to attempt all three problems at Pasadena, California, with the aid of the splendid equipment of the Observatory at Mount Wilson and of the laboratory facilities at Pasadena.

EARTH TIDES.

The first of these problems had already been brought to a satisfactory conclusion at Yerkes Observatory, with the result that the earth tides were very nearly three-tenths of the calculated value of the water tides—leading to a value for the rigidity of the earth about the same as that of steel. One object in repeating this investigation at Pasadena was the possibility of finding difference between a station well in the interior of the continent and another quite near the coast.

The method consists in the measurement of the difference in level of water in a steel pipe 6 in. in diameter, 100 ft. long, buried 6 ft. below the surface and hermetically sealed. It was found that under these conditions the effects due to changes in temperature, wind or other atmospheric disturbances were practically eliminated.

In a preliminary trial, lasting over three months, it was found that the results of measurements by the microscope (pointing on a needle point under the water surface, and on its totally reflected image) were so concordant and so little affected by earth tremors that it was decided to employ the interferometer. In the usual form, the instrument consists essentially of a plane surface upon which the incident light falls at an angle of 45 deg., part passing through and part reflected, thus separating the original pencil into two at right angles. These are reflected back on their paths, the first by a mirror in air, the second by a mirror very near and below the water surface. The two paths being the same with the exception of the thin film of water, the variation in this thickness causes a displacement of the resulting interference bands; and knowing the wave length of the incident light (the violet radiation from a mercury lamp) and the index of refraction of the water, the corresponding change of level can readily be calculated.

In the present determination a simpler arrangement is adopted, namely, a plane glass surface just below the water surface, between which two surfaces the interference of the reflected light occurs.

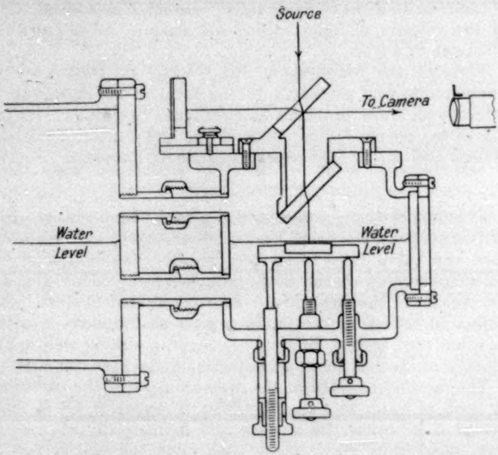

FIG. 1.—EARTH TIDES. ARRANGEMENT OF INTERFEROMETER.

With a tube 100 ft. long this gives a difference in level due to the tidal forces of sun and moon, of an amplitude of the order of 10 to 20 fringes, the position of which can be readily measured to within one-tenth of a fringe, thus giving an average error less than 1 per cent.

A continuous photographic record of the interference bands at both ends of the east-west line, and also the north-

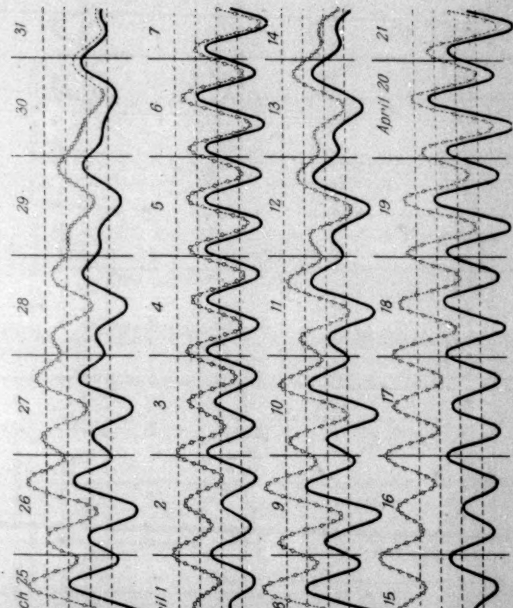

Fig. 2.—Measured Tides (E.-W.), Full Curve; Compared with Calculated Values Diminished in Proportion of 7 to 10.

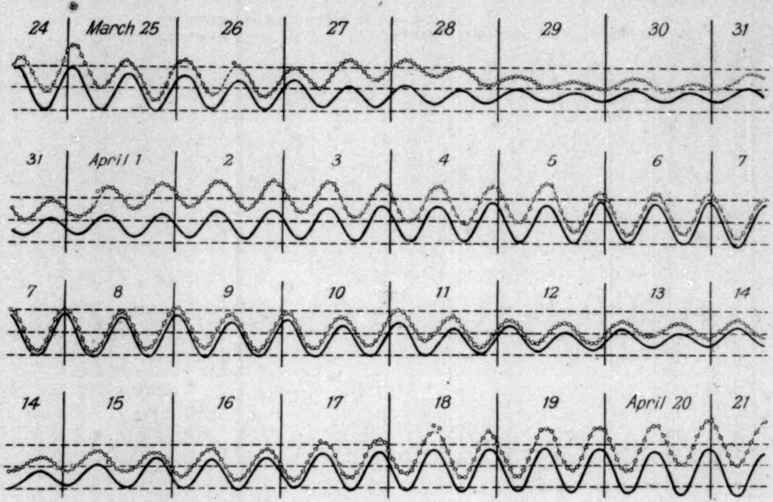

FIG. 3.—MEASURED TIDES (N.-S.), FULL CURVE; COMPARED WITH CALCULATED VALUES DIMINISHED IN PROPORTION OF 7 TO 10.

south line gave results agreeing almost exactly with the calculated values when these are diminished in the ratio of 7 to 10. There is, however, a phase retardation amounting to about 4 deg., which may be interpreted as a result of viscosity, or as due to the effect of the ocean tides; and it is probable that in the present determination (near the coast) this retardation may be found to have a different value.

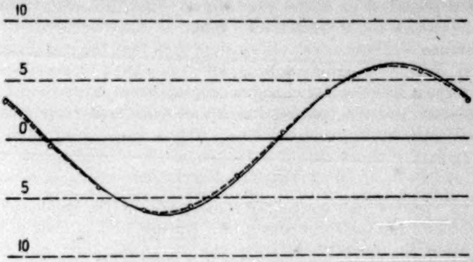

Fig. 4.—N.-S. Semi-Diurnal Lunar Tide, for Entire Year.

Dotted curve from observed values
$y = 6\cdot173 \sin(\theta + 148^\circ 24')$

Full curve from 0·7 times calculated values
$y = 6\cdot401 \sin(\theta + 147^\circ 32')$
$R = 0\cdot7\ a/b = 0\cdot675. \quad \Delta\phi = 52$

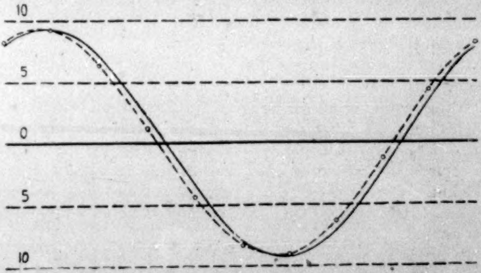

Fig. 5.—E.-W. Semi-Diurnal Lunar Tide, for Entire Year.

Dotted curve from observed values
$y = 9\cdot380 \sin(\phi + 63^\circ 56')$

Full curve from 0·7 of calculated values
$y = 9\cdot485 \sin(\theta + 58^\circ 10')$
$R = 0\cdot7\ c/d = 0\cdot692. \quad \Delta\varphi = 5^\circ 46'$

NOTE.—It may be mentioned that in this investigation a continuous record of the interference bands was obtained for a period of an entire year—interrupted, however, during three hours by an earthquake found to have occurred at a distance of 2,000 miles. It appears thus that the installation may serve as a seismograph; and it may even be possible to observe a slow secular change in the apparent level due to an inclination of the rock strata, the ultimate rupture of which causes the earthquake.

THE VELOCITY OF LIGHT.

In view of the increasing importance of the velocity of light as a fundamental constant of Nature, and in view of its relation to the ratio between the electromagnetic and the electrostatic units, it does not seem superfluous to repeat the measurement—especially as it seems possible, by a modification of the method of the revolving mirror, to obtain an order of accuracy at least 10 times as great as that hitherto reached.

Thus, while by the usual method the accuracy of the result depends upon three factors namely, the distance, the speed of the revolving mirror, and the angular deflection of the returned ray, the last measurement is in the method proposed eliminated, or preferably transferred to the fixed angles between the faces of the revolving mirror.

It is in this last process that the application of interference comes into play.

If the revolving mirror be given the form of an octagon, the angles being all equal, and if the mirror revolve at, say, 1,000 turns per second, it would require $\frac{1}{8000}$ of a second for one face to replace the preceding one; and in this time light travels 375 km.; so that the distance between stations may be under 20 km. Or, with 40 km., the speed need be only 500 turns per second.

In order to secure equality between the angles of the octagon, these are compared by interference tests with a standard angle of 135 deg. When the difference is the same for all, they must, of course, be equal to 135 deg. In the mirror actually constructed in this manner, the average error has been found to be of the order of one part in a million.*

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* It may be worth pointing out that this may furnish a method for dividing a circle to the same order of accuracy.

The experiments have only been carried far enough, at a distance of four miles, to indicate a strong probability of success at a much greater distance, and it is hoped that the investigation may be continued during the coming summer.

THE DIAMETER OF STARS.

The application of interference to astronomical measurements was indicated in an article published some 30 years ago (" Phil. Mag.," July, 1890) : a trial of the method applied to the measurement of the diameter of Jupiter's satellites gave results of an order of accuracy hardly to be attained by the use of the large telescopes of to-day. It was then pointed out that the method might give important results in the measurement of double stars, and even in estimating the diameter of some of the nearer stars if these should be considerably larger than our sun.

Briefly, the method (at least, in the earlier and simpler form) consists in employing only two small portions of the objective, at opposite ends of a diameter, instead of the whole objective.

In both cases it is, of course, an interference pattern which is observed ; but whereas with the complete objective this is a bright " disc " surrounded by a number of coloured rings, in the former case it will consist of a number of coloured bands symmetrically arranged about the central achromatic band. This if the source is a point.

But if the source is a double star, then, as is well known, these can be " resolved," using the full objective, when by Rayleigh's formula, $a=1{\cdot}22\lambda/d$, where a is the angular separation, λ the mean wave-length and d the diameter of the objective. But the actual " resolution " will depend a good deal on the observer, and in still greater degree on the atmospheric disturbances, which, indeed, are usually so strong as to obliterate all traces of the diffraction fringes.

In the proposed method, if d now represents the distance between the apertures, the formula becomes $a=0{\cdot}5\lambda/d$, so that, apart from other considerations, the accuracy is more than doubled. But here it is not a more or less vague appearance of " resolution " which is observed, but an actual vanishing of the fringes (supposing the components of the double star to be of equal brightness), and this vanishing can be observed with remarkable accuracy, so that, in fact, the increase in accuracy is at least tenfold.

The same considerations can be shown to apply, with modification, to the measurement of the diameter of a star; in this case the vanishing of the interference fringes occurs when $\alpha=1{\cdot}22\lambda/d$, α now representing the angular diameter.

Before making an actual attempt to obtain measurements, it was clearly desirable to ascertain whether atmospheric disturbances, which make good "seeing" so exceptional, would not have a still more drastic influence on the interference fringes by the new method. Accordingly, observations were made using the 40 in. refractor of the Yerkes Observatory, with the result that fringes were obtained even when the seeing was but 2 on a scale of 5.

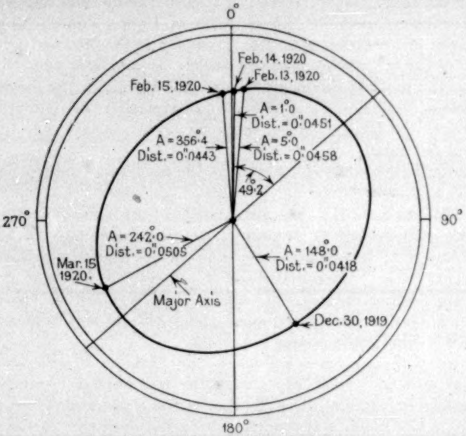
0°
Feb. 14. 1920
Feb. 13, 1920
Feb. 15, 1920
A = 1°·0
Dist. = 0''·0451
A = 356°·4
Dist. = 0''·0443
A = 5°·0
Dist. = 0''·0458
49°·2
270°
90°
A = 242°·0
Dist. = 0''·0505
A = 148°·0
Dist. = 0''·0418
Mar. 15
1920.
Major Axis
Dec. 30, 1919
180°

FIG. 6.—VISUAL ORBIT OF CAPELLA.

A still more severe test was made at the Mount Wilson Observatory, first with the 60 in. and then with the 100 in. reflector.* In both cases the fringes were remarkably clear even when the seeing was 2 on a scale of 10.

The explanation of this very satisfactory but rather unexpected result is probably as follows: At any very brief instant the fringes should be distinct. At any subsequent brief instant they should be equally clear, but in a different position; but if the motion is not too rapid for the eye to follow, they remain distinct at all times, whereas in the use of the entire objective there is an integrated disturbance which may obliterate the diffraction fringes entirely.

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* The screen with apertures in the case of the 100 in. was applied only a short distance from the focus, instead of in front of the objective, which made measurements much more convenient.

Shortly after these tests, Mr. Anderson made a series of observations on Capella, which was known to be a double from spectroscopic evidence, and succeeded in measuring both distance and position angle so accurately that the differences between the observations and the computed orbit differed by quantities of the order of one ten thousandth of a second, the whole distance being of the order of 0·″04.

The Double Star Capella. Observed and Computed Orbits.

Date of Observation.	Distance.	Position Angle.
	″	°
1.—December 30·6, 1919	0·0418	(153·9)
2.—February 13·6, 1920	0·0458	5·0
3.—February 14·6, 1920	0·0451	1·0
4.—February 15·6, 1920	0·0443	356·4
5.—March 15·6, 1920	0·0505	242·0
6.—April 23·6, 1920	(0·0439)	107·0

t−T.	a.	i.	r.	Pos. Angle.
d	″	° ° ′	″	°
1.—39·7 ...	1 and 5 0·05249	180−39 31	0·04180	153·9
2.—84·7 ...	2 and 5 0·05250	−39 36	0·04583	4·6
3.—85·7 ...	3 and 5 0·05248	−39 22	0·04506	1·0
4.—86·7 ...	4 and 5 0·05249	—39 31	0·04430	357·3
5.—11·7 ...	———	−39 30	0·05050	242·4
6.—50·7 ...	———	———	0·04391	107·2
	Means 0·05249	180−39 30		

It was then proposed, under the direction of Mr. Pease, to build an interferometer, with a base of 20 ft. in the form of a structural iron beam attached to the 100 in. tube, and provided with two plane mirrors at 45 deg., one at each end of the beam, which reflect the light from the star radially toward the axis of the tube. Two similar mirrors near the axis thence reflect the two pencils down the tube to the 100 in. mirror, whence they proceed to the focus, producing the interference fringes which are observed under rather high magnification (about 2,000 diameters).

In order to secure equality in path, a double wedge of glass is inserted in the path of one of the pencils, compensated by a plane parallel plate in the other path, and the wedges moved by a slow motion screw until, with the aid of a small direct-vision

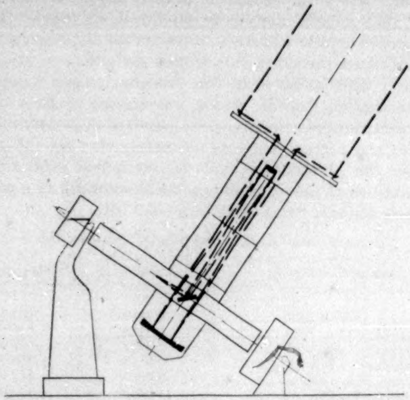

FIG. 7.—ILLUSTRATING PATH OF THE TWO INTERFERING PENCILS OF LIGHT FROM A STAR.

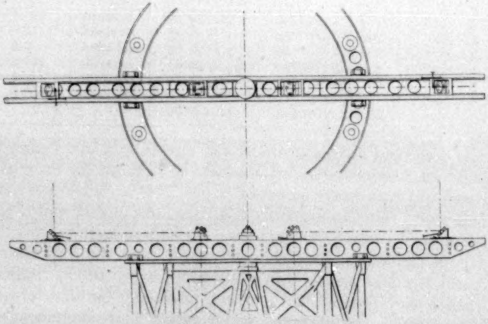

FIG. 8.—PLAN AND ELEVATION OF INTERFEROMETER OF 20 FT. BASE ATTACHED TO TUBE OF 100 IN. REFLECTOR.

prism at the eyepiece, the resulting spectrum is traversed by dark bands, when the prism is removed and the wedges still further adjusted until interference fringes appear in white light.

These were, in fact, observed in August, 1920. In December the actual vanishing of the interference fringes was observed by Pease when pointing at α-Orionis, the distance between the outer mirrors being 10 ft., while, as a check, fringes were still visible on adjacent stars. It follows from the known parallax $\pi=0{\cdot}016$, that the diameter of this star is about 300 times as great as that of the sun, which confirms in a remarkable degree the estimates given by Eddington and Russell.

The construction of an interferometer with a base of 50 ft. to 100 ft. is contemplated, which may make it possible to measure the diameter of Sirius.

XXV. *A Method for the Micro-analysis of Gases by the Use of the Pirani Pressure Gauge.* *By the* RESEARCH STAFF OF THE GENERAL ELECTRIC CO., LTD., *London.* (*Work conducted by* N. R. CAMPBELL.)

RECEIVED FEBRUARY 28, 1921.

(COMMUNICATED BY C. C. PATERSON, O.B.E., M.I.E.E.)

SUMMARY.

(1) A method of analysis of gases at a pressure between 0·1 and 0·001 mm. is described, based on the characteristic vapour-pressure temperature curve of any substance.

(2) The method demands a gauge which will measure the pressures of vapours, as well as of permanent gases, over the range mentioned. The Pirani gauge is suitable for this purpose, especially if it is used in a way which is not new, but of which the advantages have not been sufficiently recognised.

1. PRINCIPLE OF THE METHOD.

IT is well known that the relation between the vapour pressure (p) of any substance and the temperature (T) can be represented approximately by

$$\log p = A - \frac{B}{T - T_0}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which gives

$$\frac{1}{p}\,\frac{dp}{dT} = \frac{(\log p - A)^2}{B^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence $\frac{1}{p}\frac{dt}{dp}$ becomes infinite as the vapour pressure approaches zero at the temperature T_0, called the condensation temperature, which is characteristic of the vapour.

The least change of pressure which any gauge will measure is fixed by the value of dp/p rather than by the value of dp; accordingly by measurements of vapour pressure it should be possible to fix with great accuracy the condensation temperature T_0 and thus to identify the vapour. If the gauge contains several vapours, but no permanent gases, and its temperature is increased from a value below the T_0 for the least condensible vapour, we should get a pressure-temperature curve of the form shown in Fig. 1. The temperatures at which the sudden rises in the curve occur serve to identify the vapours present, for they are the values of T_0 for these vapours, while the change in the ordinate following a sudden rise upward (such as o to p_1, p_1 to p_2, or p_2 to p_3) will serve to measure the quantity of the vapour present, if the calibration curves for the various vapours are known.

If the whole gauge were immersed in a cooling bath in order to condense the vapours, the pressure of the permanent gases and uncondensed vapours would also vary with the temperature, and the parts of the curve in Fig. 2 between the sudden rises would not be horizontal. This difficulty can be easily overcome by cooling, not the whole gauge, but only the tip of a small side tube attached to the main vessel. The vapours will condense in this side tube at the appropriate temperatures, while the readings of gauge corresponding to the permanent gases present, and to those vapours which are not condensed at the temperature of the side tube, will remain entirely unaffected. If permanent gases are present, which are not condensed at the lowest temperature attainable (which in most laboratories may be taken as that of liquid air), we shall get a curve precisely similar to Fig. 1, except that it is raised above the horizontal axis by a distance corresponding to the amount of permanent gas present.

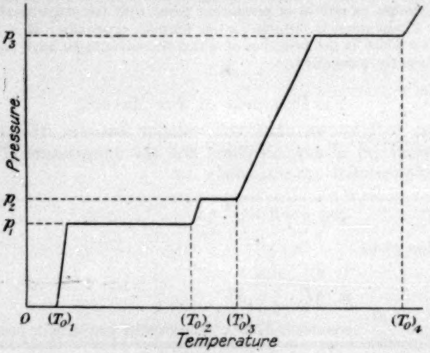

FIG. 1.

In this statement it is assumed that the pressure of each vapour present rises from zero to the maximum, fixed by the total amount of the substance present, in the interval between

T_0 for that substance and T_0 for the substance of next higher condensation temperature. If this assumption were not correct, the beginning of one sudden rise would be superimposed on the end of the rise preceding, and distinction between the two vapours might be difficult. But in practice at the pressures and with the substances that are likely to be present the assumption is always correct. The vapour pressure of a substance often reaches 0·1 mm. at a temperature only 10°C. above its characteristic T_0; accordingly, unless either the condensation temperatures are within 10 deg. or the amount of one substance present corresponds to a pressure greater than 0·1 mm., the separate steps in the curve will be perfectly sharp. Thus, if CO_2 and H_2O alone are present, the first can reach a pressure nearly atmospheric before the second begins to appear; in none of the vapours that we have desired to distinguish has there been the slightest overlapping so long as the total pressure is not more than 0·2 mm.

Little more need be said to indicate the principle of the method of micro-analysis described. If permanent gases are present, they can often be analysed by converting them into vapours, after the original vapours have been analysed, and repeating the analysis. Thus, if there is exposed to the mixture under analysis a copper wire, coated with cuprous oxide, H_2, CO, and O_2 can be identified and estimated by simply heating the wire to redness. The first two are converted into H_2O and CO_2; the last is absorbed. The method is quite satisfactory for H_2 and O_2, but CO does not seem always to be converted wholly into CO_2. But the exact methods adopted must depend largely on the nature of the problem to which the method is applied; and they will be such as are common to all methods of microanalysis.

2. The Pirani Pressure Gauge.

The first requisite for the practical application of the method is a means of measuring the pressure of vapours with some accuracy up to about 0·1 mm. The Pirani gauge* has proved very suitable. It depends on the cooling of a heated wire by the surrounding gases. In the gauge as developed by Hale† a fine wire with a high temperature-resistance coefficient is connected in a bridge with three resistances of negligible temperature coefficient; a constant potential, sufficient to heat the manometric wire some degrees above its surroundings is applied to the bridge; the change in resistance of the wire due to its change of temperature with the pressure of the surrounding gas is measured.

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* M. V. Pirani. Deutsch. Phys. Gesell. Verb., 8, 24, pp. 684-694, 1906.

† C. F. Hale. Amer. Electrochem. Soc. Trans., 20, pp. 243-258, 1911.

This form of the gauge has been used chiefly for very low pressures (less than 0·001 mm.) for which the McLeod gauge is inconvenient. At such low pressures and, indeed, up to 0·01 mm. (with most forms of the instrument) the change of resistance is very nearly proportional to the pressure, and a straight line is obtained for the calibration curve. But at higher pressures the curve ceases to be straight, and becomes much less steep; for, as the temperature of the wire decreases, the sensitiveness of the instrument also decreases.

The shape of the calibration curve at these higher pressures cannot be represented by any simple formula; it varies in a complicated manner with the nature of the gas and the temperature of the surroundings. The calibration is purely empirical, and the instrument is not reliable in any conditions but those in which it has been calibrated. These objections can be greatly lessened by a simple change in the manner of using the gauge which was mentioned, but not fully discussed, by Pirani, and does not seem to have been widely adopted. In place of measuring the change of resistance of the wire when the potential applied to the bridge is constant, we may measure the potential which must be applied to the bridge to keep the resistance (and therefore, the temperature) of the wire constant. This potential will, of course, increase with the pressure of the gas.

In this method of using the gauge the three manganin resistances of the bridge are invariable. They are chosen so that the bridge is balanced when the manometric wire is at a convenient temperature, say, 100°C.; the wire attains such a temperature when its resistance is about one-third greater than at room temperature. A good voltmeter is connected to the terminals of the bridge, and the potential across the whole bridge varied by means of a rheostat in the battery circuit, until a balance is obtained.

The loss of heat by the manometric wire is due to radiation, to conduction along the supports, and to convection or conduction in the gas. All these losses are approximately proportional to the difference of temperature between the wire and its surroundings (θ). Let the loss due to radiation and conduction along the supports (which is independent of the pressure of the gas) be $r\theta$, that due to the gas $c.f(p).\theta$, where p is the pressure. The heat supplied to the wire, since the resistance of the bridge is always the same, is proportional to V^2, where V is the potential applied to the bridge. Let it be aV^2. Then

$$(r+c\,.\,f(p))\theta=aV^2. \quad \ldots \ldots \quad (3)$$

If V_0 is the potential required for a balance when $p=0$,

$$r\theta=aV_0^2\,. \quad \ldots \ldots \quad (4)$$

and

$$\frac{V^2-V_0^2}{V_0^2}=\frac{c}{r}\cdot f(p). \quad \ldots \ldots \quad (5)$$

Now c/r is constant for considerable variations in the instrument. It does not change rapidly with the length of the wire (so long as it is so great that cooling by conduction to the supports is small); and it is surprisingly independent both of its diameter and of its material (platinum or tungsten). Moreover, so long as the changes of the temperatures of the wire and its surroundings are so small that all heat losses may be taken as proportional to θ, c/r does not change with those temperatures. Within the range of all the changes that are likely to occur if reasonable care is taken to make two gauges the same, the function $\frac{V^2-V_0^2}{V_0^2}$, which will be written (V^2), should be independent of everything except the nature and the pressure of the gas.

This independence is actually found. The manometric wire that has been employed is the tungsten filament of a 200-240-volt 40-watt lamp. The cold resistance of the lamps used varied by as much as 8 per cent., and these variations, in conjunction with changes in the room temperature (for the gauge was simply exposed to the air of the room), caused V_0 to vary between 1·3 and 1·95 volts. But, in spite of these considerable variations in the instrument, if the same setting of the bridge had been used throughout and the same calibration curves used for determining p from (V^2), the error in p, up to 0·15 mm., would not have exceeded 10 per cent. By adjusting the bridge so that the temperature of the wire was always the same, these errors could be reduced to 3 per cent.

Another great advantage of this method of using the gauge is that $f(p)$ is approximately proportional to p, and the calibration curve approximately straight. In Fig. 2 are shown the curves for H_2, air, CO, CO_2, H_2O. The pressure of the first four gases were measured by a McLeod gauge. For the first two the curve is straight within experimental error; initial curvature appears with CO, and is more pronounced with CO_2; but since it is known that a McLeod gauge is not really accurate for CO_2, it is possible that some of the curvature in CO_2 is due to errors in measuring the pressure. The

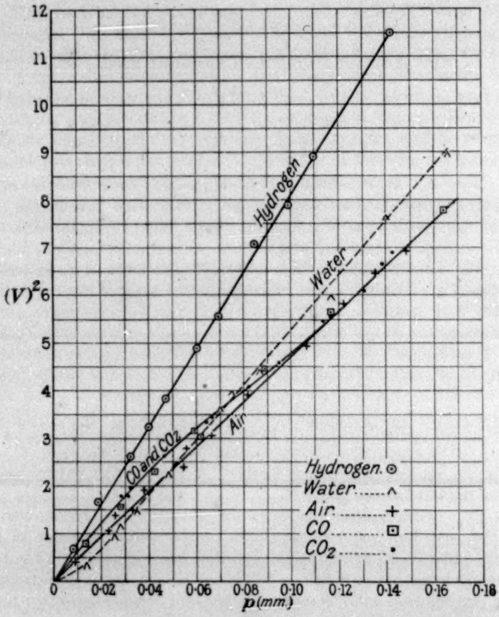

FIG. 2.

curve for water is interesting. There is no very simple method of measuring water pressures between 0·001 and 0·1 mm., and the following device was adopted. A mixture of H_2 and O_2 was introduced into the vessel at a pressure measured by the McLeod; it was then burnt by a hot platinum wire, and the water formed frozen in a side tube. The pressure of the residual gas was measured (for it was difficult to secure that the gases were present in exactly the right proportion) and from the difference the amount of water formed calculated. The residual gas was pumped away, the side tube warmed, and the water liberated as vapour. The estimate of the pressure of the water made in this way assumes that none is condensed on the walls of the vessel; and this assumption is known to be untrue. The condensation of water doubtless accounts for the deviation of the curve from a straight line at low pressures, and by extrapolating the line at higher pressures to the axis, an estimate of the limiting amount of vapour condensed is obtained. The straight line probably represents the relation between p and (V^2) for the water actually present as vapour.

It will be observed that the curves for the gases which do not contain hydrogen lie very near together. For the rough measurements for which the gauge was used they may be taken as coincident. For many purposes we think that the Pirani gauge used in the manner described would have considerable advantages over the McLeod. It may, therefore, be pointed out that it has not the advantage of a greater working speed; for a period of a minute or so must be allowed for the gauge to reach thermal equilibrium.

3. Practical Methods.

A few practical suggestions may be offered. If a qualitative analysis of the gases is being made, the best and most convenient way to identify the vapours is not to use any thermometer of an ordinary type but to compare the T_0's of the unknown mixture with the T_0's of known vapours. A series of gauges may be made up containing between them all the vapours likely to be present; any gauge of this series may, of course, contain more than one vapour so long as no interaction between constituents can occur. The side tubes from the series of known vapours and the side tube from the gauge under analysis are then immersed in a little mercury in a small Dewar flask, and liquid air poured in until the whole is cooled to the temperature of that liquid. The conductivity of the mercury is sufficient to maintain all the side tubes at the same temperature. The mercury is then warmed by blowing air at a suitable rate on its surface, and the unknown gauge watched until a sudden change in (V^2) occurs. The air stream is then stopped temporarily and the known gauges examined to discover in which of them (V^2) has changed. If several have changed from their liquid air values, the constituent in the unknown gauge is, of course, that one in these known gauges which has the highest T_0.

This method is extraordinarily sensitive and certain. One per cent. of water in CO_2 may be detected—an unfavourable case because the main constituent evaporates before the impurity; 0·0005 mm. of CO_2 may be detected in an unlimited quantity of vapours with higher T_0, or in 0·05 mm. of permanent gases. On the other hand, mercury vapour is not easy to detect. Apparently its cooling effect is very small. But it is usually known without analysis whether mercury is present, and, if so, how much.

If the constituents are known, the estimate of their amounts can be made very rapidly. A series of baths are prepared at temperatures which lie between the T_0's for the constituents; the side tube is then placed in the baths in succession, and the change in (V^2) in passing from one bath to another gives the amount of the vapour with a T_0 between the temperatures of the two baths.

Thus, in order to analyse a mixture containing (1) phosphorus vapour, (2) water, (3) a vapour not chemically identified from "vacuum" oil, (4) CO_2, (5) H_2, (6) CO, the following baths will serve in conjunction with the oxidised copper wire:—

Freezing mercury (—39°) condenses phosphorus but not water.

Freezing acetone (—95°) condenses water but not oil vapour.

Freezing ethyl alcohol (—117°) condenses oil vapour but not CO_2.

Liquid air (—183°) condenses CO_2 but not permanent gases.

For some purposes an interpolation between the first two baths might be necessary. Then it is convenient to know that freezing chloroform (—65°) is just above and solid CO_2 (—78°) just below T_0 for water. Greater difficulty arises in interpolating between the last two baths in the table, but, so far, we have not found need for such interpolation. In order to prepare the baths a few drops of the liquid are placed in a minute Dewar flask and liquid air poured in (very gently with the lighter liquids) until the solid separates.

After the values of V have been measured with the side tube immersed in the various baths, the side tube is opened and sealed to a pump, and V_0 measured when the gauge is completely exhausted. If accuracy is required, V_0 must be measured while the gauge is still on the pump, for in sealing it off, a small quantity of water vapour is always liberated from the melted glass. It is desirable for accuracy that the whole process should be completed as quickly as possible, in order that the room temperature may not change. It can be completed quite easily with practice in half an hour, and with reasonable care the variations during that time are not likely to be important. The most important source of error lies in the assumption, necessary unless very elaborate calculations are to be made, that the calibration curves are all straight and that the value of (V^2) for a mixture is the sum of those for its constituents. The truth of these two assumptions is probably closely connected.

Finally, we may give details of the analysis of a mixture containing H_2, CO, H_2O, CO_2.

	V.	V^2.	(V^2).	Diff.
Side-tube at temp. of room………	5·09	25·91	6·87	3·75
Freezing acetone …………………	3·68	13·54	3·12	
Liquid air ………………………	3·24	10·54	2·20	0·92
After heating copper oxide—				
Room ……………………………	5·00	25·00	6·60	4·83
Freezing acetone …………………	3·02	9·12	1·77	
Liquid air ………………………	$1{\cdot}82_2$	$3{\cdot}31_9$	$0{\cdot}00_8$	1·76

Gauge exhausted on pump to < 0·00001 mm.

$$1{\cdot}81_5 = V_0.$$

From Fig. 2, $p = \alpha(V^2)$, where

$$\begin{aligned} \alpha &= 0{\cdot}0121 \text{ for } H_2, \\ &= 0{\cdot}0174 \text{ for } H_2O, \\ &= 0{\cdot}0211 \text{ for CO and } CO_2. \end{aligned}$$

$\therefore$ H_2O present originally

$$=3{\cdot}75\times0{\cdot}0174=\underline{0{\cdot}065_2}\ (0{\cdot}061)$$

CO_2 present originally

$$=0{\cdot}92\times0{\cdot}0211=\underline{0{\cdot}019_5}\ (0{\cdot}018)$$

H_2O produced by burning=

H_2 present originally

$$=(4{\cdot}83-3{\cdot}75)\times0{\cdot}0174=\underline{0{\cdot}018_8}\ (0{\cdot}020)$$

CO_2 produced by burning=

CO present originally

$$=(1{\cdot}76-0{\cdot}92)\times0{\cdot}0211=\underline{0{\cdot}017_6}\ (0{\cdot}020)$$

Total.................. $0{\cdot}121_1$

The figures in brackets are the pressures of the gas estimated when the gauge was filled for this test.

Observe that the value of (V^2) for the permanent gases originally present should be

$$\frac{0{\cdot}0188}{0{\cdot}0121}+\frac{0{\cdot}0176}{0{\cdot}0211}=1{\cdot}55+0{\cdot}84=\underline{2{\cdot}39}.$$

The value found was 2·20.

DISCUSSION.

Dr. J. S. G. **Thomas** asked if the method would discriminate between the higher hydrocarbons in coal gas, or between ethane and methane. It was assumed that the heat lost by conduction at the ends of the supports was constant. He did not think this was justified, as it had been shown that at low pressures this loss depended on the pressure of the gas. Would the method discriminate between dry and wet gas at the same total pressure ?

Dr. D. **Owen** expressed his admiration of the method, which appeared to be entirely new. What pressures had the author gone down to in these experiments ? In addition to the Pirani gauge the McLeod gauge is referred to, but no others. Had Dr. Campbell any experience of other forms of low-pressure gauge and their properties ?

Dr. **Campbell**, in reply, said it was easy to analyse any mixture of components which could be frozen out by liquid air. They had analysed the vapours from the oil used in their pumps, and he thought the higher hydrocarbons in coal gas would present no difficulty. He was afraid neither ethane nor methane would freeze out, and so could not be treated by this method. As regards the validity of the assumptions of the heat loss, for accurate work every gauge is separately calibrated, and no assumptions are made. The instrument could be used to discriminate between wet and dry gas, provided the total pressure is under 0·2 mm. He did not know how low pressures the Pirani gauge would do for, but for very low pressures the Ionisation gauge would be more accurate.

XXVI. *On the Reflection of the X-Ray Spectrum of Palladium from Fluorspar.* *By* H. PEALING, *M.Sc.*

RECEIVED MARCH 25, 1921.

ABSTRACT.

An examination of the odd order spectra reflected from the 100 plane of fluorspar, using palladium X-rays, has been made with a view to test the Lewis-Langmuir theory of the motion of valency electrons in compounds. Evidence, in partial confirmation of the theory, has been obtained.

THE reflection of X-rays by fluorspar has been investigated by Sir W. H. Bragg and Prof. W. L. Bragg, and its space lattice mapped out. The fluorine atoms are arranged in a cube lattice and the calcium atoms in a face-centred cube lattice. Each fluorine atom is surrounded by four calcium atoms and each calcium atom by eight fluorine atoms.

Calcium has atomic number 20 and fluorine 9. When all the calcium and fluorine atoms are acting so as to give a strong spectrum they will contribute to the amplitude of that spectrum in the ratio of 20 to 2×9, the total amplitude being proportional to 38, the arrangement of electrons within the atom being supposed, for the moment, to have no special effect. When we get a weak spectrum the calcium atoms are in opposition to the fluorine atoms, and if the atomic number law is true the amplitude of such a weak spectrum should be $20-18$ or 2. A comparison of the intensities of strong and weak spectra, account being taken of the obliquity or order of the spectra, should give us the ratio $38^2/2^2$, or 361/1. That is, the weak spectra should be exceedingly small. The Braggs found that the first spectrum in the 1, 0, 0, plane of fluorspar was absent. The order of accuracy of the experiments could not establish whether there was a weak spectrum or none at all. This investigation was an attempt to find fresh information about the weak spectra reflected by fluorspar. It was thought that the experimental evidence of the size of these spectra would throw light on the Lewis-Langmuir theory of the structure of the atom. This theory postulates that the electrons in an atom occupy relatively fixed positions with regard to the nucleus, and are arranged in a series of shells with the nucleus as centre. The first complete shell contains two electrons, the second eight, the the third eight, the fourth eighteen and the fifth eighteen.

The atoms of helium, neon, argon, krypton, and zenon contain complete shell systems, and other elements incomplete shell systems. The chemical properties of the elements depend on the number of electrons over or below a complete shell system, and in combining with other elements the tendency always is to form complete shell systems.

An electro-positive element has rather more electrons than the nearest complete shell system, while in an electro-negative element the electrons are not quite numerous enough to form a complete shell system. If an electro-positive element combines with an electro-negative element then the electrons over and above the complete shell system, the valency electrons as they are sometimes called, can pass over to the electro-negative element to enable it to form a complete and therefore stable shell system. A calcium atom having two valency electrons will combine with two fluorine atoms and the resulting combination will give a calcium atom with an argon shell system, and two fluorine atoms with neon shells, the atomic number of argon being two less than that of calcium and the atomic number of neon being one more than that of fluorine.

In what way will the calcium atom with the argon shell of electrons scatter X-rays? If it scatters so that the amplitude of the wavelet depends only on the number of electrons in the atom, then it will behave as an atom of argon and the fluorine atom should behave as an atom of neon. Consequently when the calcium and fluorine atoms are acting in opposition when giving a spectrum, we should get an amplitude proportional to $\{(20-2)-2(9+1)\}$, or one proportional to 2. It was considered possible to detect whether a small spectrum of this magnitude was present or not. In magnitude it is 1/361 of the intensity of a full spectrum. We shall give the data later, which we consider establishes the fact that a small first 1, 0, 0, spectrum is present, but only about one-third of the value expected. It does not seem possible to reconcile this fact with the scattering law stated above, without qualifications. It seems possible, however, to modify slightly the Lewis-Langmuir theory so as to explain it. Although the argon and neon shells may be formed, it does not follow that the shell systems will be identical with those characteristic of those elements. They may be the argon and neon shells slightly distorted, that is to say, with the electrons slightly out of position. It does not seem reasonable to suppose that the nuclear charges of the calcium and fluorine atoms change. If they do not change we must suppose that the unbalanced positive charges, on the nucleus of the calcium atom, produced by the absorption of the valency electrons into the fluorine atoms, must displace the remaining electrons in the rings, slightly, from their true positions.

In the 1, 0, 0, plane of fluorspar the calcium and fluorine atoms lie on alternate planes which are all equidistant. If a distortion takes place, then the phase of the disturbance contributed by the fluorine atoms in any one plane, may not be just the mean of the disturbances from the two calcium planes, on either side of the fluorine plane. Further, the amplitude of the disturbance contributed by each atom will be altered to a slight extent.

In the experimental arrangements just the same method and apparatus was used as in the previous work of Sir W. H. Bragg and of his son, Prof. W. L. Bragg, an excellent account of which is given in their book "X-rays and Crystal Structure." The specimen of fluorspar used was a colourless variety, and the 1, 0, 0, face had been carefully worked. The writer attempted to find the α lines of the 1st, 3rd, 5th and 7th spectra reflected from that plane. In the first place only the 3rd was observed. On taking greater care the 1st was afterwards observed. The third spectrum is quite easy to observe. Two separate sets of observations taken with the bulb and electroscope under entirely different conditions gave the magnitude of the 3rd spectrum as 4·1 per cent., and 3·6 per cent. of the 2nd spectrum. We may take the value 4 per cent. as very nearly right. Other sets not taken with the same closeness of observation gave similar values. There is a slight uncertainty with regard to the position of the line. The angle of reflection was a little less than we should expect.

In observing the first order spectrum, care was taken to eliminate variations in the bulb and electroscope while a set of observations was being taken. After preliminary observations had indicated the line, the following procedure was adopted. In order to test any variations in the bulb or electroscope while the experiments were in progress, observations of the large 2nd order spectrum were made before and after a set of observations on the 1st order spectrum. The writer was unable to avoid a slight hardening of the bulb as the experiment proceeded, but this only caused a 5 per cent. reduction in the line spectrum. As the hardening of the bulb was progressive, in the writer's experiments, four sets of observations were taken. The 1st and 3rd sets were made with gradually increasing glancing angle, and the 2nd and 4th with decreasing angles. The average of one set of readings is shown in graph form in Fig. 1. The ordinates represent the deflections of the electroscope, and therefore very approximately the intensities of the reflected rays, and the abscissæ the glancing angles. The time for each deflection was 25 seconds. The bulb used was of the gas type. It will be seen that the curve is unusually flat, except for the glancing angles, where we would expect an α line if such existed. Several other sets gave similar results. The intensity is very small indeed, about 1/800 of a full spectrum, and the difference between this and the value expected—viz., 1/361—is somewhat greater

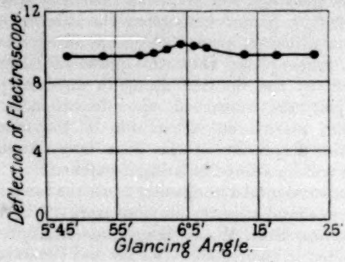

Fig. 1.—α Line, 1st Order.

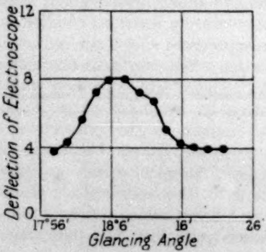

Fig. 2.—α Line, 3rd Order.

than the order of experimental error. The divergence in the case of the third order spectrum was far greater. The form of the *a* line is shown in Fig. 2. The scale is not far different from that of Fig. 1. The value observed is about thirty times that expected.

The distortion theory the writer has put forward would

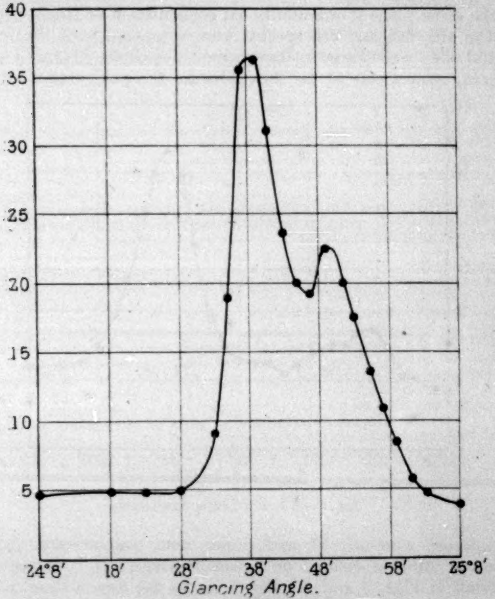

FIG. 3.—α LINES, 4TH ORDER.

easily account for the magnitude of the 1st order observed, but it could only be made to account for the size of the 3rd order by supposing that the ratio of reflecting powers of the two sets of atom planes altered according to the order of the spectrum; in other words, that it depended on the obliquity of the rays. This would be so, if the unit of reflection was that from each shell electron, and not that from the atom as a whole. In this case the contribution of each electron in any one atom will vary according to the position of the electron with regard to the nucleus and the wavelets from each electron will not be in the same phase. The total from each atom will be the vector sum of the wavelets contributed by each electron from its shell system. A study of the even order spectra in the same plane affords additional confirmation of these views. The 4th, 6th and 8th spectra were compared with the 2nd, and they were found to have intensities of 40, 11 and 3 per cent. respectively of the 2nd, whereas the percentages to be expected were 25, 11 and 6 per cent. respectively. The fourth order is seen to be unusually large. It is shown in detail in Figs. 3 and 4. Fig. 3 shows the two α lines, and Fig. 4 the α and β lines. The time taken for each reading was 20 seconds. It will be seen that lines usually regarded as faint are quite easy to observe by this method in high order spectra.

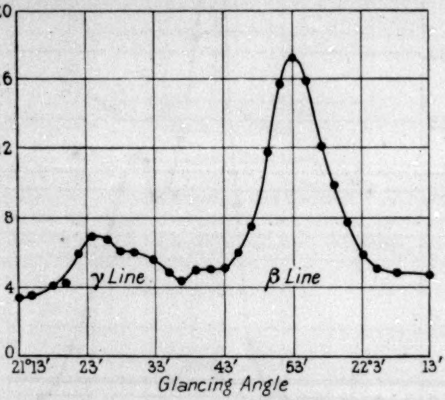

FIG. 4.—β AND γ LINES, 4TH ORDER.

We have tried above to explain the large third order spectrum by supposing that the electron is the source of the diffracted wavelets. For an obliquity corresponding to the 3rd order we must suppose that the wavelets from the electrons of one of the atoms—say, calcium—are more than usually in agreement so far as phase is concerned. For a slightly greater obliquity, the other atom—say, fluorine—exhibits the same sort of behaviour, and consequently we must expect an augmented effect. The third spectrum on this view is due to the increased reflecting power of the one set of atoms, and the large fourth order due to the increased reflecting power of the other atoms.

Sir W. H. Bragg suggested this investigation to me, provided me with the spectrometer and crystals, and gave me much kind help during the course of the work.

DISCUSSION.

Prof. **Bragg** said that the relatively high magnitude of the third order spectrum was most curious. There was evidently something diffracting strongly about a third of the way between the calcium and fluorine atoms : It was possibly only a coincidence, but this was where the junction of these atoms occurred.

Prof. **Rankine** asked what was supposed to be the source of the diffraction, the nucleus or the electrons distributed round it. If the latter, what was the justification for assuming the diffraction to originate from the planes containing the nuclei ?

Prof. **Bragg** said the origin of the diffraction was what they were all trying to settle. Possibly the Paper he was just about to read would answer Prof. Rankine's question.

Dr. **Hopwood** was not clear as to what the Paper proved. Assuming the reflecting power of the calcium and fluorine atoms to be proportional to the atomic numbers the same magnitude for the faint spectra was apparently to be expected with or without the transference of electrons of the Lewis-Langmuir theory. This being so, he did not see that the result was evidence one way or the other.

Mr. **Pealing** said the discrepancy of the larger third order spectrum was more easily accounted for on the basis of the Lewis-Langmuir theory.

Dr. **Hopwood** thought this might be due to a single distortion rather than to a transfer of electrons from the calcium to the fluorine.

XXVII. *The Intensity of X-Ray Reflection by Diamond.*

By SIR W. H. BRAGG, *K.B.E., F.R.S.*

RECEIVED MAY, 13, 1921.

ABSTRACT.

The Paper describes an investigation of the relative intensities of the reflections of monochromatic X-rays by the various crystallographic planes of diamond. The special difficulties due to the small size of available crystals and the modification of the usual method required to obviate them are discussed. An interesting feature of the results is that they lie very closely on smooth curves, which indicates that if the outer electrons of the carbon atom lie at any considerable distance from the centre they must be in motion over a wide range, or for some other reason must contribute little to the reflection.

It is shown that the properties of the carbon atom in diamond are based on a tetrahedral and not a spherical form. The tetrahedra point away from any (111) plane in the case of half the atoms, and towards it in the case of the other half. Consecutive 111 planes are not exactly of the same nature and consequently some slight second order reflection from the tetrahedral plane might be expected. This effect, though slight, has been found.

A KNOWLEDGE of the relative intensities of the reflections of monochromatic X-rays by the various planes of diamond might be expected to give useful information for several reasons.

The most important is the general reason that such relative intensities for any crystal depend on the arrangement of the atoms round each point on the lattice and on the arrangement of the electrons in each atom, and are markedly characteristic of each crystal. Nevertheless, because of insufficient knowledge, it is not easy to interpret the results.

Consequently, a study of these intensities in the case of diamond might be particularly useful, because there is but one kind of atom and because the structure is simple and exactly known. Interpretation might be easier than in the case of more complicated crystals.

There is, however, one difficulty which is not met with in the case of rock-salt or other crystals which can show faces of large area, either natural or prepared. When a fine pencil of X-rays is allowed to fall on a large enough face the dimensions of the crystal need not enter into the calculations; because the face may be so large that the pencil cannot miss it, and the depth of the crystal below the face may be so great that the rays are wholly absorbed.

This cannot be realised in the case of diamond, and a different plan must be followed. Instead of making the pencil of rays small, and using a large crystal, it is necessary to make the pencil so broad that the diamond is entirely bathed in it. All the slits of the spectrometer are opened wide, and the dimensions of the reflected pencil are determined by the size of the crystal.

The diamond is mounted on a stand somewhat similar to that of a goniometer, and its position is so adjusted by means of the X-ray reflections that any convenient zonal axis coincides with the axis of the spectrometer. It is then easy to revolve the crystal from one position to another, and to test the intensity of reflection by any of the planes belonging to the zone.

The zonal axis most used was that which is determined by the intersection of (110) and (001); all the important planes pass through it. In measuring the intensity for any particular plane, the ionisation chamber was first set at the proper angle, and the crystal was then turned at a uniform and definite rate through the small range of angle for which it reflects. The intensity was measured by the calibrated movement of the electroscope leaf during the sweep. Allowance was made for the small amount of general radiation which entered the chamber during the time of observation. The method has been described in the "Philosophical Magazine," May, 1914.

The results shown in Table I. were obtained with a diamond weighing 9·8 milligrams. When the current in the Coolidge bulb was 1 milliampere and the potential about 40,000 volts, and when the electroscope was adjusted to a moderate sensitiveness, such as 20 divisions to the volt, the reflection from the 111 plane caused the leaf to move at a rate of about 5 divisions in a second. The divisions were such as could be readily divided into 10 by eye. The bulb had a rhodium anticathode.

The diamond* was chosen of a form as nearly as possible spherical; it showed no well-marked faces. This was done because it was expected that the reflected intensities would show the influence of absorption within the crystal, as actually proved to be the case. If the diamond had been very irregular in shape, the intensities corresponding to the different planes would have been affected thereby in different degrees, and it would have been very difficult to make the correct allowances. This difficulty was largely avoided by choosing a diamond nearly spherical in form, and by measuring the intensities of all the planes belonging to three out of the six different zonal axes of the type already mentioned. The intensities agreed among themselves to about 10 per cent. The figures in Table I. are averages, and are probably correct to 5 per cent. They are given exactly as they were found, without allowance for polarisation, temperature or other influence.

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* This and other stones were most kindly given to me by Mr. Alphonse Abrahams.

TABLE 1.

Indices of plane.		Cosec θ.		Intensity.
111	......	6·72	......	200 (standard)
110	......	4·10	......	145
311	......	3·51	......	84
222	......	3·36	......	4
100	......	2·91	......	97
331	......	2·67	......	62
211	......	2·38	......	81
333 511	......	2·24	......	45
220	......	2·05	......	62
310	......	1·83	......	52*
533	......	1·76	......	29
444	......	1·68	......	38
711 551	......	1·63	......	21·5
321	......	1·56	......	36*
553	......	1·51	......	19
200	......	1·45	......	28
733	......	1·42	......	16·5
411 330	......	1·37	......	26
555	......	1·34	......	11·5

The results are plotted in Fig. 1, the ordinates being the intensities and the abscissæ the cosecants of the angles of reflection.

The curves show certain well-marked features. All the planes which have any even numbers in their indices lie on one smooth line; the rest lie on another. This is obviously due to the fact that the spacings of planes of the first kind are all equal to each other. The spacings of the second kind are alternately as 1 to 3. This is most easily realised in the case of the (111) or tetrahedral plane, and is a consequence of the fact that every atom is at the centre of gravity of its four nearest neighbours.

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* Only single instances of these planes were examined.

In the case of the first and third order reflections from any plane whose indices are all odd, the reflections from alternate sheets of atoms are in quadrature with each other. In the case of the fourth order reflection all sheets act together. For these reasons the (444) point lies on the upper curve in the figure, but the points representing the intensities of (111) and (333) lie on the lower.

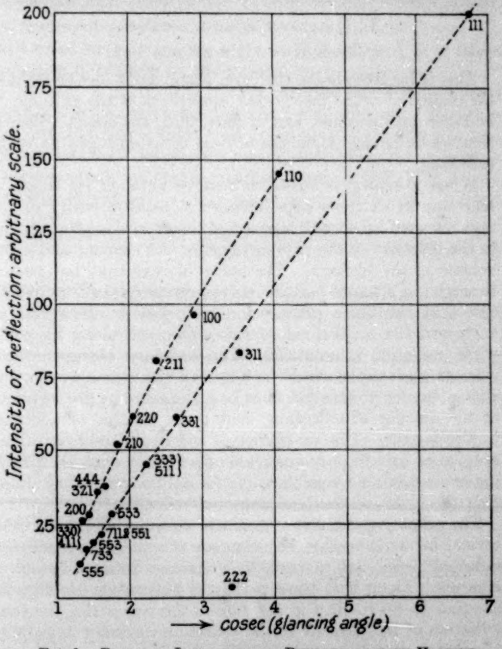

FIG. 1.—RELATIVE INTENSITIES OF REFLECTION FROM VARIOUS PLANES OF DIAMOND.

The influence of the arrangement which is characteristic of diamond structure and which causes alternate sheets of a set to act at one time in unison with each other and at another time in quadrature should cause the ordinates of the upper curve to be twice those of the lower. For if we start with a set of equal and equidistant sheets, and then take half the atoms out of each sheet to form a new set of sheets which divide the spacings of the old set in the ratio 1 : 3, the intensity of the reflection of the nth order is diminished in the ratio

$$1 : \{(1+\cos n\pi/2)^2+\sin^2 n\pi/2\}/4.$$

If $n=1$ or $= 3$, the ratio is 2 : 1.

This is far from being the case, particularly on the right of the diagram. The fact is that absorption of the rays has to be taken into account, and in particular, the special absorption which occurs when the crystal is so arranged as to be reflecting.

When a crystal is made to revolve through its range of reflection as in these experiments the total intensity of the rays reflected by a small element of the crystal is proportional to the intensity of the rays incident on the element and to the volume of the element. The pencil of rays that has passed through the element has lost energy on account of the reflection that has taken place. A similar loss or absorption of energy of the original pencil takes place all along its path, while reflection is occurring. The ordinary absorption co-efficient ρ is supplemented by a special absorption co-efficient which may for present purposes be assumed to be proportional to the amount of reflection; and to be capable of addition to the former. This co-efficient r will be different for different planes, and for any one plane it will vary with the glancing angle, having a sensible value for only a very small range on either side of the true angle of reflection.

The primary pencil has to traverse a certain distance in the crystal before it reaches any element of volume dv; and the reflected pencil has to traverse a further distance before it emerges. Along both these paths the absorption co-efficient is equal to $\rho+r$. If d is the sum of the two paths, the contribution of the element to the reflection intensity is proportional, for any given value of the glancing angle, to

$$rdve^{-(r+\rho)d}.$$

Secondary and further reflections are neglected.

The intensity of the whole reflection is obtained by integrating this expression over the volume of the crystal. The result depends on the shape of the crystal as well as on its volume, and also on the values of r and ρ. For our present purpose, a very approximate estimate is enough. It is sufficient to suppose the crystal to be so small that only first powers of $(r+\rho)\,d$ need be retained.

If $(r+\rho)\,d$ is neglected altogether the intensity of reflection is proportional to rV where V is the volume of the crystal. If first powers are retained the integration of $rdv\,(1-\overline{r+\rho}.d)$ will yield an expression $rV\,(1-k.\overline{r+\rho})$ where k is a linear quantity depending on the form of the crystal.

If, then, a point on the upper curve of Fig. 1 is compared with a point on the lower for which the abscissa is the same, the ratio of the ordinates ought to be less than two to one. The ratio would be exactly two to one if the crystal was extremely small, but, taking in first powers of $(r+\rho)d$ it is now, for the particular glancing angle in question, of the form

$$2.\ \frac{1-k(2r+\rho)}{1-k(r+\rho)},$$

which is always less than 2 and diminishes as r increases.

When, as in these experiments, the crystal is made to revolve through an angular range which includes the true angle of reflection, the numerator and denominator of this fraction require separate integration over that range; but the final ratio is still less than 2, and the more so the stronger the reflection.

If the diamonds are so large that it is not enough to limit the calculation to first powers of $(r+\rho)d$, the general conclusion is not affected. So also, if diamonds of different weight are compared with each other the larger crystals should show the effect of absorption more than the smaller, since $k(r+\rho)$ increases with the dimensions of the diamond.

The comparative intensities for a few diamonds of different weight are shown below :—

TABLE II.

Weight in mgm.	(111)	(333)	(444)
61·4	100	33·5	26
9·8	45	10·4	8·5
4·55	34	7·0	5·4

The largest stone weighs more than thirteen times the smallest, but the (111) reflection is only three times larger, and even the (444) reflection, for which r is smaller, is only five times larger than the corresponding reflection of the small stone. In the case of a large diamond of a roughly triangular form, and of more than 2 mm. thickness, the relative intensities for the three planes of Table II. were 100 to 45 to 39. This stone was kindly lent to me by Sir George Beilby.

Perhaps the most remarkable feature of the results represented in the diagrams is that such smooth curves can be drawn through the various points. If the outer electrons of the carbon atom lie at any considerable distance from the centre they must be in motion over a wide range,* or for some other reason they must contribute little to the reflection. For if not, their effects on different planes would vary greatly. If they all lie close to the centre such large motions need not be assumed. The question may be settled by an absolute measure of the intensity of diamond reflections, but the experiments are not yet complete.

Another point of interest is the existence of a small 222 reflection. This has been looked for previously but without success. The structure of the diamond cannot be explained on the hypothesis that the field of force round the carbon atom is the same in all directions : or in other words, that the force between two atoms can be expressed simply by a function of the distance between their centres. If this were so the spheres, which would then represent the carbon atoms appropriately, would adopt the close-packed arrangement. Each atom would surround itself with twelve others, all similarly placed. As a matter of fact, each atom is surrounded by four neighbours only, and the structure is so hollow that it is possible to add to the atoms in a given space others in number equal to those already there. The arrangement then becomes that of the centred cube, and each atom has eight neighbours instead of four.

It is necessary, therefore, to suppose that the attachment of one atom to the next is due to some directed property, and that the carbon atom has four such special directions : as indeed the tetra-valency of the atom might suggest. In that case the properties of the atom in diamond are based upon a tetrahedral not a spherical form. The tetrahedra point away from any (111) plane in the case of half the atoms in the diamond and towards it in the case of the other half. Consecutive 111 sheets are not exactly of the same nature ; and it might reasonably be expected that they would not entirely destroy each other's effects in the second order reflection from the tetrahedral plane. It is this effect which is now found to be quite distinct, though small.

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* *See* Papers by Coster, Proc. Roy. Acad. Sci., Amsterdam, Oct., 1919; and by Kolkmeijer, *ibid.* Jan. 1920.

DISCUSSION.

Dr. **Andrade** asked if surface tension effects entered into the problem.

Dr. **Rayner** asked if there was any chance of obtaining the arrangement of carbon atoms in the benzene ring.

Prof. **Bragg** explained that surface tension effects did not come in. In reply to Dr. Rayner, he exhibited a model of the benzene group.

Exhibit of Photographs by M. Le Duc de Broglie.

Prof. **Bragg** exhibited and explained photographs by M. le Duc de Broglie of the " Magnetic Spectra " obtained when the β-rays produced when X-rays strike copper are deflected on to a photographic plate by a magnetic field.

XXVIII. *Capacity and Eddy Current Effects in Inductometers.*

By S. Butterworth, *M.Sc.* (*The National Physical Laboratory.*)

Received March 23, 1921.

ABSTRACT.

1. A theoretical investigation has been made upon the effects of self and intercapacities and of eddy-current losses in mutual inductances for which the coils have a common point. The following are the main conclusions arrived at.

(*a*) Small capacities introduce variations in the effective self and mutual inductances which change as the square of the frequency.

(*b*) Corresponding variations are introduced in the effective resistances of the coils.

(*c*) The mutual inductance is no longer "pure," that is, the secondary E.M.F. is no longer in exact quadrature with the primary current. The primary and secondary circuits behave as if they have a resistance in common which varies as the square of the frequency.

(*d*) Eddy currents produce impurity and changes of effective resistance which are initially of the same nature as those due to capacity.

(*e*) Eddy current and capacity effects may be so combined that the variations of mutual inductance and of impurity cancel simultaneously, so that it is theoretically possible to obtain a "pure" mutual inductance having no variation with frequency over the whole range of audio frequencies.

2. The conditions of balance of three alternating-current bridges, viz.:—

(*a*) The Heaviside inductance bridge.
(*b*) The Carey-Foster bridge.
(*c*) A modified Campbell frequency bridge

have been worked out on the basis that the mutual inductance is not pure and the mutual and self-inductances are not invariable with frequency. It is shown how combined observations on these bridges may be used to determine the frequency corrections of a given inductometer.

3. An experimental study has been made upon an inductometer of the Campbell type. The results show that if such an inductometer is used at the higher audio-frequencies for the measurements of the effective resistance of coils or of condensers, the inductometer corrections may in certain cases be larger than the quantities measured.

4. A method for automatically correcting an inductometer is suggested in order that it may be used for the measurement of power factors of condensers without having recourse to tables of corrections.

1.

It is usually assumed when mutual inductances are employed in alternating-current bridge measurements that the E.M.F. developed in the secondary coil is in exact quadrature with the primary current. This assumption is practically correct at low frequencies, but at the higher telephonic frequencies a component of secondary E.M.F. which is in phase with the primary current begins to become appreciable.

Mutual inductances in which this in-phase component is developed are said to be "impure," and the magnitude of the in-phase component when unit current flows in the primary may be taken as the measure of the impurity.* Impurity, as defined in this way, is of the dimensions of a resistance, and if it is denoted by σ, the vector relation connecting secondary E.M.F. E_s with primary current I_p must be written

$$E_s=(\sigma+j\omega M)I_p, \quad \ldots\ldots\ldots\ldots \quad (1)$$

M being the mutual inductance, $\omega/2\pi$ the frequency and j the operator rotating through a right angle.

2. *Factors Producing Impurity.*

In the most general case, the factors bringing about impurity in a mutual inductance composed of coils with air cores are self and intercapacities, leakage, and eddy-current losses. If the coils have a common point, impurity might also arise due to a small resistance in common with the primary and secondary circuit, unless proper precautions are taken in regard to the mode of connection. The Carey-Foster bridge used in measurements of the capacity and effective resistance of condensers is particularly liable to error owing to this form of impurity, as the common point of the mutual inductance is a four branch point. (*See* Fig. 7.) Leakage is usually of little importance with proper insulation. However, a non-uniform distribution of resistance or inductance, combined with distributed capacity, may cause an inductance to behave as if it possessed an end to end capacity having leakage. This is readily seen by considering the case of a pure inductance, L, shunted by a capacity, C, and connected in series with a resistance, R. If C and $R/\omega L$ are small, then the capacity C across L alone is equivalent to a capacity C, having leakance $-2CR/L$, and acting as a shunt across both R and L.

The effects of self and intercapacity are treated in section 4 for the case where the two coils have a common point. It is there shown that small capacities produce impurities varying as the square of the frequency, and this variation is accompanied by variations of effective resistance and inductance following the same law.

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* Silsbee, Bull. Bureau of Standards, Vol. XIII., p. 414, 1916.

The effects due to eddy currents are considered in section 7 *et seq.* The effects are mainly variations in impurity and effective resistance, the changes in inductance due to eddy currents being small. If the frequency is not too high, the changes in impurity and effective resistance are proportional to the square of the frequency. Thus it is impossible by effective resistance measurements alone to separate the effects due to capacity from those due to eddy currents, but some estimate of the relative importance of the two effects might be obtained by determining the frequency coefficients of the inductances and assuming these to be due mainly to capacity.

3. *Preliminary Network Transformations.*

The theory to be given is much simplified if use is made of the following network transformations.*

(A) Two inductive coils (Fig. 1*a*), with resistance operators

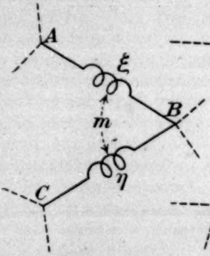

Fig. 1*a*.

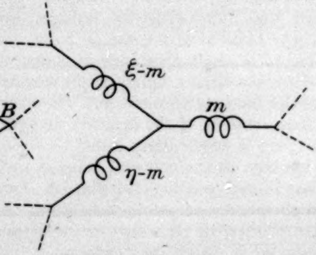

Fig. 1*b*.

ξ, η and mutual operator m, and having a common point, may be replaced by three inductive coils having resistance operators $\xi - m$, $\eta - m$, m, and no mutual operators radiating as a "star" combination from the common point. (Fig. 1*b*.)

In this transformation, m must be taken as positive when the directions of winding are such that if current is applied to the combination at the open ends the mutual inductance will oppose the self-inductances.

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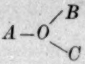
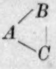
* G. A. Campbell, Trans. Am.I.E.E., Vol. XXX., p. 891, 1911.

(B) Three arms, OA, OB, OC, having resistance operators a, b, c respectively, and forming a "star" combination



may be replaced by a triangular "mesh,"



provided that the resistance operators X, Y, Z of the arms BC, CA, AB have the values $(bc+ca+ab)/a, b, c$.

Conversely, the mesh may be replaced by a star, and then

$$a,\ b,\ c=(YZ,\ ZX,\ XY)/X+Y+Z.$$

If the symbols represent *conductance* operators, the above relations are interchanged.

4. *The Effect of Self and Intercapacities in Mutual Inductances.*

Let the two coils have a common point. Let their resistance operators, apart from capacity, be ξ, η, and let their mutual operator be m. Suppose the self capacities to be sufficiently represented by condensers of capacities c_1, c_2 in parallel with the coils ξ, η, and let the intercapacity be represented by a condenser of capacity c_{12}, joining the open ends of the coils. The system is then as shown in Fig. 2a.

By application of transformation (A), the system is converted to that of Fig. 2b, in which

$$x=m,\ y=\xi-m,\ z=\eta-m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and α, β, γ are the conductance operators of the capacity arms c_{12}, c_2, c_1.

By application of transformation (B) to the star combination x, y, z, the system of Fig. 2b is converted to that of Fig. 2c, in which

$$\begin{gathered}\alpha'=\alpha+x/\mu,\ \beta'=\beta+y/\mu,\ \gamma'=\gamma+z/\mu\\ \mu=yz+zx+xy \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)\end{gathered}$$

By the converse of transformation (B), the system of Fig 2d is then obtained, in which

$$\left.\begin{aligned}X,\ Y,\ Z&=\frac{x+\alpha\mu,\ y+\beta\mu,\ z+\gamma\mu}{1+\alpha(y+z)+\beta(z+x)+\gamma(x+y)+\lambda\mu}\\ \lambda&=\beta\gamma+\gamma\alpha+\alpha\beta\end{aligned}\right\} \quad . \quad . \quad . \quad (4)$$

Finally, by the converse of transformation (A), the system of Fig. 2*d* is converted to that of Fig. 2*e*, in which

$$\xi'=X+Y,\ \eta'=Z+X,\ m'=X \quad . \quad . \quad . \quad . \quad (5)$$

The result is thus a system in which the capacities have been suppressed, so that ξ', η', m' are the modified operators to employ to include the effects of capacity. If c_1, c_2, c_{12} are small, quantities involving their squares or products may be neglected, and in this case it is found from (2), (4) and (5) that

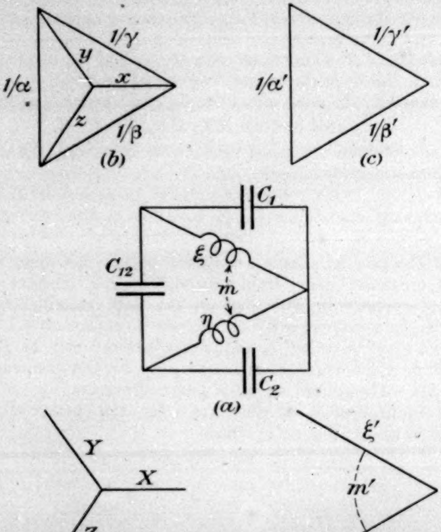


FIG. 2.

$$\left.\begin{aligned} m'&=m+\alpha(\xi-m)(\eta-m)-\beta m\eta-\gamma m\xi \\ \xi'&=\xi-\alpha(\xi-m)^2-\beta m^2-\gamma\xi^2 \\ \eta'&=\eta-\alpha(\eta-m)^2-\beta\eta^2-\gamma m^2 \end{aligned}\right\} \quad . \quad . \quad . \quad (6)$$

Now, in the case of currents alternating with frequency $\omega/2\pi$, resistance operators may be replaced by vector impedances, so that we may then write

$$\left.\begin{array}{lll} \xi=R_1+j\omega L_1, & \eta=R_2+j\omega L_2, & m=j\omega M \\ \alpha=j\omega C_{12}, & \beta=j\omega C_2, & \gamma=j\omega C_1 \end{array}\right\}, \quad . \quad . \quad . \quad (7)^*$$

in which R_1, R_2 are the resistances of the coils, L_1, L_2, M the inductances.

Applying (7) to (6) and separating the real and imaginary parts, the following formulæ are obtained.

$$m_c=C_{12}R_1R_2+\omega^2\{C_1L_1M+C_2L_2M-C_{12}(L_1-M)(L_2-M)\} \quad (8)$$

$$\sigma_c=\omega^2[C_1R_1M+C_2R_2M-C_{12}\{R_1(L_2-M)+R_2(L_1-M)]. \quad (9)$$

$$l_{1c}=-(C_1+C_{12})R_1^2+\omega^2\{C_1L_1^2+C_2M^2+C_{12}(L_1-M)^2\} \quad (10)$$

$$l_{2c}=-(C_2+C_{12})R_2^2+\omega^2\{C_1M^2+C_2L_2^2+C_{12}(L_2-M)^2\} \quad (11)$$

$$r_{1c}=2\omega^2R_1\{C_1L_1+C_{12}(L_1-M)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$r_{2c}=2\omega^2R_2\{C_2L_2+C_{12}(L_2-M)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In these formulæ m_c, l_{1c}, l_{2c} are the increases in mutual and self inductance, σ_c is the impurity introduced by the capacities, r_{1c}, r_{2c} are the added resistances due to capacity.

5. *Measurement of Self and Intercapacity.*

If it may be assumed that the variations of self and mutual inductance are due to capacity only, equations (8), (10), and (11) may be used to determine the capacities C_1, C_2, C_{12}.

With both coils in position and having a common point, the effective self inductances of the two coils separately and in series are determined at a number of different frequencies. Then, if for brevity we denote the coefficients of ω^2 in (8), (10), (11) by k_{12}, k_1, k_2 and express C_1, C_2, C_{12} in terms of these coefficients, we obtain

$$\left.\begin{array}{l} (L_1L_2-M^2)^2C_1=k_1L_2(L_1-M)-k_2M(L_1-M) \\ \qquad\qquad\qquad\qquad +k_{12}(L_1L_2-2ML_1+M^2) \\ (L_1L_2-M^2)^2C_2=-k_1M(L_2-M)+k_2L_1(L_2-M) \\ \qquad\qquad\qquad\qquad +k_{12}(L_1L_2-2ML_2+M^2) \\ (L_1L_2-M^2)^2C_{12}=k_1ML_2+k_2ML_1-k_{12}(L_1L_2-M^2) \end{array}\right\}. \quad (14)$$

---

* If leakage is also present α, β, γ are of the forms $G+j\omega C$ in which G is the leakance associated with the capacity C.

These equations determine C_1, C_2, C_{12} as k_1, k_2, k_{12} and may be deduced from the frequency variations of the self and mutual inductances.

The method fails when the coupling is very close, as, for perfect coupling, the three relations employed are no longer independent.

The values of C_1, C_2 obtained by this method are not necessarily the same as would be obtained when the coils are separated from each other, as the capacities C_1, C_2 include any intercapacity that may be regarded as acting from the inner end of one coil to the outer end of the other. Also if the common point is altered the values of C_1, C_2, C_{12} may also change because of the redistribution of the intercapacities.

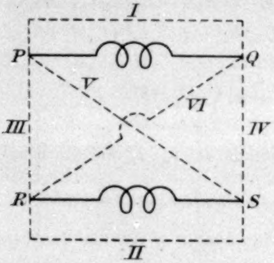


FIG. 3.

However, by a systematic alteration of the common points the assumption that distributed intercapacities may be replaced by end capacities may be put to an experimental test.

Thus, when the coils are not connected the system is as in Fig. 3, in which the broken lines represent the capacity arms. Four common points are possible, so that 12 combinations of the six capacities—I, II, III, IV, V, VI—may be measured. These are given in the following table :—

Common Point.	C_1	C_2	C_{12}
PR	I+VI	II+V	IV
QS	I+V	II+VI	III
QR	I+III	II+IV	V
PS	I+IV	II+III	VI

Since the values of C_{12} give the four intercapacities, the remaining observations give four independent determinations of the self capacities I and II respectively. If the end to end capacity theory is adequate, these should be consistent and in agreement with the self capacities as determined when the coils are separated.

6. *Conditions for Zero Impurity and for No Variation of Mutual Inductance with Frequency.*

By equation (8) the frequency coefficient of mutual inductance is zero when

$$C_{12}=M(C_1L_1+C_2L_2)/(L_1-M)(L_2-M). \quad . \quad . \quad . \quad . \quad (15)$$

By equation (9) the impurity due to capacity is zero when

$$C_{12}=M(C_1R_1+C_2R_2)/\{R_1(L_2-M)+R_2(L_1-M)\}. \quad . \quad (16)$$

If condition (15) is inserted in equation (9) an essentially negative value is obtained for σ_c, so that a mutual inductance invariable with frequency is always associated with a negative capacity impurity. As, however, the whole impurity includes both capacity and eddy current terms, and as the latter may be positive or negative it is theoretically possible to construct a mutual inductance invariable with frequency and free from impurity. This conclusion only holds when the common point is such that the two coils in series have their minimum inductance.

The procedure indicated by theory to bring about this result is to secure the condition of invariability of mutual inductance by artificial addition of self or intercapacity and then the condition of zero impurity by introduction of artificial eddy losses.

EFFECTIVE RESISTANCES AND IMPURITIES DUE TO EDDY CURRENT LOSSES.

7. *Eddy Current Losses in a Long Cylinder.*

Let a long non-magnetic cylinder of diameter d and of material of resistivity ρ be placed in a uniform field alternating with frequency $\omega/2\pi$. Let H' and H be the maximum values of the components of the field along and perpendicular to the axis of the cylinder. Also write

$$z^2=\pi\omega d^2/\rho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Then it may be shown that the rate of dissipation of energy in unit length of the cylinder due to the eddy currents set up by the alternating field is

$$W_0=\frac{\rho}{2\pi}G(z)(H^2+\tfrac{1}{2}H'^2) \quad . \; . \; . \; . \; . \quad (18)$$

in which for brevity $G(z)$ is written for

$$\tfrac{1}{4}z\,\frac{\operatorname{ber} z \operatorname{ber}' z+\operatorname{bei} z \operatorname{bei}' z}{\operatorname{ber}^2 z+\operatorname{bei}^2 z} \quad . \; . \; . \; . \; . \quad (19)$$

When z is less than 1

$$G(z)\doteq z^4/64, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (20)$$

When z is greater than 4

$$G(z)\doteq(\sqrt{2}z-1)/8, \quad . \; . \; . \; . \; . \; . \quad (21)$$

while generally the following table may be used:—

$z=$	0·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0	4·5	5·0
$G(z)=$	0·000975	0·01519	0·0691	0·1724	0·295	0·405	0·499	0·584	0·669	0·755

8. *Eddy Current Losses in a Coil.*

Equation (18) may be applied to determine the eddy current losses in a coil placed in an alternating magnetic field, provided that the values of H and H' are practically constant over the section of an individual wire, that their variation is slow along the axis of the wire and that the radius of curvature of the wire is large compared with the radius of the wire itself. Then assuming for simplicity that the field is acting transversely so that $H'=0$, the rate of energy dissipation in the whole coil is

$$W=\frac{\rho}{2\pi}G(z)\int_0^l H^2dl, \quad . \; . \; . \; . \; . \; . \quad (22)$$

where the integration is taken throughout the length l of the wire of the coil.

If $H_m{}^2$ is the mean square value of the transverse field acting on the coil and R is the direct current resistance of the coil, equation (22) may be written

$$W=\tfrac{1}{8}d^2RG(z)H_m{}^2. \quad . \; . \; . \; . \; . \; . \quad (23)$$

9. *Coil in Presence of Two Other Coils.*

Let the field H be due to two currents of vector values, $\mathbf{I}_1$, $\mathbf{I}_2$, and of phase difference ψ, flowing in two neighbouring coils. The vector fields due to $\mathbf{I}_1$ and $\mathbf{I}_2$, which act on an element of the coil, will be of the forms $a_1\mathbf{I}_1$, $a_2\mathbf{I}_2$, and will be inclined in space at an angle φ.

The components of the field along and perpendicular to $a_1\mathbf{I}_1$ are

$$\mathbf{H}_1=a_1\mathbf{I}_1+a_2\mathbf{I}_2\cos\varphi$$
$$\mathbf{H}_2=a_2\mathbf{I}_2\sin\varphi,$$

so that if H_1, H_2 are the amplitudes of $\mathbf{H}_1$, $\mathbf{H}_2$, I_1, I_2 the amplitudes of $\mathbf{I}_1$, $\mathbf{I}_2$, the value of H^2 is

$$\begin{aligned}H^2&=H_1^2+H_2^2\\&=a_1^2I_1^2+a_2^2I_2^2\cos^2\varphi+2a_1a_2I_1I_2\cos\varphi\cos\psi+a_2^2I_2^2\sin^2\varphi\end{aligned}$$

and its mean value throughout the whole coil is

$$H_m^2=\frac{I_1^2}{l}\int_0^l a_1^2dl+\frac{I_2^2}{l}\int_0^l a_2^2dl+2\frac{I_1I_2}{l}\cos\psi\int_0^l a_1a_2\cos\varphi\, dl. \quad . \qquad (24)$$

The rate of dissipation of energy in the coil is therefore obtained by using this value of H_m^2 in (23).

10. *Resistance System Imitating Eddy Current Losses.*

Let the two coils (I and II) which carry the currents I_1 and I_2 have their circuits connected by the resistance system shown in Fig. 4. The rate of dissipation of energy in this resistance system is

$$W'=\tfrac{1}{2}r_{1e}I_1^2+\tfrac{1}{2}r_{2e}I_2^2-\sigma_eI_1I_2\cos\psi, \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

and the system will dissipate the same energy as the eddy currents in the coil just considered provided that

$$\left.\begin{aligned}r_{1e}&=\tfrac{1}{4}d^2RG(z)\left\{\frac{1}{l}\int_0^l a_1^2dl\right\}\\ r_{2e}&=\tfrac{1}{4}d^2RG(z)\left\{\frac{1}{l}\int_0^l a_2^2dl\right\}\\ \sigma_e&=-\tfrac{1}{4}d^2RG(z)\left\{\frac{1}{l}\int_0^l a_1a_2\cos\varphi\, dl\right\}\end{aligned}\right\} \quad . \quad . \quad . \qquad (26)$$

If by the converse of transformation (A) we convert σ_e to a mutual operator, the common point may be removed and r_{1e},

r_{2e}, σ_e will then represent the increments of resistances and the impurity introduced by the eddy current losses in a neighbouring coil. If there are a number of neighbouring coils, then since the losses are additive, the eddy current resistances and impurities are sums of expressions such as (26). In particular the neighbouring coils may simply be the coils *I* and *II* and the eddy current resistances and impurities are got by performing the integrations in (26) throughout the lengths of wire in the two coils.

The increments in resistance obtained in this way are to be added to the ordinary "skin" increments of resistance, the latter being due to the currents flowing in the individual elements of the coil.

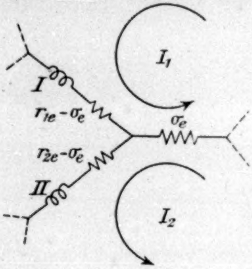


FIG. 4.

11. *Conditions Governing Sign of* σ_e.

In applying equation (26) some care is required in order to avoid the wrong sign for σ_e. In Fig. 4, from which the equations have been deduced, the directions of the currents are such that if $I_1=I_2=I$ the current I will flow only through the resistances $r_{1e}-\sigma_e$, $r_{2e}-\sigma_e$, so that when the coils are connected in series the total eddy current resistance is $r_{1e}+r_{2e}-2\sigma_e$. In order therefore to make σ_e consistent with the defining equation (1) the mutual inductance M between the coils *I* and *II* must, under the same conditions, oppose the self-inductances. Now a_1, a_2 are fields due to unit currents in coils *I* and *II* respectively, so that in order to obtain the correct sign for σ_e the directions of these unit currents must be so chosen that the mutual inductance opposes the self-inductances. With this convention σ_e will be positive if the sum of the eddy current resistances is less than the eddy current resistance of the coils in series and helping.

12.

It is readily seen from equations (26) that the inequalities

$$r_{1e}+r_{2e}>2\sigma_e$$
$$r_{1e}r_{2e}>\sigma_e^2$$

hold for the eddy current resistances, relations analagous to those between self and mutual inductances.

13. *Systems of Parallel Wires.*

The theory finds its simplest application when the coils *I* and *II* consist of systems of parallel wires. The results in these cases may be applied to coaxial coils provided that winding sections of the coils and their distance apart are small compared with their radii.

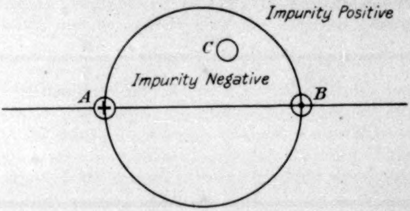


FIG. 5.

In order to illustrate the properties of the impurity σ_e, consider three parallel wires *A*, *B*, *C* (Fig. 5). Let *A* be the coil *I* and *B* the coil *II*. Let *C* carry no current. The impurity introduced by the presence of *C* will be negative, zero or positive according as the angle of inclination of the fields at *C* produced by opposite currents in *A* and *B* is less than, equal to, or greater than $\pi/2$. If a circle be drawn with *AB* as diameter, then since the two fields are normal at all points on this circle, the impurity due to the presence of *C* changes sign from negative to positive as *C* is moved out of the circle. Along the line AB, the fields either help or oppose each other, so that when C is collinear with AB the condition $r_{1e}r_{2e}-\sigma_e^2=0$ is satisfied. This is a case of perfect eddy current coupling.

The above conclusions still apply when the wires A and B are replaced by bundles of wires, provided that the distance of C is sufficient to warrant the assumption that the currents in A and B act as if concentrated at a point. It is thus possible to counteract an already existing impurity of any sign by introducing a third coil carrying no current in the proper position with respect to the two coils forming the mutual inductance. An example of this use of eddy currents to correct impurity is given in Section 19.

14. *System of Four Wires.*

Let there be four coplanar wires, two constituting the coil I and two constituting the coil II, spaced at equal distances D. Numbering the wires in order 1, 2, 3, 4, there are three possible combinations, (1, 2), (3, 4); (1, 3), (2, 4); (1, 4), (2, 3), each giving different effective resistances and impurities. By a simple application of the formula $H=\frac{2I}{r}$ we obtain the following values for a_1, a_2, $\cos\varphi$ for the combination (1, 2), (3, 4).

Wire	a_1D	a_2D	$\cos\varphi$
1	2	5/3	−1
2	2	3	+1
3	3	2	+1
4	5/3	2	−1

Using these values in (26) we find

$$r_{1e}=2{\cdot}472\frac{d^2}{D^2}G(z)R=r_{2e},\ \sigma_e=-0{\cdot}667\frac{d^2}{D^2}G(z)R$$

in which R is the direct-current resistance of one pair of wires. By similar methods for the combination (1, 3), (2, 4) we find

$$r_{1e}=1{\cdot}139\frac{d^2}{D^2}G(z)R=r_{2e},\ \sigma_e=+0{\cdot}667\frac{d^2}{D^2}G(z)R$$

and for the combination (1, 4), (2, 3)

$$r_{1e}=0{\cdot}361\frac{d^2}{D^2}G(z)R,\ r_{2e}=3{\cdot}250\frac{d^2}{D^2}G(z)R,\ \sigma_e=0.$$

In all three cases $r_{1e}+r_{2e}+2\sigma_e$ has the same value, viz., $3{\cdot}611\frac{d^2}{D^2}G(z)R$, which is, of course, the proper value for the proximity resistance of a system of four parallel wires carrying currents in the same direction. The example also shows that the sign and magnitude of the impurity depends upon the interlacing of the wires of the two coils, the tendency being for the impurity to increase positively as the interlacing increases. This point may be further illustrated by a system of 12 equidistant wires, of which six constitute one circuit and six the other circuit. If we write

$$\sigma^e=u_{12}\frac{d^2}{D^2}G(z)R,$$

u_{12} takes the following values for different arrangements :—

Arrangement	u_{12}
1 1 1 1 1 1 2 2 2 2 2 2	$-0{\cdot}80$
1 1 1 1 1 2 2 2 2 2 2 1	$-0{\cdot}70$
1 1 1 1 2 2 2 2 2 2 1 1	$-0{\cdot}66$
1 1 1 2 2 2 2 2 2 1 1 1	$-0{\cdot}65$
1 1 1 2 2 2 1 1 1 2 2 2	$-0{\cdot}35$
1 1 2 2 1 1 2 2 1 1 2 2	$-0{\cdot}00_3$
1 2 1 2 1 2 1 2 1 2 1 2	$+0{\cdot}66$

Interlacing the coils will increase intercapacity effects, so that if the coils are used with their common point such that the inductance in series is a minimum, the effect of increasing intercapacity will cause a capacity impurity acting oppositely to the eddy current impurity.

15. *Short Single Layer Coils in Contact.*

The eddy current impurity between two short coils of equal radius in end contact may be calculated if their separate effective resistances r_{1e}, r_{2e} and their effective resistance r_s in series are known for $r_s=r_{1e}+r_{2e}+2\sigma_e$.

The value of r_s has been shown by the author in another Paper to be of the form

$$r_s=u_n\frac{d^2}{D^2}G(z)R. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Where R is the direct-current resistance of the whole combination, $G(z)$ is defined in Section 7, d is the diameter of the wire, D the distance of consecutive wires and u_n is a factor depending on the number of wires n in the two coils. The following values when plotted against $1/n$ enable u_n to be determined in any case :—

$n=$ 1 2 3 5 10 inf.
$u_n=$0·000, 1·000, 1·500, 2·014, 2·514, 3·290

The values of r_{1e}, r_{2e} are, however, greater than would be estimated by the above method because of the eddy currents induced in one coil by the current in the other. If the number of wires is finite in both coils, the additive term due to the presence of the other coil may be calculated by methods similar in principle to those employed in the case of the four-wire system. When the number of wires is large, this method becomes laborious, but the extreme case of an infinite number of wires may be obtained by integration. Treating the narrow coils as straight-wire systems, the field at any point in the plane of a strip of width a is $2\frac{I}{D}\log\frac{a+x}{x}$ where I is the current in each wire and x the distance of the point from the edge of the strip. If the strip representing the other coil has width b, the mean square value of the field over this coil is

$$\frac{4I^2}{D^2}\cdot\frac{1}{b}\int_0^b \log^2\frac{a+x}{x}\,dx=\frac{4I^2}{D^2}\frac{a}{b}f\left(\frac{b}{a}\right)$$

in which
$$f\left(\frac{b}{a}\right)=\int_0^{\frac{b}{a}}\log^2\frac{1+\mu}{\mu}d\mu.$$

The values of $f(\mu)$ may be obtained in the form of series. Thus suitable series are :

$\mu<2$

$$f(\mu)=-\mu\log^2\nu+2\log\nu\log(1-\nu)+2\sum_1^{\infty}\frac{\nu^r}{r^2}\quad . \quad . \quad (28)$$

in which
$$\nu=\mu/(1+\mu);$$

$\mu>2$

$$f(\mu)=\frac{\pi^2}{3}-\sum_{r=1}^{r=\infty}\frac{(-1)^{r+1}}{r\mu^r}\sum_{s=1}^{s=r}\frac{1}{s(s-r+1)}\quad . \quad . \quad . \quad (29)$$

By (26) the term in r_{1e} for the coil (a), which is due to the eddy currents in (b) is

$$\frac{d^2}{D^2}G(z)\frac{a}{b}f\left(\frac{b}{a}\right)R=\frac{d^2}{D^2}G(z)f\left(\frac{b}{a}\right)R_a,$$

where R_a, R_b are the direct-current resistances of R_a and R_b.

Hence

$$\left.\begin{aligned} r_{1e} &= \left\{u_n(a)+f\left(\frac{b}{a}\right)\right\}\frac{d^2}{D^2}G(z)R_a \\ r_{2e} &= \left\{u_n(b)+f\left(\frac{a}{b}\right)\right\}\frac{d^2}{D^2}G(z)R_b \end{aligned}\right\} \quad . \quad . \quad . \quad (30)$$

These equations, together with (27) and the tables of u_n and $f(b/a)$ enable the eddy current resistances and impurities of short single layer coils in end contact to be calculated. In the tables for $f\,(b/a)$ the values for the case b/a infinite but n finite are calculated from the formula

$$f\left(\frac{b}{a}\right)=\frac{\pi^2}{6}+1+\frac{1}{2^2}+\frac{1}{3^2}+\;\ldots\;+\frac{1}{(n-1)^2}$$
$$-\frac{1}{n}\left(1+\frac{1}{2}+\frac{1}{3}+\;\ldots\;+\frac{1}{n-1}\right).$$

Where n is the number of wires in a.

When n is also infinite $f(b/a)=\pi^2/3$.

Values of $f(b/a)$ in Equations (30).

n_a=number of wires in system a.

n_b= „ „ „ „ „ b.

$b/a < 1$.

n_a	n_b	$f\,(b/a)$	n_a	n_b	$f\,(b/a)$.
2	1	1·125	5	1	1·042
3	2	1·511	10	4	1·704
3	1	1·120	10	3	1·523
5	4	1·858	10	2	1·265
5	3	1·701	10	1	0·858
5	2	1·462			

$n_a=n_b$=infinity.

$b/a=$	0·2	0·4	0·6	0·8
$f\,(b/a)=$	1·375	2·089	2·334	2·492

$b/a > 1$.

n_a	$b/a=1$	2	3	4	5	inf.
1	1·000	1·250	1·361	1·423	1·463	1·645
2	1·472	1·743	1·859	1·923	1·963	2·145
3	1·715	1·992	2·108	2·172	2·209	2·395
5	1·970	2·248	2·365	2·429	2·470	2·652
10	2·218	2·497	2·613	2·674	2·710	2·901
inf.	2·606	2·886	3·002	3·067	3·119	3·290

For interpolation $f(b/a)$ may be plotted against b/a for different values of n_a, and against $1/n$ for different values of b/a.

16.

As an illustration of the application of these results let the coil (a) have 20 turns, and the coil (b) 30 turns.

Then by graphical interpolation from the tables we find for coil (a)

$$f(b/a,\, n_a)=f(3/2,\, 30)=2\cdot 55$$

$$u_n(a)=\quad u_{20}\quad =2\cdot 84,$$

so that by (30)

$$r_{1e}=5\cdot 39\times 20\, G(z)\, \frac{d^2}{D^2}\, R, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (\text{A})$$

R being the direct-current resistance of one turn of wire.

For coil (b)

$$f(a/b,\, n_b)=f(2/3,\, 30)=2\cdot 24$$

$$u_n(b)=\quad u_{30}\quad =2\cdot 98$$

and

$$r_{2e}=5\cdot 22\times 30G(z)\, \frac{d^2}{D^2}\, R. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (\text{B})$$

For coils (a) and (b) in series, helping

$$f=0 \quad u_n=u_{50}=3\cdot 10$$

and

$$r_{1e}+r_{2e}+2\sigma_e=r_s=3\cdot 10\times 50G(z)\, \frac{d^2}{D^2}\, R. \quad . \quad . \quad . \quad . \quad . \qquad (\text{C})$$

From (A), (B) and (C)

$$r_{1e} : r_{2e} : \sigma_e=10\cdot 8 : 15\cdot 7 : 5\cdot 5$$

σ_e being negative.

The absolute magnitudes will depend upon the frequency, the diameter of the wire and the spacing. If we assume the wire to be of copper of resistivity 1,600 C.G.S. units, and of diameter 1 mm., then by (17) the value of z is unity when the frequency is approximately 8,000 cycles per second, and then $G(z)=0\cdot 01519$. Therefore if we take $d/D=0\cdot 8$

$$\frac{r_{1e}}{20R}=5\cdot 39\times 0\cdot 01519\times (0\cdot 8)^2=0\cdot 0523,$$

so that at this frequency the value of r_{1e} is 5·2 per cent. of the direct-current resistance of coil (a).

The ordinary skin effect for copper wire of diameter 1 mm.

at this frequency causes an increase of 0·52 per cent., and this must be added to the above value. If coil (*b*) had been absent the value of r_{1e} would be reduced in the ratio $2{\cdot}84/5{\cdot}39$, so that slightly less than half of the above increase is due to the presence of the coil (*b*).

For frequencies less than 8,000 cycles per second the eddy current resistances would fall as the square of the frequency, as by section (7) z^2 is proportional to frequency, and $G(z)$ is proportional to z^4.

At greater frequencies the increase is not so rapid, and may be deduced from the values of $G(z)$ given in section (7), z being proportional to the square root of the frequency.

In certain applications the values of $r_{1e}/L_1\omega$, $r_{2e}/L_2\omega$, $\sigma_e/M\omega$ are of importance, the latter quantity giving the want of quadrature of the mutual inductance. In the above example if we suppose the coils such that R/L for coil (*a*) is about 1,000, we have $r_{1e}/L_2\omega=0{\cdot}001$, and this also gives the order of $\sigma_e/M\omega$. Hence at a frequency of 8,000 cycles per second the secondary E.M.F. will lead the primary current by an angle which exceeds a right angle by about 0·001 radian, *i.e.*, about 3′.

This angle does not increase indefinitely with frequency, as from (27) and (17) $\sigma_e/M\omega$ is proportional to $G(z)/z^2$. From the table of values for $G(z)$ it is seen that $G(z)/z^2$ is maximum when $z=2{\cdot}6$, and (for the above example) the maximum value for the want of quadrature is about 10′.

It should be noted that for a given spacing factor this maximum angle is independent of the diameter of the wire, but is pushed further up the scale of frequency as the diameter of the wire diminishes.

17. *Short Many-layered Coils Wound Over Each Other.*

The investigation will be confined to the case in which the winding depth is small compared with the winding breadth, and the latter small compared with the coil radius. The results apply whether the coils are solenoidal or flat.

For a single coil of winding breadth, b, having m layers and n turns per layer of wire of diameter d, and direct-current resistance R_1, the term to be added to the effective resistance due to the increased eddy currents produced by coiling, has been shown by the author to be

$$r_e=\tfrac{1}{3}\pi^2\left(\frac{nd}{b}\right)^2(2m^2-1)G(z)R_1 \quad . \quad . \quad . \quad . \quad . \quad (31)$$

If a second coil of M layers is wound over this coil there is a further increase in resistance due to the eddy currents induced in the wire of the second coil.

The components of the field due to the current I in the first coil acting along and perpendicular to the winding breadth have the values $2\pi mnI/b$, and $2mnI \log \frac{b-x}{x}/b$ respectively at a point on its surface distant x from the edge. If the second coil has very small winding depth, and is close on the first, the mean square value of the field having the above components may be used in (23) to determine the required additive term.

Since

$$\frac{1}{b}\int_0^b \log^2 \frac{b-x}{x}\,dx = \frac{\pi^2}{3},$$

the result is

$$r'_e = \frac{4}{3}\pi^2\left(\frac{nd}{b}\right)^2 MmG(z)R_1, \quad . \quad . \quad . \quad . \quad (32)$$

remembering that R in (23) refers to the second coil, and is R_1M/m.

Combining with (31) we have

$$r_{1e} = \tfrac{1}{3}\pi^2\left(\frac{nd}{b}\right)^2 G(z)R_1(2m^2+4Mm-1). \quad . \quad . \quad (33)$$

Similarly

$$r_{2e} = \tfrac{1}{3}\pi^2\left(\frac{nd}{b}\right)^2 G(z)R_2(2M^2+4Mm-1), \quad . \quad (34)$$

and for the two coils in series helping (31) immediately gives

$$r_{1e}+r_{2e}+2\sigma_e = \tfrac{1}{3}\pi^2\left(\frac{nd}{b}\right)^2 G(z)(R_1+R_2)\{2(M+m)^2-1\} \quad (35)$$

Solving for σ_e with $\frac{R_1}{m}=\frac{R_2}{M}$,

$$\sigma_e = \tfrac{1}{3}\pi^2\left(\frac{nd}{b}\right)^2 G(z)(R_1+R_2)Mm. \quad . \quad . \quad . \quad . \quad (36)$$

Comparing with (31) $\sigma_e/r_e = M(M+m)/(2m^2-1)$ from which it is seen that in the case of equal coils the impurity is slightly greater than the term due to the coiling when the two coils are separate. Since the electromagnetic coupling is fairly close, the value of $r_e/L\omega$ for either coil may be taken as a measure also of $\sigma_e/M\omega$, the want of quadrature.

It should be noted also that σ_e is positive for coils with their widest edges in contact, thus differing from the coils in end contact treated in section 15.

18.

Some notion of the magnitude of the eddy current impurities in coils of the type just considered may be obtained by taking wire of a diameter of 1 mm. and a frequency of 8,000 cycles per second. Suppose also that for a single layer $R_1/L_1=1{,}000$. By the example of section (16) the value of $r_e/L_1\omega$ for a single layer is of the order 5/10,000. Now as the layers increase L increases as m^2, R_1 as m, so that by (31) $r_e/\mathrm{L}_1\omega$ will increase as $(2m^2-1)/m$. If m is large the value of $r_e/L_1\omega$ will therefore be $m/1{,}000$, and this may also be taken as a measure of $\sigma_e/M\omega$. Errors in quadrature due to this cause will therefore amount to one per cent. (34′) at a frequency of 8,000 cycles per second. For lower frequencies and diameters the error is proportional to the frequency and the fourth power of the diameter, so that the formula

$$\frac{\sigma_e}{M\omega}\doteq\frac{mf}{8\times10^9}\cdot\frac{R}{L}d^4$$

may be used to give a very rough estimate of the want of quadrature. In this formula R/L is the time constant of a single layer, d the diameter of the wire in mm., f the frequency in cycles per second, m the number of layers in one of the two equal coils.

Separation of the coils will not necessarily determine the value of σ_e/M as both σ_e and M will diminish.

19. *Balancing of Impurity and Frequency Co-efficient of a Mutual Inductance.*

It will now be shown that it is possible to introduce intercapacity and eddy current losses of such a magnitude as to balance simultaneously an already existing frequency coefficient and impurity in a mutual inductance, provided that the inductance is used with a common point in such a way that the coils are in opposition when in series. Let the two coils be equal and coaxial. Let the radius of each coil be 10 cm., their separation 3 cm. and let there be 240 turns with a square winding section of 1 cm. side. Each coil would then have an inductance of approximately 24 millihenries and their mutual inductance would be approximately 10 millihenries. The frequency co-efficient of mutual inductance will be assumed to be positive and of amount 4/1,000 at a frequency of 2,000 cycles per second.* The impurity at the same frequency will be taken to be $+0{\cdot}08$ ohm.*

If intercapacity is introduced then by (8) and (9) with $L_1=L_2=L$, the frequency co-efficient of M is reduced by $\omega^2C_{12}(L-M)^2/M$ and the impurity by $2R\omega^2C_{12}(L-M)$. Using the above values of L, M the value of C_{12} required to wipe out the already existing frequency co-efficient is approximately 1,280 $\mu\mu F$. This will also introduce negative impurity of amount $0{\cdot}0057R$ ohms. If the resistance of the coils is less than 14 ohms resistance may be added until $0{\cdot}0057R=0{\cdot}08$, and this will be sufficient to balance the impurity.

If R is greater than 14 ohms, the impurity has become negative by the addition of capacity, and in this case an auxiliary coil of the same radius as the other coils can be placed coaxially with them to introduce positive impurity. Thus suppose the auxiliary coil to consist of wire 1 mm. diameter placed at 1·5 cm. separation from the nearer coil, the positive impurity introduced by the auxiliary coil is $\dfrac{N^2d^2}{D_1D_2}G(z)R'$,

where N=No. of turns in the inductometer coils=240.
a=diameter of wire=0.1 cm.
D_1, D_2=separations of coil from inductometer coils. =1·5, 4·5 cm.
R'=resistance of auxiliary coil.

Using these values and remembering that $G(z)=0{\cdot}01519$ at a frequency of 8,000 cycles per second and falls as the square of the frequency, the value of R' to give 0·01 ohm impurity at 2,000 cycles per second is

$$R'=0{\cdot}124 \text{ ohm.}$$

This corresponds to about 10 turns of wire of radius 10 cm.

This balancing will hold over the range of frequencies, for which the square law holds. If the diameter of any wire used does not exceed 1 mm. the range is up to 8,000 cycles per second.

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* These are the order of magnitude of these quantities in the Campbell type of inductometer.

20. *Effect of Impurity, &c., in Alternating Current Bridges.*

The effect of impurity in alternating current bridge measurements in which mutual inductances are employed may be included by replacing the mutual impedance $j\omega M$ in the vector equations of balance of these bridges by $\sigma + j\omega M$. In interpreting these equations account should of course be taken of the variations of inductance and resistance of the various arms with frequency. Three bridges will be considered.

(1) The Heaviside Inductance bridge with equal arms.
(2) The Carey Foster Inductance-capacity bridge.
(3) A modified Campbell frequency bridge.

21. *The Heaviside Inductance Bridge.**

The bridge is shown in Fig. 6. P, P are two similar coils of small residual inductance. L, R is the inductive coil to be measured by means of the inductometer M. R is a constant inductance rheostat.

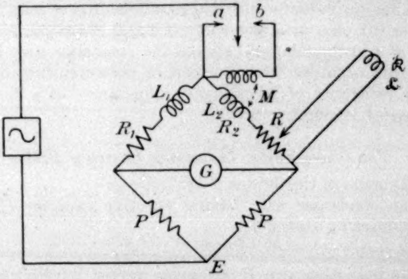

Fig. 6.—Heaviside Inductance Bridge.

Let the procedure be as follows :—

With the source connected to a and $\mathcal{L}, \mathcal{R}$ removed, L_1 and R are adjusted for balance. Then with $\mathcal{L}, \mathcal{R}$ in position and the source connected to b, M and R are adjusted for balance. Let the two values of R be R', R''.

If residuals and impurities are ignored $\mathcal{L} = 2M$ and $\mathcal{R} = R' - R''$.

As regards residuals, if observations are repeated with the arms P, P reversed the residuals of P, P cancel in the mean. Also, R being of constant inductance the residuals of the other arms cancel when the above procedure is used.

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* Heaviside, "Phil. Mag.," p. 173, Vol. XXIII., 1887.

If the mutual inductance has impurity σ a simple application of transformation (A) shows that the two conditions of balance are $\mathcal{L}=2M$, $\mathcal{R}=2\sigma+R'-R''$

It should be noted that the value of R_2 for the coil L_2 only remains the same in the two adjustments provided that the inductive action of M is introduced by the method of transference of the source from a to b *keeping the primary coil of the inductometer in position.* This is the procedure used in the Campbell form of inductometer, the fine adjustment only being made by a moving coil. In inductometers in which the whole of the adjustment is made by a moving coil, the eddy current and capacity conditions may be different in the two balances, so that R_2 will have different effective values.

Since for an inductometer of 10 millihenries σ may be of the order 0·1 ohm at a frequency of 2,000 cycles per second, neglect of σ in the relation between the resistance may introduce errors of order two per cent. in measurements of the effective resistance of 20 millhenry inductances of a nominal resistance of 10 ohms.

22. *The Carey-Foster Inductance Capacity Bridge.*[*]

The elements of this bridge (Fig. 7) are :—

(a) The condenser arm having effective capacity C and effective series resistance S.[†]

Let its vector impedance be $Z=S-j/\omega C$.

(b) The inductive arm P, L having vector impedance $\xi=P+j\omega L$;

(c) The resistance arm R having residual inductance l and therefore vector impedance $Y=R+j\omega l$;

(d) The mutual inductance M having impurity σ and acting on the inductive arm. Its mutual impedance is $m=\sigma+j\omega M$.

The condition of balance is readily found by applying transformation (A) to the mutual inductance. The result is a Wheatstone bridge of which the upper arms have vector impedances m, $\xi-m$, and the lower arms have vector impedances Y, Z. The vector equation of balance is therefore

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[*] A. Campbell, Proc. Phys. Soc., Vol. XX., p. 626 (1907). Also Phil. Mag., October, 1907.

[†] The effective capacity and resistance refer to the whole arm. Thus C includes the equivalent capacities of the leads and any residual inductance in S, while S includes the effective resistance of the condenser and leads as well as any added resistance.

$$mZ=Y(\xi-m) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Inserting the values of m, ξ, Y, Z and separating real and imaginary parts,

$$M/C+\sigma S=(P-\sigma)R-\omega^2 l(L-M) . \quad . \quad . \quad . \quad . \quad (38)$$

$$MS-\sigma/\omega^2 K=R(L-M)+Pl . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

In these equations M, K, σ, P, L, S may vary with frequency. Also σ, l and the frequency variations may be taken to be so small that their squares or products may be ignored. Thus in the small terms of (38) and (39) we may assume with sufficient accuracy that the relations

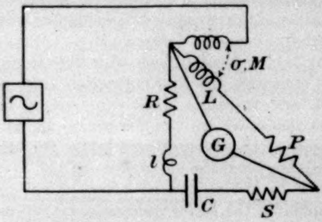

FIG. 7.—THE CAREY-FOSTER INDUCTANCE CAPACITY BRIDGE.

$$M/C=PR, S=R(L-M)/M \quad . \quad . \quad . \quad . \quad . \quad (40)$$

will hold.

In regard to equation (38) if we write $P=P_0+r$, where P_0 is the direct current resistance of P and $C_0=M/P_0R$ we obtain to the first order of small quantities

$$C=C_0\left\{1-\frac{r-L/M}{P_0}+\frac{\omega^2 l(L-M)}{P_0 R}\right\}, \quad . \quad . \quad (41)$$

which shows that if we calculate C_0 with the value of M proper to the frequency of measurement there is a further frequency correction due not only to residual inductance in R, but also due to the change of the effective resistance of P and to the change of impurity with frequency. The latter correction is less the greater the value of P_0, so that different values of C_0 may be obtained if measurements are made with different values of P_0 keeping P_0R constant. Even if the values of C_0 are consistent with various values of P_0 this is no proof that r and σ are negligible, but merely that they are of the same sign and that r/L and σ/M are approximately equal.

Since r and σ normally vary as the square of the frequency it is probably preferable to write (41).

$$C=\frac{M_0}{P_0R}(1+\gamma\omega^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

in which $\gamma\omega^2$ is the frequency correction peculiar to the particular arrangement of capacity inductance bridge used, and M_0 is the geometrical mutual inductance.

It is possible by means of Anderson's bridge to determine the true variation of the capacity of a condenser with frequency so that the value of γ may be determined.

Further, by taking observations with different values of P_0 keeping P_0R constant, the first correction term in (41) may be separated, and therefore since the residual inductance l of R may be measured we can determine by means of the capacity balance of the Carey-Foster bridge the true variation of M with frequency and the value of $r-\sigma L/M$.

Turning now to the second equation of balance (39) the resistance S includes the series resistance, (s) representing loss in the condenser, so that putting $S=S_0+s$ equation (39) may be written

$$s-\sigma/\omega^2MC=R\left(\frac{L}{M}-1\right)+\frac{Pl}{M}-S_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

Now L/M may vary with the frequency, but otherwise the quantities on the right-hand side of (43) may be regarded as known.

Writing $L/M=L_0/M_0+\mu\omega^2$ . . . . (44)

and $$s_0=R(L_0/M_0-1)+\frac{Pl}{M}-S_0, \quad . \quad . \quad . \quad . \quad . \quad (45)$$

s_0 being capable of being precisely determined, we have

$$s-\sigma/\omega^2MC-\mu\omega^2R=s_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Thus the second equation in the Carey-Foster bridge balance merely gives a relation between three unknown quantities.

However, by taking observations with different values of R keeping PR constant, the value of μ may be determined.

If σ varies as the square of the frequency equation (46) shows that even if the correction $\mu\omega^2R$ is applied the apparent series resistance of a condenser as measured by the Carey Foster method is still in error by a constant amount.

As regards the above determination of the correction μ, a check determination may be made if the variation of the series resistance of a condenser with frequency can be determined independently. This may be done by a proper use of Anderson's bridge*. A comparison of the variations of s_0 in equation (46) with those of s then gives the value of $\mu\omega^2R$, the assumption being that σ/ω^2MC is constant, or that σ varies as the square of the frequency. As has already been pointed out, the four-point contact where the galvanometer and R are connected to the inductometer may cause trouble in regard to the impurity. If this causes a true resistance coupling it will introduce a term varying as $1/\omega^2$ in the apparent series resistance of the condenser, and is thus of importance at the lower frequencies.

23. *The Modified Campbell Frequency Bridge.*†

This bridge is shown in Fig. 8, the modification being the addition of an adjustable condenser C' across the open ends of the inductometer M.

If the mutual inductance and capacity C are both perfect the only condition of balance is

$$\omega^2MC=1. \qquad (47)$$

If these two elements are not perfect a second balance is required. Equation (9) shows that intercapacity may be used to cause the impurity of a mutual inductance to assume a negative value. This suggests the introduction of the condenser C' in Fig. 8. The addition of C' will modify the original condition (47). The new conditions are found as follows:— Apply transformation (A) to the mutual inductance to convert it to a "star" combination without mutual inductance. Then apply transformation (B) to the mesh formed by C' and the two arms of the star which it connects. The result is that the generating and detecting circuits are connected only by a single arm and the conditions of balance follow from the condition that the vector impedance of this arm shall be zero. Thus after transformation (*A*) the circuits are as shown in Fig. 8*a* in which the vector impedances are

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*The use of Anderson's Bridge for determining the true variation of the series resistance and capacity of a condenser will form the subject of a separate Paper.

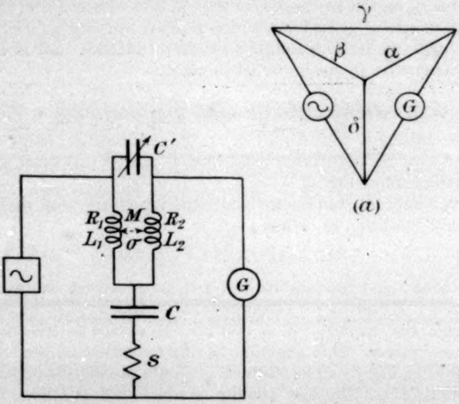
† A. Campbell, Proc. Phys. Soc., Vol. XXI., p. 69 (1908).

$$\alpha = R_1 - \sigma + j\omega(L_1 - M),\ \beta = R_2 - \sigma + j\omega(L_2 - M),$$
$$\gamma = -j/\omega C',\qquad \delta = \sigma + s + j(\omega M - 1/\omega C).$$

Applying transformation (*B*) to the mesh α, β, γ the arm common to the generating and detecting circuits will then have vector impedance $\alpha\beta/(\alpha+\beta+\gamma)+\delta$, and the condition of balance is that

$$\alpha\beta/(\alpha+\beta+\gamma)+\delta=0. \qquad (48)$$

Now for small values of σ and s, δ will be small compared with α and β in the neighbourhood of balance. Hence γ must be large, so that to the first order of small quantities (48) becomes



(*a*)

FIG. 8.—MODIFIED CAMPBELL FREQUENCY BRIDGE.

$$\alpha\beta/\gamma+\delta=0. \qquad (49)$$

Inserting the values of α, β, γ, δ the two conditions are

$$\omega^2 MC = 1 - \omega^2 CC'\{R_1R_2 - \omega^2(L_1-M)(L_2-M)\} \qquad (50)$$
$$\sigma + s = \omega^2 C'\{R_1(L_2-M) + R_2(L_1-M)\}. \qquad (51)$$

σ being ignored where it is multiplied by the small quantity C'.

In regard to equation (50) it should be noted that if the relation

$$R_1R_2=\omega^2(L_1-M)(L_2-M). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

is satisfied then to the first order of small quantities, the original condition

$$\omega^2MC=1$$

is the condition for this balance. At the usual frequencies for which the bridge is suitable (1,000 cycles per second upwards) the relation (52) is practically L_1 or $L_2=M$. When this condition is satisfied the balances do not react on each other if adjustments are made by C and C'. Otherwise the adjustments may be made independent by inserting a small adjustable resistance ($=S$, say) in series with the condenser C, and adjusting by C and S.

As regards equation (51) it is not necessary to know the separate values of R_1, R_2, &c., if the following procedure is adopted. Using the adjustments C and S repeat the balances with different values of C'. Then since $\sigma+s+S$ is proportional to C', the latter when plotted against S will give a straight line cutting the negative side of the axis of S at the value $\sigma+s$. This method may be used if C' is so large that the approximation used in obtaining equations (50) and (51) is no longer valid. For then by expansion of $\alpha\beta/(\alpha+\beta+\gamma)$ in powers of $1/\gamma$ and extracting the real part we obtain instead of (51) an expression of the form

$$\sigma+s+S=AC'+BC'^2+\ldots \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

By using a sufficient number of values of C' the quantities $\sigma+s$, A, B . . . may be found. The same procedure holds in regard to (50), the equation being of the form

$$\omega^2MC=1+A'C'+B'C'^2+ \text{- - -} \quad . \quad . \quad . \quad . \quad . \quad (54)$$

The modified frequency bridge will therefore yield the values of $\sigma+s$ and MC at a known frequency.

24. *Determination of Impurity of a Mutual Inductance and Series Resistance of a Condenser by two sets of Bridge Measurements.*

It has been shown in section 22 that the value of $s-\sigma/\omega^2MC$ may be determined by means of the Carey-Foster bridge, and in section 23 that the value of $\sigma+s$ may be found by means of the modified Campbell frequency bridge if the frequency is such that $\omega^2MC\doteqdot1$. Hence the two sets of determinations yield $s-\sigma$ and $s+\sigma$ respectively at this frequency and enable

s and σ to be determined separately. An essential point in this and similar methods is that the common point of the mutual inductance shall be the same in both bridges, as a change in common point will alter the intercapacity relations and, therefore, alter σ.

25. *Measurement of Impurity and Effective Resistance by Combined Observations on the Heaviside Bridge and the Carey-Foster Bridge.*

If observations of the capacity balance are taken on the Carey-Foster bridge with different values of P, keeping PR constant, the results enable $(r-\sigma L/M)$ to be determined, where r is the added resistance in the arm P due to the alternations of the current. Again, from the Heaviside bridge, if r' is the added resistance of another inductive coil (the coil L of Fig. 6), the measurements give $r'-2\sigma$. Hence, if two inductometers are used we can measure $r-\sigma L/M$, $r'-\sigma' L'/M'$, $r'-2\sigma$, $r-2\sigma'$, the accented letters referring to the second inductometer. These four relations will, therefore, give the required quantities r, r', σ, σ'.

26. *Experimental Study of the Frequency Variations of a Mutual Inductometer.*

The inductometer studied was one of the Campbell type* having a range of 10 millihenries sub-divided by tappings in steps of 0·1 millihenry for coarse adjustment, while fine adjustments were secured by a moving coil having a range somewhat greater than 0·1 millihenry. The scale of the moving coil was capable of being read to 0·1 microhenry, so that an accuracy of setting of one part in 100,000 of the whole range could be secured.

The method of sub-division of the fixed coils is important. This is obtained by twisting 10 equal insulated strands together, connecting in series through tappings, winding as one coil (the primary coil) and adjusting the *whole* to have a mutual inductance of 10 millihenries with regard to the fixed secondary coil. By this method the sub-division into millihenries is found to be correct to a fraction of a microhenry.

This mode of sub-division, however, introduces large capacity effects in the coil because the first and last sections are very close together. When one section only is used, the presence of the remaining nine sections, almost perfectly coupled, causes the system to behave as a 9 to 1 transformer with a capacity load, and thus introduces a large effective self capacity in the coil in use.

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* A. Campbell, Proc. Phys. Soc., Vol. XXI., p. 69 (1908).

27.

The frequency variations to be determined are those of the mutual inductance and the self inductance of the secondary coil, the impurity, and the effective resistance of the secondary coil. These quantities are not usually required for the primary coil as this coil is in the source arm in alternating current bridges. The methods to be given are independent of any assumption in regard to the perfection of inductive coils and condensers. All that is assumed is that " non-inductive " resistances and the residual inductances of these resistances are invariable with frequency.

28. *Source Employed.*

The source employed was the simple valve circuit, shown in Fig. 9.

The filament current of the valve was about 4 amperes, and the filament voltage 12 volts, while the anode voltage was 400 volts. The anode inductance was 20 millihenries, and the anode capacity was adjustable continuously to 5 mfds., so that frequency could be varied from 500 ~ per second upwards. The grid inductance was about 1 henry and was not usually tuned.

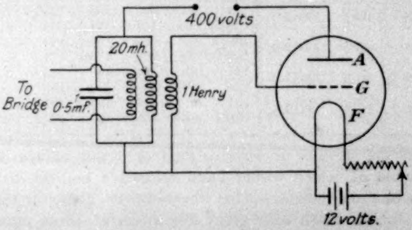

FIG. 9.—SOURCE OF ALTERNATING CURRENT.

A number of experiments were made using the modified Campbell frequency bridge, partly to test the modification, but mainly to form some notion of the steadiness of the valve source and its reaction to various factors.

With a frequency bridge for which $M \doteqdot 10$ millihenries, $C=0{\cdot}5$ mfd., so that the frequency was about 1,600~ per second, it was found that by placing an air condenser (c) in parallel with C, and making the final adjustments by this condenser, the sensitivity with a 60 ohms Brown telephone was such as to enable a change of 1 $\mu\mu F$ to be detected. This means that the arrangement is capable of holding a frequency constant to one part in a million if the other elements of the bridge can be kept constant to this accuracy.

The following observations illustrate a test of the drift in frequency which occurred during a three hours' continuous run of the source, the bridge being kept on continuously :—

Time.	Temperatures		M Setting	Readings.					
h. m.	Source Table	Bridge Table.	μH.	μμF.					
	°C.	°C.							
5 15	18·8	20·5	...	...					Valve on
5 30	...	...	...	...					Bridge on
5 43	...	...	10010·0	129	135	141	147	159	minute
5 52				165	170	178	185	190	readings.
6 15	18·7	20·5	10012·0	132	134	136	138	139·5	½ minute
6 20				142	143·5	146	148	150	readings
6 47	18·8	20·5	10013·0	124	125	127	128	129	,,
6 52				129·5	131	131·5	134	134	,,
7 15	18·7	20·5	10013·0	166·5	167	167·5	167·5	168	,,
7 20				168·5	169	170	170·5	171	,,
7 45	18·65	20·5	10013·0	187·5	188	188	189	189·5	,,
7 50				189	189	189·5	190·5	190	,,
8 13	18·6	20·5	10013·0	197	197	196	196·5	196	,,
8 18				196·5	196·5	196·5	197	197	,,

From the groups of observations of c the values of the inductance M which would have secured a balance without change of c were deduced for the moments given in the following table, which also gives the drifts at these moments expressed in terms of M. Since for the frequency bridge $\omega^2 MC=1$, the changes in adjustments of M with time are proportional to $1/\omega^2$.

Time. h. m.	Corrected M. μH.	Drift per min.
5 45	10011·42$_1$	0·071
6 15	13·32$_0$	0·040
6 45	14·20$_0$	0·022
7 15	14·66$_2$	0·010
7 45	14·87$_9$	0·004$_8$
8 15	14·96$_8$	0·000$_8$

The results are plotted in Fig 10, the tangential lines representing the observed drifts. The curve *B* shows the observed drift on a Carey-Foster bridge under similar conditions. The balance for this bridge is practically independent of frequency, so that this curve gives some indication as to what portion of the drift is contributed by the elements of the bridge.

The curve shows the extremely regular behaviour of a valve source when working under favourable conditions.

The whole change in 2½ hours is only 2 parts in 10,000 when expressed in terms of frequency, the change in the last hour being 15 in a million, and during the whole period there was no detectable departure from a regular drift. Other tests were made showing that the source was susceptible to various factors. A change of valve condenser after the circuit had been brought to a steady state immediately produced a further drift in frequency towards a new state of equilibrium (apart altogether from that due to any change in the value of the capacity). Some condensers produced a drift of opposite sign.

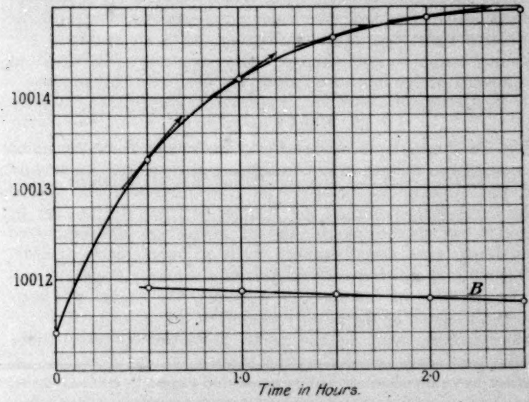

Fig. 10.—Drift in Frequency of a Valve Source as Indicated by the Modified Frequency Bridge.

An increase in condenser current of 2 per cent. produced an increase in frequency of two parts in 100,000, and a reduced filament current also increased the frequency by a few parts in 100,000, these changes taking place immediately. By running the valve on full filament current for some time it was found possible to bring about a steady state for a reduced filament current much earlier than is indicated by Fig. 10. The source frequency is also susceptible to variations in load on the bridge, the change of frequency being such as would be expected from the reduced effective inductance of the valve circuit when coupled to the bridge circuit. It is therefore necessary when bridge measurements are being taken which involve extreme steadiness of frequency and also considerable changes in the effective resistance of the bridge to have an auxiliary frequency bridge to hold the frequency constant. The modified frequency bridge has been found very useful for this purpose.

29. *Earth Capacities.*

It was found to be necessary to make corrections for earth capacities in certain of the bridge measurements. In bridges of the Wheatstone bridge type (the Heaviside bridge and the Carey-Foster bridge) the Wagner earthing device may be used,* but actually a method of correction was employed involving two measurements with different earth points. This method will form the subject of a separate Paper.

In the case of the frequency bridge, the bridge was connected to earth at the point common to the source, detector and condenser arm as the earth capacities (assumed to be condenser arms connecting the branch points to earth) will either have no effect or act as an added capacity to C (Fig. 8). As this bridge was simply used to measure *changes* in frequency or to determine quantities in which C was involved, only to the first order of small quantities, no appreciable error could arise from this cause.

30. *Frequency Coefficient of Mutual Inductance.*

For this purpose a Carey-Foster bridge method was employed, using a condenser of nominal value 0.5 mfd., whose variation with frequency had been determined by Anderson's bridge. The curve of variation for this condenser is shown in Fig. 11 in which the variations ΔC in micro-microfarads are plotted against frequency. Since the leads to the condenser and the resistance S are included in the condenser arm (Fig. 7) the effect of their residual inductances must be included in the value of C in equation (41). Let the residual inductance of the condenser arm be λ. Then if C in (14) is replaced by $C(1+\omega^2\lambda C)$ the equation becomes

---

* *See* Laws, "Electrical Measurements."

$$C=\frac{M}{P_0R}\left\{1-\frac{r-\sigma L/M}{P_0}+\frac{\omega^2 l(L-M)}{P_0R}-\omega^2\lambda C\right\}. \quad . \quad (55)$$

Writing $M=M_0(1+\gamma'\omega^2)$, (55) becomes

$$C=\frac{M_0}{P_0R}(1+\gamma\omega^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

in which

$$\gamma\omega^2=\gamma'\omega^2-\frac{r-\sigma L/M}{P_0}+\frac{\omega^2 l(L-M)}{P_0R}-\omega^2\lambda C, \quad . \quad . \quad . \quad (57)$$

and is the frequency correction of the whole bridge outside the condenser.

γ may be determined by the variations in the M settings required to balance the known variations in C at different frequencies. Actually coarse adjustments only were made by the moving coil of M, the fine adjustments being made by

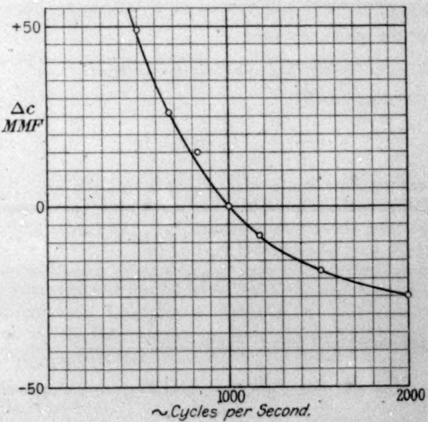

Fig. 11.—Change of Capacity of ½ μF Condenser with Frequency.

an air condenser C' in parallel with C. By subtracting $C'P_0R$ from the reading of M (viz., M_0, since the inductometer is calibrated at low frequencies) the value of M_0 could be deduced which would give the reading when C' was absent, and by further subtracting $\triangle C \times P_0R$ (Fig. 11) the value of M_0 if C had remained constant at its value at a frequency 1,000 could be obtained.

This procedure gave the following results :—

$P_0=400$ ohms. $R=50$ ohms.

Frequency $f \sim$ per second.	M_0 setting μH.	C' $\mu\mu F$	ΔC $\mu\mu F$.	M_0 corrected μH.	M_0 deduced μH.
1,650	10,025	113	-20	$10023{\cdot}1_4$	$10023{\cdot}1_7$
1,170	10,040	202·5	-8	$10036{\cdot}1_1$	$10036{\cdot}0_8$
825	10,047	203·5	$+12$	$10042{\cdot}6_9$	$10042{\cdot}6_7$

The values in the last column are such as to satisfy the law $M_0=10049{\cdot}1_7\{1-9{\cdot}51\times10^{-4}\ (f/1{,}000)^2\}$, so that by (56)

$$\gamma\omega^2=9{\cdot}51\times10^{-4} \text{ when } f=1{,}000.$$

For the resistances used

$$\lambda=6{\cdot}5\mu H,\ l=2{\cdot}8\mu H,\ L=26{\cdot}1mH.$$

Using these in (57)

$$\gamma'\omega^2-\frac{r-\sigma L/M}{P_0}=9{\cdot}9_1\times10^{-4}.$$

Measurements made with $P_0=200$, $R=100$ and $P_0=100$, $R=200$ gave the values $9{\cdot}9_8\times10^{-4}$ and $10{\cdot}1_1\times10^{-4}$ respectively. By plotting these results against $1/P_0$ the values of $\gamma'\omega^2$ and $r-\sigma L/M$ may be separated leading to

$$\gamma'\omega^2=9{\cdot}8\times10^{-4}.$$

$$r-\sigma L/M=-0{\cdot}003_5 \text{ ohm}$$

at a frequency of 1,000 ~ per second.

31. *Measurements of Impurity and Frequency Coefficient of L/M.*

By the theory given in Section 22, if s_0 is the apparent series resistance of the condenser in the Carey-Foster bridge as defined by equation (45), s_0 is of the form

$$s_0=a-bR, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

when the values of s_0 are determined with different values of R, keeping P_0R constant, the values of a and b being by (46)

$$a=s-\sigma/\omega^2MC,$$

$$b=\mu\omega^2,$$

s being the true series resistance of the condenser C, σ the impurity of the mutual inductance M, and $\mu\omega^2$ the frequency correction of L/M. The values of a and b were determined using the condenser already employed in section 30 for three different frequencies. The results are given in the following table, the columns marked "s_0 calculated" being obtained from (58), using the best values of a and b for the observations.

TABLE.

Carey-Foster Bridge.
Variation of P and R.

$M_0 \doteq 10$ mh. $L_0=26{\cdot}110$ mh.
$C \doteq 0{\cdot}5\ \mu F$. $PR=20{,}000$.
f=frequency ∿ per second.

R ohms.	$f=1{,}000.$		$f=1{,}600.$		$f=2{,}000.$	
	s_0 observed ohms.	s_0 calculated.	s_0 observed ohms.	s_0 calculated.	s_0 observed ohms.	s_0 calculated.
50	0·098	0·098	0·096	0·096	0·128	0·130
100	0·128	0·127	0·167	0·165	0·143	0·241
200	0·185	0·185	0·302	0·303	0·464	0·462
$a=$	0·069		0·027		0·019	
$-b=$	0·00058		0·00138		0·00222	
$-\frac{b}{\omega^2}=$	$1{\cdot}42\times10^{-11}$		$1{\cdot}38\times10^{-11}$		$1{\cdot}40\times10^{-11}$	

Hence, as a mean value

$$\mu=-1{\cdot}40\times10^{-11}.$$

A check on this determination may be obtained by combining the results of observations on the Carey-Foster bridge with observations on Anderson's bridge.*

For the Carey-Foster bridge we have equation (46), viz.,

$$s_0=s-\sigma/\omega^2MC-\mu\omega^2R, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

and for Anderson's bridge,* if s' is the series resistance used with the condenser,

$$s'=A-s,$$

---

* The use of Anderson's bridge for determining the true variation of the series resistance and capacity of a condenser will form the subject of a separate Paper.

where A is a quantity independent of frequency involving the remaining residuals in Anderson's bridge.

Hence

$$s_0+s'=A-\frac{\sigma}{\omega^2MC}-\mu\omega^2R,$$

or, since σ is proportional to ω^2

$$s_0+s'=A'-\mu\omega^2R, \quad \ldots\ldots\ldots \quad (59)$$

from which μ may be found by observations at different frequencies.

The results obtained by this method for three different condensers are given in the following table :—

TABLE.

Anderson's Bridge and Carey-Foster Bridge.

For the Carey-Foster Bridge—

$M \doteqdot 10$ mh. $C \doteqdot 0{\cdot}5\ \mu F$. $L_0=26{\cdot}110$ mh. $P=400$ ohms. $R=50$ ohms.

$$s_0=R\left(\frac{L_0}{M_0}-1\right)+\frac{Pl}{M}-S.$$

s'=series resistance in Anderson's Bridge.

Frequency ~ per second.	——		Condenser I.	Condenser II.	Condenser III.
1,000	s_0		0·100	0·077	−0·016
	s'		0·625	0·650	0·715
	s_0+s'	observed	0·725	0·727	0·699
		calculated	0·726	0·726	0·700
1,600	s_0		0·096	0·072	+0·001
	s'		0·675	0·700	0·740
	s_0+s'	observed	0·771	0·772	0·741
		calculated	0·771	0·771	0·745
2,000	s_0		0·127	0·096	0·042
	s'		0·685	0·715	0·751
	s_0+s'	observed	0·812	0·811	0·793
		calculated	0·812	0·812	0·786

The calculated values of s_0+s' were obtained from formula (59), using as the values A' for the three condensers.

$$A'=0{\cdot}698 \quad 0{\cdot}698 \quad 0{\cdot}672,$$

and for μ the common value $-1{\cdot}42_5\times10^{-11}$, which are the values agreeing best with the observations. This value of μ differs from the previous determination by 2 per cent.

As regards A', the capacity of condenser III. was a few per cent. higher than those of condensers I. and II., which differed only by 2 or 3 parts in 10,000. The differences in A' are sufficiently accounted for by the changes in the residuals in Anderson's bridge brought about by rebalancing for condenser III.

Determination of σ.

Using the known value of μ in equation (46) the values of $s-\sigma/\omega^2MC$ at a frequency of 1,600 cycles per second for the three condensers are

$$+0{\cdot}025, \quad +0{\cdot}001, \quad -0{\cdot}070.$$

Since $\omega^2MC=\frac{1}{2}$ at this frequency, these are the values of $s-2\sigma$.

Another relation between s and σ can be found by the modified frequency bridge (section 23). For this bridge $\omega^2MC\doteqdot1$, and this condition may be secured by paralleling two of the condensers, say, I. and II.

By the usual laws of combination, and by (53) the modified frequency bridge measures

$$(s_1+s_2)/4+\sigma.$$

The value obtained with Condensers I. and II. was

$$(s_1+s_2)/4+\sigma=0{\cdot}096.$$

Now,

$$s_1-2\sigma=0{\cdot}025$$

$$s_2-2\sigma=0{\cdot}001,$$

so that, solving for s_1, s_2, σ, $s_1=0{\cdot}115$, $s_2=0{\cdot}091$, $\sigma=0{\cdot}045$ ohms. Using this value of σ for the Condenser III.,

$$s_3=0{\cdot}020.$$

Since σ varies as the square of the frequency, the value of σ for the inductometer studied is $0{\cdot}017_5$ ohm, at a frequency of 1,000 cycles per second.

32. *Summary of Inductometer Corrections.*

From the results obtained in sections 30 and 31 the corrections for frequency for the inductometer studied are as follows:

At a frequency of 1,000 ~ per second the mutual inductance has a positive correction of 9·8 parts in 10,000,* the self-inductance of the secondary coil has a positive correction of 7·6 parts in 10,000, the impurity is $0{\cdot}017_5$ ohm, and the effective resistance of the secondary coil is greater than its direct current value by 0·042 ohm.

All the corrections vary as the square of the frequency.

These corrections only hold when the mutual inductance has a common point such that when the coils are in series, the mutual inductance opposes the self-inductances.

33. *Determination of Impurity by Measurements of Effective Resistance.*

If an inductive coil L is placed in series with a condenser C, non-inductive resistance and an alternating-current ammeter so as to form a simple circuit, the current induced in the circuit by an E.M.F. E, alternating with frequency $\omega/2\pi$, is $I=E/\{R^2+(\omega L-1/\omega C)^2\}^{\frac{1}{2}}$, so that if C is adjusted to resonance with L, the current is a maximum and has the value $I_{\text{max.}}=E/R$.

By observing the maximum current with different values of the non-inductive resistance the resistance of the remainder of the circuit may be deduced. The method therefore gives the resistance of the coil and condenser in series. Assuming the condenser to have the same power-factor at all frequencies, then remembering that the value of the capacity is altered for different frequencies the series resistance of the condenser will increase as the frequency, while if the coil is wound with thin wire the resistance of the coil increases as the square of the frequency, so that the losses in the condenser will become less and less important as the frequency increases. Moreover, at the higher frequencies the resonating capacity may be made so small that an air condenser may be used and thus the condenser losses may be still further reduced.

In order to determine the impurity of a mutual inductance by this method, the two coils are connected at one point (B in Fig. 7A) and the effective resistances measured across the points AB, BC, CA. If at a particular frequency the values of the effective resistances are R_1, R_2, R_3 respectively, then since $R_3=R_1+R_2-2\sigma$ we have $\sigma=\frac{1}{2}(R_1+R_2-R_3)$.

The following values were obtained for a fixed mutual inductance of 10 millihenries, wound on marble with No. 26 d.s.c. wire.

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* This result is in good agreement with an earlier determination on the same inductometer by Campbell. (Proc. Roy. Soc., Vol. LXXXVII., 1912, p. 406.)

Primary coil	$L=21{\cdot}9$ millihenries.	D.C.R. $=12{\cdot}54$ ohms.
Secondary coil	$L=21{\cdot}5$ millihenries.	D.C.R. $=12{\cdot}25$ ohms.

Alternating Current Resistances.

Frequency. ~ per second.	Primary.	Secondary. ohms.	Series opposition.
17,300	36	34	59
25,000	62	59	99
33,300	105	98	154

The resistances are plotted against the square of the frequency in Fig. 12, and from the mean straight lines we find at $f=1{,}000$ ~ per second, the following corrections :—

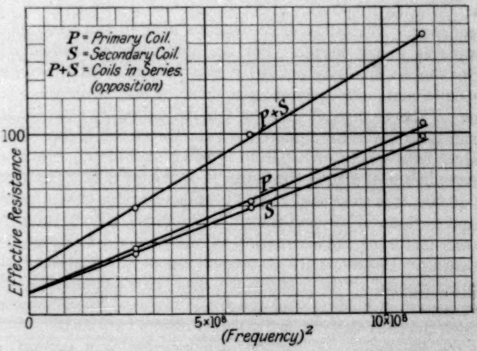


FIG. 12.—EFFECTIVE RESISTANCE OF COILS OF INDUCTOMETER.

Primary	0·081 ohm,
Secondary	0·075 ohm,
Series opposition	0·118 ohm,

from which the impurity is 0·020 ohm.

The same inductance was tested at a frequency of 1,600 ~ per second upon the modified frequency bridge, using a $1\mu F$ condenser of known power-factor. The deduced impurity at a frequency of 1,000~ per second by this method was 0·0207 ohm.

34. *Errors in Power Factor Measurements due to Imperfections of Mutual Inductance.*

Equation (46) may be written

$$\omega s_0 C = \omega s C - \sigma / \omega M - \mu \omega^3 M / P$$

and since $\omega s C$ is the power factor of the condenser, $\omega s_0 C$ will be apparent power-factor of the condenser as deduced from Carey-Foster bridge observations upon the assumption of a perfect mutual inductance. It is seen that the apparent power-factor is subject to two errors, one due to impurity and proportional to the frequency, and the other due to the change of L/M with frequency, and proportional to the cube of the frequency. The latter error depends also on the resistances employed in the bridge. For the inductometer studied, the error due to impurity is $2{\cdot}8/10^4$ at a frequency of 1,000, and tends to make the apparent power-factor low, while the error due to the change of L/M is $0{\cdot}9/10^4$ (when $P=400$ ohms), tending to make the apparent power-factor too high. These values hold whatever capacity is being measured. Since the power-factor of a good condenser is of the order of a few parts in 10,000, it is clear that the Carey-Foster bridge measures the imperfections of its own mutual inductance, rather than the power-factor of the condenser at the higher telephonic frequencies. However, by a proper use of intercapacity the imperfections of the inductance may be compensated, the compensation holding over the whole range of frequencies for which the eddy current and capacity effects vary as the square of the frequency. A method of compensating a mutual inductance for change of mutual inductance with frequency and for impurity has already been given in section 19. This compensation is suitable for a mutual inductance used for the measurement of self-inductance upon a Heaviside bridge, but when using the inductance upon a Carey-Foster bridge for power-factor measurements, what is required is that the impurity shall be zero, and the variation of L/M shall be zero.

Let a condenser C' connect the open ends of the inductometer. Then by equations (8), (9), (11), the change in the frequency coefficient of L_2/M is $\omega^2 C'(L_2-M)(L_1L_2-M)^2/ML_2$ and the change in σ is

$$-\omega^2 C' \{R_2(L_1-M)+R_1(L_2-M)\}.$$

For the inductometer studied the values of L_1, L_2, R_1, R_2, M were

$L_1 \doteqdot 7$ millihenries, $L_2 \doteqdot 26$ millihenries, $M = 10$ millihenries,

$R_1 \doteqdot 10$ ohms, $R_2 \doteqdot 18$ ohms.

Using these values we find that if $C' = 1{,}100$ $\mu\mu F$ the change of L_2/M due to C' is $+2{\cdot}2/10^4$ at a frequency of 1,000 ∽ per second, and this will just balance the existing variation of $-2{\cdot}2/10^4$. Further with this value of C' a negative impurity of amount 0·0175 ohm will be introduced if R_1 is made equal to 21·5 ohms, so that by adding 11·5 ohms to the primary coil and using an added intercapacity of 1,100 $\mu\mu F$ across the whole combination the corrections for this inductometer will be automatically compensated.

The experimental work was carried out at the Nationa Physical Laboratory. I wish to thank a number of my colleagues, particularly Mr. D. W. Dye, for valuable suggestions during the course of the experimental investigation.

DISCUSSION.

Dr. A. Russell said he presumed the Paper was on an experimental basis, as the author had mixed theory and experimental approximation throughout, *e.g.*, in treating the variation of mutual and self-inductance due to what he terms the mutual capacity and self-capacity of the two coils. He thought G. A. Campbell's method was probably not good for high frequencies. The author neglected skin effects. He supposed he was familiar with Curtis's Paper on skin effects in parallel wires in which formulæ are given for the eddy-current losses which are fully verified by experiment. Did these results agree with Mr. Butterworth's? He fully agreed with the author's remarks on the Carey Foster Bridge. In the equation for $\omega^2 C$ the equating of the last factor to zero is the condition that the phase difference between the two currents is a maximum.

Dr. D. Owen said the Paper should prove of great value in inductance measurements of high precision. The first impression on reading it might perhaps be that alternate-current methods introduced complications: but this was really a measure of the great accuracy which these methods of measurement afforded. The Paper showed that many conditions have to be borne in mind in obtaining definite results; one would like to know whether the effects of capacities to earth had also been kept in view.

Mr. Albert Campbell (communicated remarks): Mr. Butterworth's elaborate and thorough research will be of great value to all who are interested in precision measurements in which inductometers are used. The effects which he has investigated in general only give trouble in the rather extreme cases, *e.g.*, at the higher audio-frequencies, or when extremely minute power losses are being measured (in condensers of low power factor). In such cases it is probably better to use a more elaborate form of inductometer, wound with multiple pancake coils or otherwise so as to minimise the capacitance effects. Such construction would, of course, increase the volume of the instrument and add to its cost. The inductometer which Mr. Butterworth has studied is of early type, in which the higher subdivisions are all obtained by stranding the wires. In the more modern inductometers of this kind the higher coils are not stranded together; each subdivision consists of an individual coil separately adjusted. Stranding is still employed for the lower sections where the capacitance effects are of much less relative importance. The methods described by the author for applying automatic corrections will be valuable in practice and the whole Paper contains much that may ultimately be of importance to the radio engineer.

Mr. BUTTERWORTH, in reply to Dr. Russell, said he was unaware of the criticism of G. A. Campbell's method which he had mentioned, but it was all right if the frequency was under 2,000 or so. He had seen Curtis's Paper. As far as skin effects are concerned it should be taken in conjunction with a Paper by himself (at present in the course of publication), in which a much shorter formula was obtained.

In reply to Dr. Owen the question of earth capacities is dealt with in a Paper originally included as an appendix to the present one, but to be published later as a separate Paper.

XXIX. *New Specific Heat Apparatus. By* EZER GRIFFITHS. *D.Sc.* (*The National Physical Laboratory.*)

RECEIVED APRIL 11, 1921.

ABSTRACT.

The Paper is a description of an apparatus for the determination of the specific heats of materials such as cork, charcoal, &c., which are difficult to deal with by the ordinary methods. The calorimeter is a closed cylinder rotating on a horizontal axis. The heating coil and the thermo-elements project into the calorimeter and are maintained stationary. The material under test fills about one-third of the calorimeter and is mixed by being carried round by the fins attached to the internal surface. To ensure equalisation of the temperature, the material is ground to a state of fine sub-division before test.

By reducing the weights of all parts to a minimum a calorimeter of 4 litres capacity has been constructed whose water equivalent is less than 100 grams. The jacket surrounding the calorimeter is made of copper wound with a heating coil. In an experiment the temperature of this jacket is maintained equal to that of the calorimeter, thus eliminating the corrections for heat loss by radiation and convection.

THE usual "method of mixtures" for specific heat determinations is simple, and gives satisfactory results when the test specimens happen to take the form of metallic blocks. But when materials such as cork, chocolate, concrete, &c., have to be examined, serious experimental difficulties are encountered when an attempt is made to use the above method, so it is better to design an apparatus specially adapted to the work in hand. The apparatus described below was constructed for the study of the thermal capacity of light, bulky materials.

Cork, charcoal, slag wool, and diatomaceous earth are materials extensively used for cold storage insulation, and it is sometimes necessary to make calculations as to the penetration of heat through a wall when the temperature of one of its faces is suddenly raised.

In the course of an investigation on heat insulators for the Food Investigation Board, it became necessary to determine the specific heats of various heat insulating materials.

An electrical method was employed, because it was desirable to obtain the specific heat over a narrow range of temperature. Heating materials, such as charcoal, to 100 deg. alters the moisture content of the samples, and some preliminary tests proved that the charcoal and diatomaceous earth on heating to 105°C. showed a loss of weight of 6·9 and 8·8 per cent. respectively, due to the water driven off. Hence, any method which necessitated the heating of the sample through a wide interval of temperature was inadmissible.

Description of Apparatus.

The calorimeter (Fig. 1) was a large cylindrical vessel of thin sheet aluminium. It was carried on a horizontal axis, and rotated at a uniform rate by an electric motor through a gear box. The usual speed of rotation was about one revolution in two seconds.

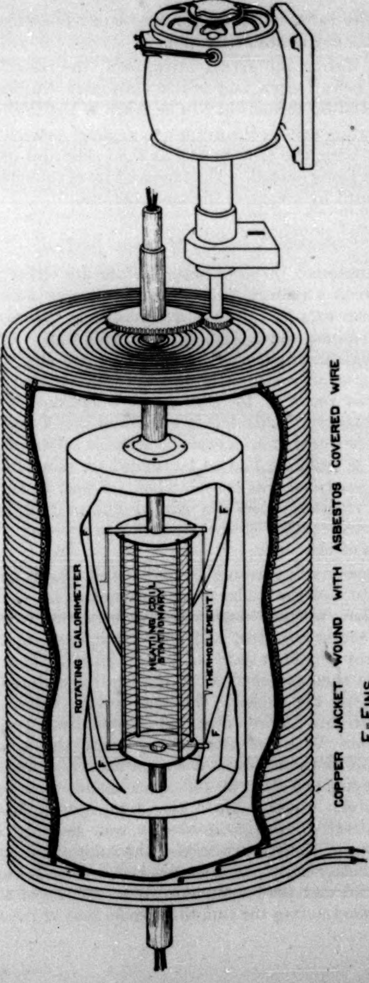
The material, in the state of fine sub-division, occupied about one-third to one-half the volume of the calorimeter. Helical fins were riveted to the internal surface, which, by the rotation of the cylinder, carried the contents around, and produced a very thorough mixing up of the powder.

A heating coil projected into the cylinder, and it was held stationary. The watts dissipated in this coil were measured by observations of the current and the potential difference at its ends.

Temperature was measured by six thermo-junctions of iron-constantan wire suitably supported so as to give the temperature at various points, but the observed differences were found to be negligibly small.

The rotation of the cylinder carried the material up, and, on reaching various heights, it dropped off the fins and fell through the heating coil on to the thermo-junctions. The helical fins were so arranged that half the set tended to carry the material in one direction by screw motion, and the other half in the opposite direction, the object being to produce mixing of the contents in the longitudinal direction. It was deemed advisable to assist the mixing by arranging the apparatus so that it would be swayed about an axis at right angles to that of the rotation, the effect of this being to throw the material from end to end of the cylinder.

By reducing the weights of the various parts to a minimum, it has been possible to construct the apparatus with a heat capacity of only 103 gram calories, whilst the calorimeter has a cubical capacity of 4,300 c.c.s. A calorimeter of this volume with such a small thermal capacity would have a relatively big heat loss by radiation and convection, if it were allowed to radiate to constant temperature surroundings. This correction was eliminated by arranging the rotating calorimeter within a large copper drum wound with a heating coil. During the experiment the surroundings were heated up at the same


ROTATING CALORIMETER
HEATING COIL STATIONARY
THERMOELEMENT
F
F
F
F
F
COPPER JACKET WOUND WITH ASBESTOS COVERED WIRE

F=FINS

FIG. I.

rate as the calorimeter by suitably controlling the current through the coil around the drum.

Hence, during the actual experiment, the material under test was heated from one nearly stationary temperature to another, and no account had to be taken of any heat losses.

The heating coil on the drum was made of asbestos-covered resistance wire, the ends as well as the cylindrical portions of the drums being heated. The temperature of the internal face was obtained by means of thermo-junctions.

Preparation of the Materials for Test.

These materials in their natural state are either granular or of fibrous structure. Consequently it was necessary to reduce them to a fine state of subdivision to ensure that the mixing up during the stirring produced uniformity of temperature throughout the mass. The charcoal and slag wool were ground up in an iron mortar and the iron abraded from the mortar was removed by the aid of a magnet. Cork was somewhat more difficult, but it was found that a meat-mincer effectively reduced it to a state of suitable subdivision. The operation of grinding had to be carried on slowly, otherwise the heat generated was considerable; it was, in fact, found that too vigorous operation caused charring of the cork. The moisture content of the samples was determined during the course of the work.

The procedure in carrying through a test was as follows :—

A quantity of the material was ground to an exceedingly fine powder, weighed and inserted into the calorimeter. This was set in rotation, and after an interval of time the temperature would attain a steady state. The electric supply was then switched on to the heating coil and a little previously to the jacket, the appropriate values of the heating currents having been determined from preliminary experiments. The temperatures of both the calorimeter and the jacket then rose at a steady rate, and adjustments of the supply to the jacket were made to keep them approximately equal. After the temperature had risen through about 15 deg., the supply was switched off and observations of the temperature of the calorimeter continued until the steady state had been reached. From a knowledge of the initial and final temperatures as well as of the total watts supplied during the run, the specific heat of the material is readily calculated. Experience showed that, owing to the fact that the response of the jacket temperature to the heating coil was less rapid than that of the calorimeter, it was necessary to switch on and cut down the supply to the jacket about two minutes before switching on and off the supply to the calorimeter.

The result would be that the rise and the final steady temperature of the jacket would correspond very closely with that of the calorimeter. It was, of course, necessary to maintain a small current through the coil on the jacket to compensate for the heat lost from its surface at the higher temperatures. Tests and calculations were made to determine the small correction for the heating of the charge by the stirring.

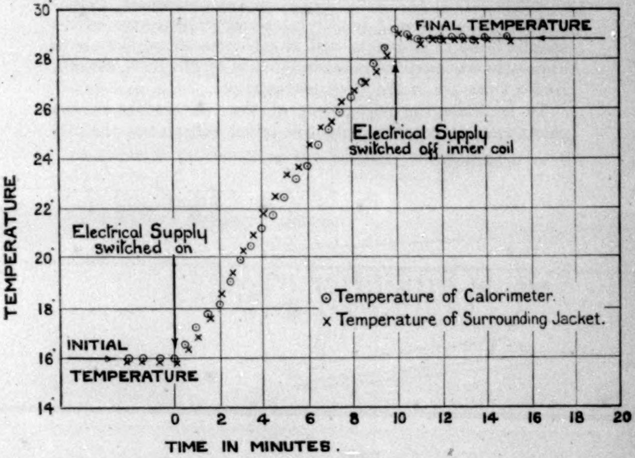


FIG. 2.

Correction was also made for the heat absorbed by the air in the space unoccupied by the charge in the calorimeter. The values of the specific heats given correspond to the material in the closely packed state and, of course, include that of a little air.

A typical series of observations are shown in Fig. 2.

Specific Heat Results.
(At mean temperature of 25°C.)

Material.	Moisture Content.	Specific Heat.
Charcoal	6·89 per cent.	$0{\cdot}29_9$
Slagwool	0·02 ,,	$0{\cdot}17_1$
Diatomaceous earth	2·09 ,,	$0{\cdot}23_2$
Baked cork (slab)	—	$0{\cdot}43_1$
Granulated cork	3·45 ,,	$0{\cdot}43_8$

During the course of some more recent experiments on the specific heats of materials used in cable insulation, the above apparatus was modified in several details by Mr. R. R. Strand, Senior Observer in the Heat Department.

To facilitate the adjustment of the temperature of the calorimeter to equality with the jacket differential thermocouples were attached to the surfaces of both calorimeter and jacket, and the method of stirring modified so that only a partial rotation is given to the calorimeter. The form of gearing required to effect this is shown diagrammatically in Fig. 3. The worm-driven wheel makes about one revolution in two seconds, and from the crank pin a string is led over the pulley fixed to the axle of the calorimeter. By suitably proportioning the diameter of the pulley and the radial distance of the crank pin the magnitude of the angular rotation of the calorimeter can be adjusted.

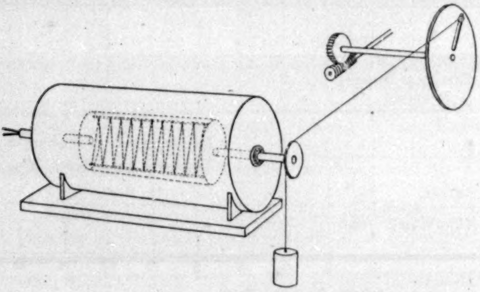


FIG. 3.

The jacket was wound with flat nichrome tape insulated by very thin micanite so as to obtain quicker response of the temperature of the surroundings to alterations in the energy supply.

In this particular calorimeter the heating coil was made of a flat, open grid of platinum wire occupying the diametrical plane. This coil was also the thermometer.

Initially a minute current was passed through the coil and the potential difference at its ends compared with that at the terminals of a standard coil connected in series with it. This gave the initial temperature rise of the calorimeter due to the stirring alone. The connections were then switched over to the heating supply circuit and the watts dissipated in the heating coil measured in the usual manner. Finally, the connections were switched back to the thermometer circuit and observations taken until the final equilibrium temperature had been attained. The advantage of the resistance thermometer is that it gives the average temperature over the area occupied.

We wish to thank our director, Sir Joseph Petavel, for his kind interest and encouragement.

XXX. *On Encounters between Non-spherical Gaseous Molecules.* By A. O. RANKINE, *D.Sc., Professor of Physics in the Imperial College of Science and Technology.*

RECEIVED APRIL 14, 1921.

ABSTRACT.

This Paper is an extension of the author's previous work on molecular dimensions and structure derived from the combined data of the kinetic theory of gases and of X-ray crystal measurements (Proc. Roy. Soc., A. Vol. XCVIII., p. 360). The cases previously considered have been confined to molecules built up of atoms all of equal size. In this Paper this limitation is removed, and unequal atoms are contemplated.

The results arrived at cover all cases of diatomic molecules and certain special cases in which the molecules are polyatomic. It will be possible to test their validity when the appropriate viscosity data become available.

1. IN dealing with the kinetic theory of gases it has hitherto been the almost invariable custom to regard the molecule as spherical, although it is well recognised that, except for monatomic molecules, this is unlikely to be true. The reason for this has, no doubt, been lack of information from which to assign probable shapes to molecules consisting of more than one atom. Recently the situation in this respect has changed, and the developments of the Lewis-Langmuir * theory of molecular constitution have provided good grounds for specifying the comparative sizes and shapes, not only of atoms themselves, but also, in many cases, of the molecules they form by combining with one another. Added to this, the dimensions in question have been reduced to a quantitative basis by W. L. Bragg's † deductions from X-ray crystal measurements, so that we are able to state with some confidence the actual distance apart of the centres of two atoms which are in chemical combination.

It has already been possible to apply this new set of data usefully to the kinetic theory. For example, a chlorine molecule, on the Lewis-Langmuir theory, is conceived to be an arrangement of electrons nearly identical in shape with that obtained by placing two argon atoms so close together that their outer electron shells are touching one another.

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* I. Langmuir, Journ. Amer. Chem. Soc., XLI., p. 868 (1919).

† W. L. Bragg, "Phil. Mag.," Vol. XL., p. 169 (1920).

The author* has been able to show that two such atoms, with this degree of proximity, may be expected to behave, from the point of view of the kinetic theory, as a molecule of chlorine in fact does. On the same lines it has been proved that molecules of nitrous oxide and carbon dioxide both behave kinetically like hypothetical molecules each consisting of three neon atoms with their centres equally spaced on a straight line and with outer electron shells contiguous, thus affording further support to Langmuir's conclusions derived from other considerations.

In all these cases, however, the molecules dealt with have been constituted of atoms of equal size. It is the purpose of the present Paper to extend the theory so as to include molecules containing unequal atoms. Unfortunately, there exist at present no reliable viscosity data by means of which to test this new extension, so that its application must await the results of further experimental investigations.

2. According to the kinetic theory of gases the atoms of a monatomic gas behave, when in thermal agitation, like hard elastic spheres, which exert slight mutual attraction on one another and frequently collide. S. Chapman † has shown that this model of the atom, viz., a hard attracting sphere of radius σ, gives, for the monatomic inert gases, results which are very closely in accordance with experiment. His formula giving the magnitude of σ is, apart from a negligible term,

$$4\pi\sigma^2 = \frac{0{\cdot}491\rho\bar{C}}{\sqrt{2}\nu\mu(1+S/T)}, \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where ρ=gaseous density, $\bar{C}$=mean molecular velocity, ν=number of molecules per unit volume, μ=viscosity, S=Sutherland's constant, and T=absolute temperature. The quantity $\pi\sigma^2$, it will be noted, is equal to the area of any and every central section of the molecule, which is supposed to be, in this case, spherical.

The measurement of the viscosity of the gas, and its variation with temperature (which determines Sutherland's constant), thus enables us to evaluate $\pi\sigma^2$ for monatomic molecules. In reality, what we find is the area which the molecule presents as a target for other molecules approaching from all directions.

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* A. O. Rankine, Proc. Roy. Soc., A., Vol. XCVIII., p. 360.

† S. Chapman, "Phil. Trans.," A., Vol. CCXVI., p. 279 (1916).

It is permissible to apply formula (1) to molecules which are not spherical provided that we interpret $\pi\sigma^2$ as being an average area, $\overline{A}$, which measures the mean target area presented by the molecule for all possible orientations. But we can go no further in the investigation of the molecular dimensions until we know how the constituent atoms are arranged.

3. If, as is evident from Chapman's work, single atoms behave in collision like hard spheres of definite radius, it is reasonable to suppose that when in chemical combination they preserve this property of apparent hardness. To a first approximation, also, we may continue to regard them as spheres. No doubt there are distortions, but there is no obvious way of estimating their extent, and we will, therefore, leave them out of account at present. According to this view we may picture a molecule as an assemblage of hard spheres arranged in a definite manner, and presenting, according to aspect, different target areas. Our purpose is to calculate the mean value of this area from all possible directions. It is evident that the problem cannot be treated conveniently by means of a perfectly general case. Each alteration of the number and size of the constituent spheres, and each variation of the positions of their centres, develops a new special case which is best examined separately. I* have already given the solution for a molecule consisting of any number of equal spheres with their centres equally spaced on a straight line, and have applied it with success to actual molecules whch probably have that form. The present extension consists of examining the new case where the spheres are of unequal radii, but still have their centres disposed rectilinearly, so that there is an axis of symmetry. This arrangement is necessarily the only possible one for all of the numerous diatomic molecules, such as, for example, HCl.

4. Let us consider first of all a molecule consisting of two hard spheres of radii σ_1 and $\sigma_2 (\sigma_1 > \sigma_2)$, the distance between their centres being $2d$, as indicated in Fig. 1. The evidence so far available from X-ray crystal measurements (which enable us to specify $O_1 O_2$) is that the centres of atoms in chemical compounds are so close together that the hard spheres to which the atoms correspond overlap to some

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* *Loc. cit.*

extent. In other words, $2d$ is never greater than $\sigma_1+\sigma_2$. For this reason the present calculation is limited to values of $2d$ between $\sigma_1+\sigma_2$ and $\sigma_1-\sigma_2$, *i.e.*, to the range between external and internal contact of the spheres.

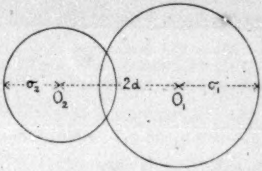

Fig. 1.

The axis $O_1\ O_2$ of the molecule may have all possible orientations distributed at random. Let Fig. 2 represent the parallel projection in a direction making an angle θ with the axis. O_1 and O_2 are projected into O_1' and O_2' respectively, and we have the relation $O_1'O_2'=O_1O_2 \sin\theta$, or,

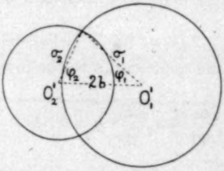

Fig. 2.

$$2b=2d\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the angles φ_1 and φ_2 are as indicated, we also have

$$\sigma_1\sin\varphi_1=\sigma_2\sin\varphi_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\sigma_1\cos\varphi_1+\sigma_2\cos\varphi_2=2b, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which, combined with (2), gives

$$\sigma_1\cos\varphi_1+\sigma_2\cos\varphi_2=2d\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The expression for A, the area of the projection, depends on whether the smaller sphere is completely eclipsed or not by the larger sphere. For values of θ from zero to θ_0, given by the relation

$$\sin\theta_0 = \frac{\sigma_1 - \sigma_2}{2d}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

A is equal to $\pi\sigma_1^2$ simply, because the small sphere is completely covered. Over the remaining range from θ_0 to $\frac{\pi}{2}$ a variable crescent appears in addition. The area of this crescent is

$$\pi\sigma_2^2 - \sigma_1^2(\varphi_1 - \cos\varphi_1 \sin\varphi_1) - \sigma_2^2(\varphi_2 - \cos\varphi_2 \sin\varphi_2).$$

Now, since the axis $O_1\,O_2$ is oriented at random, the angle θ occurs with a proportional frequency $\sin\theta \,.\, d\theta$, so that the mean area of projection for all possible orientations is given by

$$\bar{A} = \int_0^{\frac{\pi}{2}} \pi\sigma_1^2 \sin\theta \,.\, d\theta + \int_{\theta_0}^{\frac{\pi}{2}} \{\pi\sigma_2^2 - \sigma_1^2(\varphi_1 - \cos\varphi_1 \sin\varphi_1)$$
$$- \sigma_2^2(\varphi_2 - \cos\varphi_2 \sin\varphi_2)\} \sin\theta \,.\, d\theta. \quad . \quad . \qquad (7)$$

Partial integration reduces this to the form

$$\bar{A} = \pi\sigma_1^2 - \int_{\theta_0}^{\frac{\pi}{2}} \left(\sigma_1^2 \frac{d\varphi_1}{d\theta} + \sigma_2^2 \frac{d\varphi_2}{d\theta}\right) \cos\theta \,.\, d\theta + \int_{\theta_0}^{\frac{\pi}{2}} (\sigma_1^2 \cos\varphi_1 \sin\varphi_1$$
$$+ \sigma_2^2 \cos\varphi_2 \sin\varphi_2) \sin\theta \,.\, d\theta, \quad . \qquad (8)$$

which may be written, for the sake of brevity,

$$\bar{A} = \pi\sigma_1^2 - I + J. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

Elimination of φ_1 and φ_2 from I and J by means of equations (3) and (5) gives

$$J = \tfrac{1}{2}\int_{\theta_0}^{\frac{\pi}{2}} \{4\sigma_1^2\sigma_2^2 - (4d^2 \sin^2\theta - \sigma_1^2 - \sigma_2^2)^2\} \sin\theta \,.\, d\theta, \qquad (10)$$

and

$$I = \int_{\theta_0}^{\frac{\pi}{2}} \frac{\{\sigma_1^2 - \sigma_2^2)^2 - 4d^2(\sigma_1^2 + \sigma_2^2) \sin^2\theta\} \cos^2\theta \,.\, d\theta}{\{4\sigma_1^2\sigma_2^2 - (4d^2 \sin^2\theta - \sigma_1^2 - \sigma_2^2)^2\}^{\frac{1}{2}} \sin\theta}. \qquad (11)$$

The quantities I and J can be expressed in terms of complete elliptic integrals by means of the substitution

$$\cos\theta = \cos\theta_0 \cos\psi. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

With this transformation we obtain

$$J=2d\cos^2\theta_0\sqrt{\sigma_1\sigma_2}\int_0^{\frac{\pi}{2}}\left(1-\frac{d^2\cos^2\theta_0}{\sigma_1\sigma_2}\sin^2\psi\right)^{\frac{1}{2}}\sin^2\psi\,.\,d\psi. \quad (13)$$

Or, writing $k^2=\dfrac{d^2\cos^2\theta_0}{\sigma_1\sigma_2}$, which is less than unity over the range $\sigma_1+\sigma_2>2d>\sigma_1-\sigma_2$ contemplated,

$$J=\frac{2(\sigma_1\sigma_2)^{\frac{3}{2}}}{d}\int_0^{\frac{\pi}{2}}k^2\sin^2\psi(1-k^2\sin^2\psi)^{\frac{1}{2}}d\psi. \quad . \quad . \quad . \quad (14)$$

$$=\frac{2\sigma_1\sigma_2}{3q}\{(2k^2-1)E_1(k)+(1-k^2)F_1(k)\},$$

where q is written for $d/\sqrt{\sigma_1\sigma_2}$ and $F_1(k)$ and $E_1(k)$ are the complete elliptic integrals of argument k, and of the first and second orders respectively.

With the same substitution (12) the quantity I takes the form

$$I=\frac{(\sigma_1^2-\sigma_2^2)^2\cot^2\theta_0}{4d\sqrt{\sigma_1\sigma_2}}\int_0^{\frac{\pi}{2}}\frac{\cos^2\psi\,.\,d\psi}{(1+\cot^2\theta_0\sin^2\psi)\Delta(\psi)}$$

$$-\frac{(\sigma_1^2+\sigma_2^2)\sqrt{\sigma_1\sigma_2}}{d}\int_0^{\frac{\pi}{2}}\frac{k^2\cos^2\psi\,.\,d\psi}{\Delta(\psi)}, \quad (15)$$

where $\Delta(\psi)=(1-k^2\sin^2\psi)^{\frac{1}{2}}$, k having the same value as previously. If we write $n=\cot^2\theta_0$ and integrate, we obtain

$$I=\frac{(\sigma_1^2-\sigma_2^2)^2}{4q\sigma_1\sigma_2}\{(n+1)\Pi_1(n,k)-F_1(k)\}$$

$$-\frac{\sigma_1^2+\sigma_2^2}{q}\{E_1(k)-(1-k^2)F_1(k)\}, \quad (16)$$

where $\Pi_1(n,k)$ is the complete elliptic integral of the third order. The expression for $\bar{A}$ in (9) therefore becomes, using (14) and (16),

$$\bar{A}=\pi\sigma_1^2+\frac{\sigma_1^2+\sigma_2^2}{q}\{E_1(k)-(1-k^2)F_1(k)\}$$

$$+\frac{2\sigma_1\sigma_2}{3q}\{(2k^2-1)E_1(k)+(1-k^2)F_1(k)\}$$

$$-\frac{(\sigma_1^2-\sigma_2^2)^2}{4q\sigma_1\sigma_2}\{(n+1)\Pi_1(n,k)-F_1(k)\}. \quad (17)$$

Dividing both sides by $\pi\sigma_1^2$, and writing $p=\sigma_2/\sigma_1$, so that $p<1$,

$$\frac{\bar{A}}{\pi\sigma_1^2}=1+\frac{p}{\pi q}\Big[E_1(k)\Big\{\frac{1}{p}+p+\tfrac{2}{3}(2k^2-1)\Big\}$$
$$-F_1(k)\Big\{(1-k^2)\Big(\frac{1}{p}+p-\tfrac{2}{3}\Big)\Big\}\Big]$$
$$-\frac{(1-p^2)^2}{4\pi pq}\{(n+1)\pi_1(n,\, k)-F_1(k)\} \quad . \quad . \quad (18)$$

5. This expression enables us to evaluate the ratio of the mean collision area of the two spheres relative to that of the larger one only, for all values of σ_1, σ_2 and d within the limits specified. In the special case when the two spheres are equal, $p=1$, so that the last term vanishes. In that case, also, $q=d/\sqrt{\sigma_1\sigma_2}=d/\sigma$, and θ_0 now being zero, $k=d/\sigma$. For two equal spheres there thus emerges the equation

$$\frac{\bar{A}}{\pi\sigma^2}=1+\frac{1}{\pi k}\Big\{\frac{4}{3}E_1(k)(k^2+1)-\frac{4}{3}F_1(k)(1-k^2)\Big\}$$
$$=1+\frac{4}{3\pi}\{(1/k+k)E_1(k)-(1/k-k)F_1(k)\}$$
$$=1+X, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

which is the expression obtained previously and mentioned earlier.

It is worth noting that it provides the solution for any number of equal spheres which successively overlap one another, whether their centres are equally spaced or not. For clearly the excess over unity, *i.e.*, X, is the contribution to the mean area due to the crescent formed by the variable overlapping of the two spheres. If there are N equal spheres arranged on a straight line, there will be $N-1$ such crescents, and, in order to find $A/\pi\sigma^2$, we have merely to add extra terms of the same type as X. Thus, for N spheres

$$\bar{A}/\pi\sigma^2=1+(X_{1,2}+X_{2,3}+X_{3,4}+\ .\ .\ .\ +X_{(N-1)N}). \qquad (20)$$

In this expression $X_{3,4}$, for example, represents the contribution of the crescent formed by the third sphere overlapping the fourth, and in each such term the appropriate value of k must be inserted. Thus, in $X_{3,4}$, the value of k is $\frac{d_{3,4}}{\sigma}$, where $2d_{3,4}$ equals the distance between the centres of the third and fourth spheres. If all these distances are equal, *i.e.*, if the spheres are equally spaced,

$$\overline{A}/\pi\sigma^2=1+(N-1)X \quad . \quad . \quad . \quad . \quad . \quad (21)$$

for all the crescents have the same area for the same orientation.

6. This specialised aspect of the subject may, with little difficulty, be made quite general, so as to include cases in which $2d > 2\sigma$. The difference introduced is that the integration has to be subdivided into two stages in which the projections do, and do not, overlap ; otherwise the process is the same, and need not be repeated in detail. The result for two non-overlapping spheres of equal size proves to be

$$\begin{aligned}\frac{\overline{A}}{\pi\sigma^2}&=1+\frac{4}{3\pi}\left\{\left(\frac{1}{c^2}+1\right)E_1(c)-\left(\frac{1}{c^2}-1\right)F_1(c)\right\}\\&=1+Y. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)\end{aligned}$$

In this case $c=\frac{\sigma}{d}<1$, and it will be noted that the expression differs from (19). The limiting value is 2, as we should expect, when $c=0$, *i.e.*, when the diameters of the spheres are negligible compared with their distance apart, with the result that the effect of overlapping is inappreciable and each sphere makes a contribution $\pi\sigma^2$ independently to $\overline{A}$. When the spheres are just in contact, in which case $c=\frac{\sigma}{d}=1=\frac{d}{\sigma}=k$, (22) reduces to the same value as (19), viz., $1+\frac{8}{3\pi}$.

We may deduce the value of $\frac{\overline{A}}{\pi\sigma^2}$ for any number of non-overlapping equal spheres in precisely the same way as indicated in the last paragraph, and obtain

$$\frac{\overline{A}}{\pi\sigma^2}=1+(Y_{1,2}+Y_{2,3}+Y_{3,4}+\ .\ .\ .\ +Y_{(N-1)N}) \quad . \quad (23)$$

with the appropriate values of c introduced. Or we may have a mixture of overlapping and non-overlapping spheres, and therefore use in the summation an X or a Y expression according as d/σ for the pair of spheres in question is less or greater than unity.

7. Returning now to the general case of two unequal spheres, as expressed in equation (18), we have to transform the last

term for purposes of calculation. Tables for the third order complete elliptic integrals, $\Pi_1(n,k)$, are not available, but these functions can be expressed, as is well known, in terms of complete and incomplete integrals of the first and second orders. The required relation, given by Legendre, is

$$\frac{(1-b^2\sin^2\theta_0)^{\frac{1}{2}}}{\sin\theta_0\cos\theta_0}\Pi_1(n,k)=\frac{\pi}{2}+\frac{\sin\theta_0}{\cos\theta_0}(1-b^2\sin^2\theta_0)^{\frac{1}{2}}F_1(k)$$
$$-E_1(k)F(b,\theta_0)+F_1(k)\{F(b,\theta_0)-E(b,\theta_0)\}, \quad . \quad (24)$$

where $b^2=1-k^2$, and $F(b, \theta_0)$ and $E(b, \theta_0)$ are, respectively, first and second incomplete elliptic integrals of argument b and amplitude θ_0. The insertion of this relation in the last term of (18), together with application of the connections between q, p, n, and θ_0, gives the result

$$\frac{\overline{A}}{\pi\sigma_1^2}=1+\frac{p}{\pi q}\Big[\{1/p+p+\tfrac{2}{3}(2k^2-1)\}E_1(k)$$
$$-(1-k^2)(1/p+p-\tfrac{2}{3})F_1(k)\Big]$$
$$-\frac{1-p^2}{\pi}\Big[\frac{\pi}{2}+F_1(k)\{F(b,\theta_0)-E(b,\theta_0)\}-E_1(k)F(b,\theta_0)\Big]$$
$$. \quad . \quad . \quad . \quad (25)$$

Thus the expression is obtained in terms of functions for which tables are available. We may write (25) as

$$\frac{\overline{A}}{\pi\sigma_1^2}-1=Z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where Z represents the contribution to the mean area—in terms of $\pi\sigma_1^2$—of the variable crescent (Fig. 2) formed by the overlapping of sphere 1 over sphere 2. I have calculated Z for seven values of the ratio $p=\frac{\sigma_2}{\sigma_1}$, ranging from unity to 0·4. This lower limit has been adopted because there is no evidence for the existence of relatively great differences of dimensions of molecules commonly in the gaseous state. The greatest and smallest atoms for which reliable estimates have been made are xenon and helium respectively. For these atoms the ratio $\frac{\sigma_2}{\sigma_1}$ is $\frac{0{\cdot}96}{1{\cdot}76}=0{\cdot}55$. Possibly the hydrogen atom is somewhat smaller than that of helium, but not to such an extent that it would be outside the limits above mentioned. The range of calculation is in each case for values of q, *i.e.*, $\frac{d}{\sqrt{\sigma_1\sigma_2}}$, lying within limits determined by $\sigma_1+\sigma_2>2d>\sigma_1-\sigma_2$, or extending from external to internal contact of the spheres. The results are shown graphically in Fig. 3. From these curves, if necessary by interpolation, the values of Z can be obtained which correspond to specified values of d, σ_1 and σ_2. Or, conversely, we can derive q from given values of Z, and hence determine d if σ_1 and σ_2 are known.

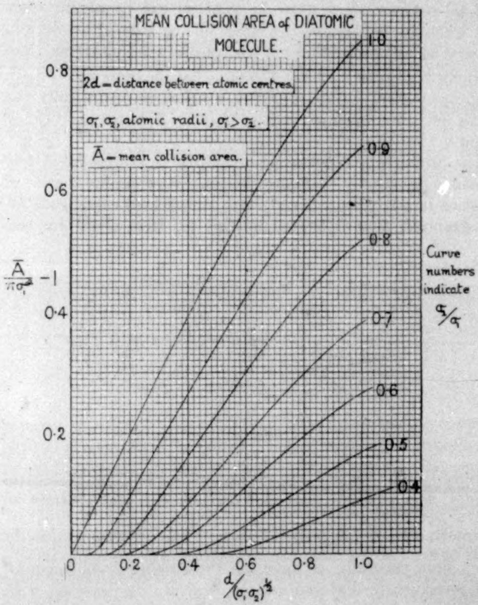

FIG. 3

8. With reference to these curves, there are several points of interest. For the particular case when $p=1$, equation (19) applies, and $q=d/\sigma=k$. The curve could be extended for

values of k greater than unity by means of formula (22), and $\frac{\bar{A}}{\pi\sigma^2}-1$ would reach unit value asymptotically at $k=\infty$. For the other curves the corresponding extension has not been worked out—*i.e.*, we have no values for $q>\frac{\sigma_1+\sigma_2}{2\sqrt{\sigma_1\sigma_2}}$. But the general courses which must be followed can be readily seen. For when q is very great, the effect of overlapping arises from a very small proportion of the possible orientations, and becomes negligible. Thus, $\bar{A}$ approaches the limit $\pi\sigma_1^2+\pi\sigma_2^2$, or $Z=p^2$. Each curve, therefore, reaches asymptotically the value p^2; for example, when $p=0{\cdot}5$, Z becomes $0{\cdot}25$ at $q=\infty$.

The use of this diagram is not limited to the model of two overlapping spheres. It also provides very simple means of solution in a large number of more complicated cases, such as, for example, that indicated in Fig. 4. Here there are two exactly similar bulges, one on each side of the large central sphere, which make equal and independent contributions to the mean area of projection. Thus $\frac{\bar{A}}{\pi\sigma_1^2}-1$ becomes equal to $2Z$. More generally, if there are three spheres, all unequal, but rectilinearly disposed and successively overlapping, with the largest sphere occupying the middle position, we have

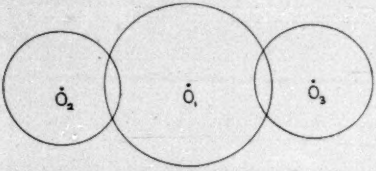

FIG. 4.

$$\frac{\bar{A}}{\pi\sigma_1^2}-1=Z_{1,2}+Z_{1,3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

where $Z_{1,2}$ and $Z_{1,3}$ are the mean areas of the crescents formed by the overlapping of the largest sphere 1 over the outer smaller spheres 2 and 3 respectively. In $Z_{1,2}$ and $Z_{1,3}$, of course, the proper values of q and p must be used.

There are other examples of extensions of this simple nature.

The simplicity appears to depend on two conditions—viz., (i.) the straight row of spheres must continually (but not necessarily uniformly) decrease in size as we pass from the largest one, either in a single direction or in two opposite directions, although, in the latter case, the decrements need not be equal on the two sides ; (ii.) the spacings of the centres must be such that the successive total eclipses, which occur as the model is rotated towards the line of view, take place in the order of the sizes of the spheres, beginning with the smallest—*i.e.*, the outermost. With these conditions we can express the mean area of projection of the model, however many spheres it contains, by means of a series of independent Z's, each of which is obtainable from the curves in Fig. 3.

9. A typical case in which these rules are departed from is shown in Fig. 5. Here we have a small sphere interposed between two larger ones, and at a certain orientation its projection is completely covered by the two outer projections taken together, and not by each independently. The result is that Fig. 3 is no longer applicable. For it becomes necessary in evaluating $\overline{A}$ to separate the integration into two stages which are not the same as those previously used, and the elliptic integrals obtained are not complete, as they are in (18). The formula is much more unwieldy, and its reduction to arithmetic much more laborious. It is not proposed, therefore, to deal here with cases of this type, except to say that the solution is derivable by the methods indicated, and that it can be applied numerically in special molecules suspected to have such shapes, and for which the necessary data become available.

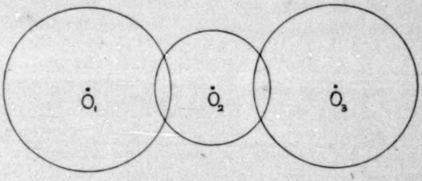

FIG. 5.

10. It remains to consider for what actual molecules there is a reasonable prospect of obtaining the viscosity data for testing the applicability of the foregoing theory. Measure-

ments of viscosities at high temperatures are of great difficulty, and it is necessary to confine attention, for the moment at any rate, to chemical compounds which are ordinarily gaseous, or at least fairly volatile. It immediately appears that nearly all such molecules which satisfy the conditions—*i.e.*, have unequal atoms—have hydrogen as a constituent.

We have, in the first place, the group of hydrogen halides HF, HCl, HBr and HI, all of which are gaseous at ordinary temperatures. Following Lewis and Langmuir, these may be regarded as systems in which the single electron of the hydrogen atom has completed the corresponding inert gas arrangement of outer electrons around the halogen nucleus, the hydrogen nucleus occupying an external position. Thus, for example, HCl may be pictured as having the shape of the argon atom with the hydrogen nucleus as a protuberance. The work of W. L. Bragg and H. Bell* has recently provided information regarding the distance between the nuclei of the hydrogen and the combined halogen, it being obtained simply by adding $0{\cdot}26 \times 10^{-8}$ cm. to the radius of the outer electron shell of the corresponding inert atom. But we do not yet know the correct way of regarding a single hydrogen atom from the point of view of molecular collision, since monatomic hydrogen is not available for viscosity measurements. If we may proceed upon the tentative assumption that it is spherical, a knowledge of the mean collision areas of the hydrogen halides deduced from viscosity data, in conjunction with the foregoing theory, will enable its magnitude to be estimated. That is to say, treating the halogen atom as a sphere having the collision radius (σ_1) of the corresponding inert atom, and the hydrogen atom as an overlapping sphere of radius (σ_2), $\bar{A}$ determines the value of σ_2 (Fig. 3). If this radius σ_2 proves to be the same for all the hydrogen halides, it will indicate that the hydrogen protuberance behaves in molecular collissions as though it were spherical. The proportion of the mean collision area of a molecule attributable to a constituent hydrogen atom will thus be available for application to other cases.

There is also the group H_2O, H_2S and, possibly, H_2Se, for which the necessary viscosity data should be obtainable. The water molecule is conceived by Lewis and Langmuir as a neon arrangement with two hydrogen nuclei, presumably on opposite sides. In the H_2S molecule the arrangement would be the same, except that argon replaces neon, and in H_2Se the central configuration would be that of the krypton atom. All these are of the type indicated in Fig. 4, and, if we know the collision size of the hydrogen atom, Fig. 3 gives the theoretical mean collision area of the molecule in the manner already described. We should expect, for example, that the mean collision area of H_2S would be greater than that of HCl by the same amount as the mean collision area of HCl itself is greater than that of argon.

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* W. L. Bragg and H. Bell. Nature, March 24, 1921.

Yet another interesting group deserves consideration—namely, NH_3, PH_3, AsH_3 and SbH_3. Presumably these have the external electron arrangement of Ne, A, Kr, and Xe respectively, with three hydrogen protuberances in each case. We may therefore regard each as a large central sphere with three smaller overlapping spheres attached, or rather, as behaving as such in molecular collision. This model is not strictly included in the above theoretical treatment, but it is clearly possible for the hydrogen spheres to be relatively so small and so disposed round the central one that they never can eclipse one another, except when the central sphere is also doing so. In that state of affairs—possibly reached in SbH_3, where the central sphere has the large size of xenon—the contributions to the mean collision area by the hydrogen protuberances would be independent of one another. We should thus expect that the mean collision area of SbH_3 would be in excess of that of xenon by three times the excess of HI over xenon. For earlier members of the group, such as NH_3, this condition would probably not hold, and, owing to the effect of additional overlapping, the mean collision area should prove to be somewhat diminished. Until the effective collision radius of a hydrogen atom is known, one cannot estimate to what extent this would be the case.

There are, finally, the gaseous compounds of carbon and hydrogen. Nearly all of these probably have molecular shapes which are much more complicated than those contemplated in this Paper. In order to deal with them, further investigations of a theoretical nature will be necessary, and the analytical treatment is likely to prove very unwieldy, and perhaps even insoluble, owing to the removal of the simplification arising from the existence of an axis of symmetry. It seems probable that graphical methods will provide the most promising mode of attack.

Suitable molecules containing no hydrogen are very scarce. Sulphur dioxide and carbon bisulphide appear to be the only likely ones. The former is supposed to have the shape of a central argon atom with two neon atoms attached ; the latter corresponds to a central neon atom and two outer argon atoms arranged as in Fig. 5. The validity of both these postulated models can be tested when the appropriate viscosity data become available.

11. In conclusion, I wish to express my thanks to Dr. H. Levy, of this College, for valuable hints regarding some of the mathematical aspects of this work.

DISCUSSION.

The author added that since the Paper was communicated, very satisfactory progress has been made in the Imperial College Physics Laboratory with the measurements of the viscosities of several gases to which the theory in the Paper would apply, including the group HCl, H_2S and PH_3. The derived molecular dimensions pointed to the probability that the hydrogen atoms, instead being at equal distances from the central nucleus in all these compounds, became more and more remote as their number increased.

The President, Sir W. H. Bragg, said he thought the idea that the hydrogen atoms were more widely separated in the more complex molecules was involved in a recent Paper by Sir J. J. Thomson. It may be possible to get at this by X-ray spectroscopic examination of some organic compounds. The molecules of some of these compounds must be very empty ; in naphthalene for example, the hydrogen atoms probably act as bridges between the carbon atoms, so that it may be possible to get at the size of the hydrogen atom by X-ray methods.

XXXI. *An Electroculture Problem.* *By* C. CHREE, *Sc.D., LL.D., F.R.S.*

RECEIVED JUNE 2, 1921.

ABSTRACT.

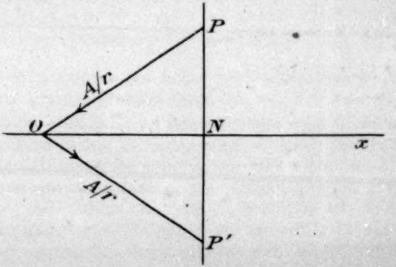
The problem is that presented by a number of fine parallel wires at the same height above the ground—treated as a plane surface—all charged to the same potential.

The chief object is to investigate how the potential gradient at ground level depends on the height and spacing of the wires. A second object is to find how the potential gradient varies with the height above the ground. The cases of a finite and an infinite number of wires are considered.

§1. THE problem considered is two-dimensional, being that presented by fine parallel wires at equal intervals $2a$ apart, all at the same height h above the ground, and at the same electrical potential.

The electrostatic potential due to a single infinitely long wire in an infinite medium is $A \log_e r$, where r is the distance from the wire, and A is a constant, proportional to the electrical charge per unit length of the wire.

In practice the earth must be treated as a conductor at zero potential. The figure is intended to represent a vertical plane perpendicular to the length of the wire, P being the section of the wire, and Ox in ground level. To maintain zero potential along Ox, we require a distribution of electricity on the ground, equivalent so far as points above ground level are concerned to a second charged wire parallel to the first, and at an equal distance from Ox. The charge per unit length of this image wire is numerically equal but of opposite sign to that in the real wire. In the figure P' represents the



section of the image wire, PP' being perpendicular to Ox and bisected at N. The electrical force at O is the resultant of A/r along PO, and A/r along OP'. The resultant is thus vertical and equals $2Ah/r^2$, or $2Ah/(x^2+h^2)$, where $x=ON$.

In an electroculture area there are a large number of parallel wires, each contributing to the vertical force at ground level. Let us suppose that the charge on any one of the more remote wires instead of being concentrated on the wire is uniformly distributed over a horizontal width a on either side of the wire. If this hypothesis is made for each of the more distant wires, we have a surface charge of uniform density at height h, extending from x_0-a to x'_0+a, where x_0 and x_0' are the horizontal distances from O of the nearest and the furthest of the wires to which the hypothesis extends. The vertical force at O contributed by these more remote wires is then, according to the hypothesis, replaceable by the integral

$$\int_{x_0-a}^{x_0'+a} \frac{2Ah}{2a} \cdot \frac{dx}{h^2+x^2} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

The value is thus $(A/a)\left(\tan^{-1}\frac{x_0'+a}{h} - \tan^{-1}\frac{x_0-a}{h}\right)$, or

$$(A/a)\left(\tan^{-1}\frac{h}{x_0-a} - \tan^{-1}\frac{h}{x_0'+a}\right) \quad . \qquad (2)$$

The expression inside the bracket obviously represents the angle subtended at O by the hypothetical distribution.

§2. Comprehension may be aided by discussing a particular case. Suppose, then, we have in all 21 wires, and wish to find the force under the central wire at ground level. The origin being supposed under the central wire, the horizontal distances of the 10 wires on one side are $2a$, $4a$ . . . $20a$. Each of the 21 wires has an image. The resultant vertical force at the origin due to the whole 42 wires, real and imaginary, is easily seen to be

$$\frac{2A}{h}\left[1+2\left\{\frac{1}{1+(2a/h)^2}+\frac{1}{1+(4a/n)^2} \quad . \quad . \quad + \frac{1}{1+(20a/h)^2}\right\}\right].$$

It is easy evaluating this for any convenient value of h/a. If, for instance, $h=2a$, we have only to sum

$$\frac{}{a}\left[1+2\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}+\frac{1}{26}+\frac{1}{37}+\frac{1}{50}+\frac{1}{65}+\frac{1}{82}+\frac{1}{101}\right)\right].$$

To four decimal places, this is $(A/a)\times 2{\cdot}9636$.

The contribution from the 10 most remote wires—five on either side—viz.,

$$\frac{2A}{a}\left(\frac{1}{37}+\frac{1}{50}+\frac{1}{65}+\frac{1}{82}+\frac{1}{101}\right) \text{ equals } \frac{A}{a}\times 0{\cdot}1690.$$

Suppose, however, that instead of regarding the charge as concentrated on these outside wires, we distribute it uniformly. It will then cover a patch on either side extending from $x=11a$ to $x=21a$. Taking the contributions from both patches, from the integrals we get $2(A/a)[\tan^{-1}(21a/h)-\tan^{-1}(11a/h)]$ or $2(A/a)[\tan^{-1}10{\cdot}5-\tan^{-1}5{\cdot}5]$. The angles, of course, are supposed to be in circular measure. But we have only to multiply by 0·017453 the angles as expressed in degrees in an ordinary book of logarithms, where we get

$$\tan^{-1}10{\cdot}5=84^\circ 33'{\cdot}58\equiv 84^\circ{\cdot}560,$$

$$\tan^{-1}5{\cdot}5=79^\circ 41'{\cdot}71\equiv 79\ {\cdot}695.$$

Thus the contribution from the integral is

$$2(A/a)4{\cdot}865\times 0{\cdot}017453, \text{ or } (A/a)\times 0{\cdot}1698.$$

It is, of course, more accurate and quite as easy in this case to obtain the contribution from the 10 outside wires direct from the series as has been done above. But our immediate object is to illustrate the integration method, and show its accuracy.

The ultimate object of the integration method was really to bring the case of an infinite number of wires readily within our reach, with an accuracy sufficient for practical purposes. Suppose, then, we have obtained a result for 21 wires, and wish to find the addition to be made to this when the number becomes infinite. Taking the integral with the superior and inferior values of x as ∞ and $21a$ respectively, we have for the sum of the contributions from both sides

$$2(A/a)\{\tan^{-1}\infty-\tan^{-1}11{\cdot}5\}=(A/a)\times 0{\cdot}1899.$$

The use of the integral increases in accuracy as the inferior limit of x increases. When the inferior limit was $11a$ the error, as we have seen, was less than $\frac{1}{2}$ per cent. With an inferior limit of $21a$ the accuracy will be much better, and we may feel assured that while the result $(A/a)\times 0{\cdot}1899$ obtained above may not be accurate in the fourth decimal place, it is exact enough for all ordinary practical purposes. Adding it to the result $(A/a)(2{\cdot}9636)$ obtained above for the 21 wires, we obtain for the force at ground level directly under a wire of an infinitely extended electroculture area, having $h=2a$, the value $(A/a)(3{\cdot}1535)$. It is hoped that this will sufficiently explain the analytical methods employed in calculating the force at ground level. The only difference in the procedure when the force is calculated for a point midway between two wires is that the estimate is made in the first instance for the nearest 20 wires. The contribution from the more remote wires on either side when their number is infinite is derived from the result of integration, viz. $(2A/a)(\tan^{-1}\infty-\tan^{-1}20a/h)$.

§3. The vertical force could easily have been calculated at a number of different heights above ground level, but it appeared sufficient to calculate it at the half-height of the wires, and to obtain in addition its mean value between this height and ground level. The vertical force at height $h/2$ due to one wire at horizontal distance x and its image wire is

$$A\left\{\frac{h/2}{x^2+(h/2)^2}+\frac{3h/2}{x^2+(3h/2)^2}\right\} \quad . \; . \; . \; . \; . \; . \quad (3)$$

Putting $x=o,\ 2a\ .\ .\ .$ or else $x=a,\ 3a\ .\ .\ .$ we get results applicable to a point directly under a wire or midway between two wires.

When the point is immediately under a wire the correction to allow for the difference between 21 wires and an infinite number is—

$$(A/a)(0{\cdot}017453)\{180-\tan^{-1}(42a/h)-\tan^{-1}(14a/h)\},$$

the angles being expressed in degrees.

For a point midway between two wires the allowance for the difference between 20 wires and an infinite number is similarly—

$$(A/a)(0{\cdot}017453)\{180-\tan^{-1}(40a/h)-\tan^{-1}(13{\cdot}\dot{3}a/h)\}.$$

§ 4. If V_0 denote the potential at height z, the vertical force, numerically considered, is $\frac{dV}{dz}$. Thus the mean value of the force between ground level and the height $h/2$ is—

$$V_{h/2}\div(h/2),$$

where $V_{h/2}$ represents the potential—which varies of course with x—at the height $h/2$.

The potential at a height $h/2$ in the vertical through the origin, due to a single wire at a horizontal distance x and its image combined, is—

$$(A/2)\log_e\frac{x^2+9h^2/4}{x^2+h^2/4}. \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

Thus, for the potential at height $h/2$ directly under the central wire of a series of 21 we have, when $h=2a$—

$$V_{h/2}=\tfrac{1}{2}A\log_e 10\left[\log 9+2\left\{\log\frac{13}{5}\times\frac{25}{17}\times\frac{45}{37}\times\frac{73}{65}\times\frac{109}{101}\times\frac{153}{145}\times\frac{205}{197}\times\frac{265}{257}\times\frac{333}{325}\times\frac{409}{401}\right\}\right]. \quad (5)$$

All the logarithms inside the square bracket are to base 10. The separate factors $13/5$, $25/17$, &c., represent the contributions from the successive 10 wires (and their images) on each side of the central wire.

An allowance is easily made for the more remote wires. The contribution to $V_{h/2}$ from a single wire and its image as given above in (4) may be written

$$(A/2)\{\log_e(1+9h^2/4x^2)-\log_e(1+h^2/4x^2)\} \quad . \quad . \quad (6)$$

Expanding in powers of h/x, and retaining the two first terms, we get—

$$A\left(\frac{h^2}{x^2}-\frac{5}{4}\frac{h^4}{x^4}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Now, suppose, as before, the charge spread over a width $2a$ for each of the more remote wires, then for the contribution to the potential from the remote wires on one side of the origin we get from (7)—

$$\int_{x_0-a}^{x_0'+a}(A/2a)\left(\frac{h^2}{x^2}-\frac{5}{4}\frac{h^4}{x^4}\right)dx, \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where x_0 and x_0' have the same significance as in (2).

Doubling the result of the integration, viz.—

$$\left[(A/2a)\left(\frac{-h^2}{x}+\frac{5}{12}\frac{h^4}{x^3}\right)\right]$$

taken between the limits, we get the contribution to $V_{h/2}$ from the more remote wires on both sides of the origin, and dividing the value of $V_{h/2}$ so found by $h/2$, we get the mean value of the vertical force between ground level and the height $h/2$.

As an example, suppose we have found the mean force due to the nearest 20 wires up to height $h/2$, midway between two wires, and wish to find the addition to be made when the number of wires becomes infinite. The limiting values of x in the integral are $20a$ and ∞, so its value, when doubled and divided by $h/2$ is simply

$$\frac{2A}{a}\left\{\frac{h}{20a}-\frac{5}{12}\left(\frac{h}{20a}\right)^3\right\}\equiv\frac{Ah}{10a^2}\left\{1-\frac{1}{960}\left(\frac{h}{a}\right)^2\right\}. \quad . \quad (9)$$

Unless h/a is large, the first term gives quite a good approximation. When $h=2a$ we have $(A/a)0{\cdot}2(1-1/240)=0{\cdot}1992A/a$.

§ 5. The original investigation had two objects in view :—

1. To ascertain whether an experimental station with only a few wires could supply information likely to be of practical service.

2. To ascertain the spacing of wires appropriate to electroculture areas.

Obviously if the wires are far apart, and near the ground, the potential gradient at ground level will be much higher at points under a wire than at points midway between wires. If a very high potential gradient were injurious, and a moderate gradient beneficial, we might have improvement of one part of the crop neutralised by injury to another. It should at least facilitate drawing conclusions, as to the merits of electroculture, if an approach to uniformity of conditions is secured throughout the area.

In the original calculations a number of values of h/a were considered, but here results will be given mainly for three values only, viz., 1, 2 and 4, as these sufficiently illustrate the main features.

Table I. contrasts the exact results obtained from the series or the logarithms with the approximate values obtained by integration for the contribution to the force arising from the most remote 10 of a series of 21 or of 20 wires. It is confined to the values of the force at ground level, and to the mean values of the force between ground level and the half-height of the wires.

TABLE I.—*Force due to the* 10 *most remote wires* (*Coefficient of* A/a).

	Force under central wire when 21 wires.				Force midway between central wires when 20 wires.			
	At "ground level."		Mean up to $h/2$		At "ground level."		Mean up to $h/2$.	
h/a	True.	By Integral.	True.	By Integral.	True.	By Integral.	True.	By Integral.
1	0·0857	0·0861	0·0857	0·0860	0·0989	0·0991	0·0988	0·0993
2	0·1690	0·1698	0·1682	0·1689	0·1944	0·1954	0·1933	0·1942
4	0·3198	0·3211	0·3148	0·3120	0·3646	0·3662	0·3581	0·3533

The difference between the exact and approximate values for a given value of a increases, as was to be expected, with the value of h. But even for $h/a=4$ this difference is a very insignificant fraction of the force due to the nearest 21 or 20 wires. For higher values of h/a it would be an improvement to extend the exact calculation to a larger number of wires than 21 or 20, especially when considering the force or potential at heights above ground level.

The principal object of calculating Table I. was to show the degree of accuracy attainable by the approximate methods of integration.

Table II. gives the results obtained, when the number of wires is infinite, for the force arising from all more remote than the nearest 21 or 20. These results being obtained by the integration methods, are all approximate. The accuracy to be expected is considerably higher, however, than for the corresponding integration results in Table I. For a given value of a the accuracy will naturally diminish as h increases. Also the numerical value of the contribution from the more remote wires varies approximately as h when a is constant. It is roughly independent of the height above ground level, so long as the half-height of the wire is not exceeded.

TABLE II.—*Contribution to force from wires outside the central* 21 *or* 20, *when number is infinite* (*Coefficient of* A/a).

h/a.	Under a wire. Additional to effect of 21			Midway between wires. Additional to effect of 20		
	1	2	4	1	2	4
At "ground level" ...	0·0952	0·1899	0·3764	0·0999	0·1994	0·3948
Mean between "ground level" and $h/2$ ...	0·0951	0·1898	0·3752	0·0999	0·1992	0·3933
At $h/2$...	0·0951	0·1895	0·3732	0·0998	0·1988	0·3911

It will be observed that each numerical result in Table II. is in excess of the corresponding result in Table I., but the excess is not large.

§ 6. Table III. summarises the results obtained for the vertical force at points exactly under a wire, or exactly midway between two wires. It is convenient to express the force in terms of A/a, but it should be remembered that for a given value of h/a an increase in h means a corresponding increase in a. Increasing the height, keeping the span-height ratio constant, always entails a fall of force.

It has also to be remembered that by "ground level" in the table is really meant the surface of zero potential. It is thus to be interpreted as the top of the crop, not the actual surface of the ground in an electroculture area. If, for example, the wires over a field of wheat are 8 ft. apart and 8 ft. above the ground, a is 4 feet throughout the season; but h diminishes from 8 ft. at the start to 4 ft. at the finish, if the wheat grows to 4 ft. high. Under these circumstances, we should have h/a varying from 2 to 1, and if we had only a single wire, and maintained its potential constant, the vertical force at crop level would double in the course of the season.

TABLE III.—*Vertical force due to overhead wires* (*Coefficient of* A/a).

—		Directly under a wire (maximum).				Midway between wires (minimum).				Mean of max. and min.
Number of wires		1	5	21	∞	2	4	20	∞	∞
Value of h/a	Level of point.									
1	At "ground level"	2·000	3·035	3·330	3·425	2·000	2·400	2·781	2·881	3·153
	Mean up to $h/2$ ...	2·197	3·201	3·495	3·590	1·911	2·303	2·682	2·782	3·186
	At $h/2$	2·667	3·604	3·896	3·991	1·723	2·098	2·473	2·573	3·282
2	At "ground level"	1·000	2·400	2·964	3·154	1·600	2·215	2·931	3·130	3·142
	Mean up to $h/2$ ...	1·099	2·440	2·996	3·186	1·609	2·197	2·900	3·100	3·143
	At $h/2$	1·333	2·553	3·093	3·282	1·600	2·133	2·813	3·011	3·147
4	At "ground level"	0·500	1·800	2·765	3·142	0·941	1·581	2·747	3·142	3·142
	Mean up to $h/2$ ...	0·549	1·832	2·767	3·143	1·001	1·622	2·747	3·141	3·142
	At $h/2$	0·667	1·897	2·773	3·147	1·124	1·699	2·745	3·136	3·141

If the effect, whether beneficial or otherwise, depends only on the potential gradient at crop level, the "ground level" values in Table III. alone matter. If h/a is 2 or larger—*i.e.*, if the distance apart of the wires is not greater than their height—the difference, when the number of wires is infinite, between the force directly under a wire (*i.e.*, the maximum) and the force midway between two wires (*i.e.*, the minimum) is less than 1 per cent. of their mean. For practical purposes, the force may then be regarded as everywhere the same and equal to $\pi A/a$.

Even with h/a as small as 1—*i.e.*, with the height only half the distance apart of the wires—when the number of wires is infinite, the least value of the force at "ground level" is 0·81 of the largest value. With further reduction in h/a the ratio of the minimum to the maximum value falls rapidly.

For values $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ of h/a, the respective values of the ratio are only 0·68, 0·43 and 0·18 approximately.

§ 7. An infinite number of wires represents, of course, an ideal state, from which a considerable fraction of any real electroculture area must differ sensibly. As an example, suppose we travel outwards in the direction perpendicular to the wires from the centre of an electroculture area in which $h=2a$. Even at first, the value of the force at ground level is not everywhere absolutely the same, but it varies only from 3·15 A/a under a wire to 3·13 A/a midway between adjacent wires. As we approach the margin, the difference between the force under a wire and the force midway between adjacent wires increases, and, at the same time, both values fall off. When we are directly under the last wire the force has fallen to about 2·1 A/a. When we pass outside the area, travelling in the same direction, the force continually diminishes, but it remains finite for quite a considerable distance. It is still about 0·1A/a when we have got to a distance of $20a$, or $10h$, from the margin.

At a considerable distance outside the area, the force for a given spacing and voltage of the wires varies approximately as the apparent height of the nearest wire. Thus, the distance to which an appreciable potential gradient effect extends increases as the height of the overhead system.

Here we have strictly confined ourselves to the electrostatic problem presented when all the charge is on the wires, and there is no appreciable excess of ions of one sign in the atmosphere. When there is appreciable free charge, with an excess of ions of one sign, the potential gradient is affected. When the overhead voltage is high enough, it must inevitably produce copious ionisation near the wires, and a strong wind will spread the ionisation to quite a long distance to leeward of the area. These and other practical considerations must be borne in mind by anyone wishing to apply the present calculations. Perhaps the most immediately useful deduction from them is that, apart from wind effects, a very uniform set of conditions can be secured at crop level if the distance between adjacent wires does not exceed the height of the wires above the crop. Also, under these conditions, the potential gradient is nearly uniform from the crop level up to at least the half height of the wires above the crop. This last point was really investigated with an eye to certain experimental investigations.

DISCUSSION.

Mr. E. V. **Appleton** (communicated): I presume that Dr. Chree is aware that Maxwell ("Electricity and Magnetism," Vol. I.) has dealt with the electrical theory of a charged grating of parallel wires. In Maxwell's treatment, however, the assumption is made that the distance between the wires and the conducting plane (the ground in this case) is large compared with the distance between successive wires, and in such a case it is a simple matter to deduce from Maxwell's equations that the potential gradient at the ground level is equal to $\dfrac{V}{h+\dfrac{a}{\pi}\log_e \dfrac{a}{\pi c}}$, where V is the maintained potential difference between the wires and the ground and c is the radius of the cylindrical wires. But Dr. Chree's treatment of the problem gives us information about the cases in which the distance between the wires and the ground is comparable with the distance between successive wires.

Workers on the physics of the triode vacuum tube meet the same type of problem in dealing with the action of a charged grating in producing a potential gradient near the cathode of such a tube. It has usually been assumed that Maxwell's equations give a fair approximation to the truth in the practical case when the distance between the cathode and grating is about equal to the distance between the grating wires. Dr. Chree shows us how far such an assumption is justifiable.

Mr. F. J. W. **Whipple** (partially communicated): I find that I was mistaken in thinking that the problem of the grating of parallel wires is one of those discussed in Sir J. J. Thomson's "Recent Researches in Electricity and Magnetism." The problem is dealt with by Maxwell himself ("Electricity and Magnetism," Vol. I., § 233). Maxwell's discussion is only directly applicable to the case in which the distance between consecutive wires is small compared with their distance from the conducting plane. The method of conjugate functions can be used effectively in working out the more general problem and leads to the following results:

If $\frac{1}{2}A$ be the line density of the charge on each of a very large number of thin parallel wires at height h above the ground, and if $2a$ be the distance between adjacent wires, then the vertical force, F, at a point on the ground at a horizontal distance, x, from one of the wires is given by the formula

$$F=\frac{\pi A}{a}\cdot\frac{\sinh\dfrac{\pi h}{a}}{\cosh\dfrac{\pi h}{a}-\cos\dfrac{\pi x}{a}}.$$

The force varies between the values

$$\frac{\pi A}{a}\coth\frac{\pi h}{2a}\quad\text{and}\quad\frac{\pi A}{a}\tanh\frac{\pi h}{2a}.$$

It will be seen that for the variation from the mean not to exceed 10 per cent. on either side $\coth\dfrac{\pi h}{2a}$ must not be greater than 1·1, and therefore h must not be less than 0·97a. The condition is satisfied if the height of the wires is at least half the distance between them.

It is easy to verify that the values given by Dr. Chree in Table III. for the vertical force at ground level with an infinite number of wires are in accordance with my formula.

The formula for V, the potential difference between the wires and the ground, involves r, the radius of the cross section of a wire; it is

$$V=A\,.\log\left[\frac{2a}{\pi r}\sinh\frac{\pi h}{a}\right].$$

For comparison with the case of a single charged wire this formula may be written

$$V=A\log\frac{2h}{r}+A\log\frac{\sin\pi h/a}{\pi h/a},$$

and for comparison with the case when a uniformly charged conductor is substituted for the wires it takes the form

$$V=\frac{\pi Ah}{a}+A\log\frac{a}{\pi r}+A\log\left(1-e^{-2\pi h/a}\right).$$

Dr. CHREE communicated the following reply: The experimental electro-culture area for which I first made calculations had only five wires. In any actual case only an approach is made to an infinite number. Thus a method was essential which applied to a finite number of wires. In the course of the investigations I hit on a simple method, with an obvious geometrical interpretation, which enabled me to take account of remote wires, and so to include the infinite number as a limit. I thought an exact mathematical solution for an infinite number probably existed somewhere. but having satisfied all my immediate wants I did not pursue the enquiry. Thirty years ago I read the proofs of the last edition of "Maxwell's Electricity" (and also of Sir J. J. Thomson's "Recent Researches"), but I had forgotten the existence of the solution* to which Mr. Appleton refers. The solution for an infinite number of wires should be specially useful for obtaining the exact variation of force at ground level with distance from the nearest wire in cases where a/h considerably exceeds 1. In applications of the mathematical formulæ, especially those involving the radius of the wire, and particularly to points near the wire, it should be remembered that with the usual fine wires there is a considerable sag, and there may be a corona.

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* 203 *et seq.*, and Fig. XIII.

Demonstration of Some Optical and Acoustical Experiments.

By PROF. C. V. RAMAN, *University of Calcutta.*

SHOWN AT THE MEETING OF JUNE 24TH, 1921.

1. The ballistic sonometer, an instrument which indicates the intensity of a noise by the extent of the movement of a spot of light.

2. Vibrations of stringed instruments. By means of a small optical lever in contact with the bridge of a 'cello and a revolving mirror the vibrations of the bridge under various conditions were shown on the screen. The effects of meeting and of playing the " wolf " note were demonstrated.

3. The modes of vibration of a Trevelyan rocker were exhibited in the same way.

4. Indian drums, 10 centuries old, in which the membrane is loaded in such a way as to produce harmonic overtones were exhibited, and their modes of vibration when struck in different ways exhibited by means of sand figures.

5. The polygonal form of an oscillating liquid drop was projected on the screen.

6. A robust ripple apparatus was described, and photographs shown illustrating the change of phase of waves on passing through a focus.

7. Various interference and diffraction effects shown when light is transmitted by heterogeneous media were exhibited.

8. A spectrum traversed by Talbot's bands was shown, to illustrate the sudden change of colour between the bands.

The Stability of Atoms. *By* SIR ERNEST RUTHERFORD, F.R.S. (*Abridged report of a Lecture delivered before the Society on June* 10, 1921.)

DURING the latter half of the nineteenth century it was generally accepted that the atoms of the chemist and physicist were permanent and indestructible, and were uninfluenced by the most drastic physical and chemical agencies available. The existence of elements in the earth that appeared to have suffered no certain change within periods of time measured by the geological epochs gave a strong support to the prevailing view of the inherent stability of the elements. The discovery at the beginning of the twentieth century that the radio-active elements uranium and thorium were undergoing a veritable transformation, spontaneous and quite uncontrollable by the agencies at our disposal, was the first serious shock to our belief in the permanency of the elements. The essential phenomena which accompanied the series of transformations soon became clear. The disintegration of an atom was accompanied either by the emission of a swift atom of helium carrying a positive charge, or of a swift electron. With the exception possibly of potassium and rubidium, only the heavy radio-active elements showed this lack of stability. The great majority of the chemical elements appeared, as before, to be inherently stable and to be unaffected by the most intense forces at our disposal.

A number of attempts have been made from time to time to test whether the atoms of the elements can be broken up by artificial methods. Some thought that they had obtained evidence of the production of hydrogen and of helium in the electric discharge tube. It is, however, a matter of such great difficulty to prove the absence of these elements as a contamination in the materials used that the evidence of transformation has not carried conviction to the minds of the majority of scientific men.

In this lecture an account will be given of some preliminary experiments which indicate the possibility of artificial disintegration of some of the ordinary elements by a new method. Before discussing the results, it is desirable to say a few words on the modern conception of the structure of the atom. The results have been interpreted on the nuclear theory of atomic constitution. According to this view, the atom is to be regarded as consisting of a minute positively charged nucleus, in which most of the mass of the atom is concentrated, surrounded at a distance by a distribution of negative electrons which make the atom electrically neutral. We know that one or more of these outer electrons can be easily removed from the atom. The atom thus undergoes a kind of transformation, but only a temporary one, for the missing electrons are readily recaptured from neighbouring atoms. It seems not unlikely that the whole of the exterior electrons might be removed from an atom without interfering sensibly with the stability of its nucleus. Under suitable conditions, the atom would promptly regain its lost electrons and be indistinguishable from the original atom. In order to produce a *permanent* transformation of the atom, it is necessary to disintegrate the nucleus. Such a disruption of the nucleus occurs spontaneously in the radio-active atoms, and the processes appear to be irreversible under ordinary conditions.

The nucleus, however, is very small, and its constituent parts are probably held together by strong forces ; and only a few agencies are available for an attack on its structure. The most concentrated source of energy at our command is a swift α-particle, and it is to be expected that an α-particle in its flight would occasionally approach so close to the nucleus as to disintegrate its structure. It is, indeed, from a study of the deflexion of swift α-particles in passing through matter that we have obtained the strongest evidence in support of the theory of the nuclear constitution of atoms. In the region surrounding a heavy nucleus, the inverse square law holds for the forces of repulsion between the charged α-particle and positively charged nucleus. The particle describes a hyperbolic orbit round the nucleus, and the amount of its deflexion depends on the closeness of its approach. It is from a study of this scattering of α-particles, combined with Moseley's interpretation of the X-ray spectra of the elements, that we know the magnitude of the resultant positive charge on the nucleus. This charge, in fundamental units, is equal to the atomic or ordinal number of the element, and varies between 1 for hydrogen and 92 for uranium. Recently Chadwick has shown by direct measurements of the scattering of α-particles that the charge on the nucleus is in close accord with Moseley's deduction, and has thus verified the correctness of this fundamental conclusion.

Some information about the dimensions of the nucleus can be obtained by studying the amount of scattering of α-particles at large angles by different atoms. The general results indicate that the nucleus of a heavy atom, if assumed spherical, cannot have a radius greater than 6×10^{-12} cm. It is not unlikely that the dimensions may be smaller than this. No doubt the size of a nucleus decreases with its atomic mass, and it is to be expected that the nucleus of the light elements should be smaller than for the heavy atoms. It is thus clear that the volume occupied by the nucleus is exceedingly small compared with that occupied by the atom as a whole.

A direct collision of an α-particle with this minute nucleus is thus a rare occurrence. It can be estimated that even in the case of heavy elements only one α-particle in about 10,000 makes a close collision with the nucleus. On account of the powerful repulsive field of the latter, the α-particle may either be turned back before reaching the nucleus, or be so diminished in energy that it is unable to effect its disruption. The case of the lighter elements, however, is much more favourable; for the repulsive forces are so much weaker that we may expect the α-particle to enter the nucleus structure without much loss of energy, and thus to be an effective agent in promoting the disintegration of the atom.

One of the most interesting cases to consider is that in which an α-particle (helium nucleus) collides with the nucleus of an hydrogen atom. Marsden showed that in some cases the *H*-atom is set in such swift motion that it can be detected by the scintillation produced on a zinc sulphide screen. The maximum speed obtainable is 1·6 times that of the incident α-particle, and such a swift *H*-atom is able to travel four times as far as the α-particle before being stopped. For example, the maximum range of a *H*-atom set in motion by an α-particle from radium *C*—range 7 cm. in air—corresponds to 29 cm. of air.

A close examination of the production of swift *H*-atoms by this method showed that the number was about 30 times greater than that to be expected if the colliding nuclei behaved as point charges repelling each other according to the inverse square law. This, and other observations, show that the law of the inverse square ceases to hold in such intense collisions, where the closest distance of approach is of the order 3×10^{-13}cm. It is probable that this distance is comparable with the actual dimensions of the structure of the α-particle itself. Some recent experiments by Chadwick and Bieler indicate that there is an abrupt change in the law of force at distances of about 5×10^{-13} cm. So far, no definite evidence has been obtained as to the nature of these forces which arise in the close collisions between nuclei. Attention should be directed to the enormous intensity of the electrical forces that come into play in such close collisions—forces much greater than can be brought to bear on an atom by ordinary laboratory methods. Unless the nucleus is a very stable structure, it is to be anticipated that it should be greatly disturbed, if not disintegrated, under the influence of such intense forces.

We must now consider the experiments which indicate that some of the lighter elements can be disintegrated by the action of α-particles. When a stream of α-particles is passed through dry air or nitrogen, a number of scintillations are observed far beyond the range of the α-particle. These scintillations are due to swift charged particles which are bent in a magnetic field like H-atoms set in swift motion by α-particles, and which, indeed, are undoubtedly H-atoms. Since this effect is not observed in dry oxygen or carbon dioxide, it appears likely that some of the nuclei of nitrogen have been disintegrated by the action of the α-particles. Recently these experiments have been repeated by Mr. Chadwick and myself under much better optical conditions for counting these comparatively weak scintillations. It has been found that, using radium C as a source of α-rays, the maximum range of the H-atoms from nitrogen atoms corresponds to 40 cm. of air, while the maximum range of the H-atoms from hydrogen, or any combination of hydrogen, is only 29 cm. under similar conditions. This result negatives the possibility that the presence of these H-atoms can be ascribed to any hydrogen contamination in the ordinary chemical sense.

This observation opened up a simple method of examining other elements besides nitrogen. Experiments were in all cases made beyond the maximum range (29 cm.) of ordinary H-atoms, so as to be quite independent of the presence of free or combined hydrogen in the material under examination. In this way it has been found that similar particles are produced in boron, fluorine, sodium, aluminium, and phosphorus. No definite effect has so far been observed for other elements of the production of particles with ranges greater than 29 cm. of air. The question of the production of slower velocity H-atoms has not so far been examined. The range of penetration of the atoms from aluminium is specially marked, being more than 80 cm. While no definite proof has yet been obtained of the nature of these ejected particles, it seems probable that they are in all cases *H*-atoms liberated from the nuclei of the elements in question. It is of special interest to note that *H*-atoms are only liberated in elements whose mass is given by $4n+2$ or $4n+3$ where n is a whole number. No *H*-atoms are observed from elements like carbon, oxygen, and sulphur, whose mass is given by $4n$. This is an indication that the α-particles are unable to liberate *H*-atoms from elements composed entirely of helium nuclei, but are able to do so from some elements composed of *H*-atoms as units as well as helium nuclei. It would appear as if the *H*-atoms were satellites of the main nuclear system and that one of them gained sufficient energy from a collision with an α-particle to escape from its orbit with a high speed. If the long-range particles from aluminium are *H*-atoms, it can be calculated that the maximum energy of motion is somewhat greater than that of the incident α-particle, indicating that the escaping fragment of the atom has gained energy from the system. It is of special interest to note that, in the case of aluminium, the direction of escape of the *H*-atom is to some extent independent of the direction of the α-particle. Nearly as many are shot in the backward as in the forward direction, but in the former case the average velocity is somewhat smaller. No element of mass greater than phosphorus (31) has been found to yield *H*-atoms. It would appear as if the constitution of the nucleus undergoes some marked change at this stage.

It should be remarked that the disintegration observed in these experiments is on a very minute scale. Only about one α-particle in a million is able to get close enough to a nucleus to effect its disintegration.

So far we have only been able to observe those fragments of atoms which escape with sufficient speed to travel further than the α-particles. Another very important method of examining the effects produced within the range of the α-particle has been recently examined by Mr. Shimizu. This depends on the discovery of Mr. C. T. R. Wilson that the tracks of ionising radiations can be made visible by sudden expansion of a moist gas, so that each ion becomes the centre of a visible globule of water. Wilson had previously observed an occasional sharp bend in the track of an α-particle, with a short spur attached, indicating the collision of an α-particle with an oxygen or nitrogen nucleus. By taking a large number of photographs of tracks of α-particles, Mr. Shimizu found a number of cases in which the track of the α-particle near the end of its range showed two nearly equal forks. It can readily be shown from the range and angle between the forks that these effects cannot be ascribed to a collision of the α-particle with a H-atom, or with a nucleus of oxygen or nitrogen. It would appear not unlikely that these forks indicate an actual disruption of the atom in which a helium nucleus is released. While this conclusion is only tentative, it will be of great interest to follow up further this new method of attack of a fundamental problem. It is remarkable that while only one α-particle in a million is able to liberate a H-atom from nitrogen, about one α-particle in 300 appears to show a forked track, indicating that this type of disintegration occurs much more frequently than the liberation of a H-atom.

If this interpretation proves to be correct, it shows that the amount of energy required to liberate a helium nucleus from a complex nucleus of a light atom is not great. Such a result is not inconsistent with modern ideas of the relation between mass and energy, for the fact that the atomic masses of carbon and oxygen are very nearly integral multiples of the mass of the helium atom is an indication that the helium nuclei are bound loosely together. On the other hand, if we suppose the helium nucleus itself to be composed of four hydrogen nuclei and two electrons, the loss of mass in the combination indicates that the helium nucleus is so stable a structure that it should not be dissociated by even the swiftest α-particle. This conclusion is supported by experimental observations as far as they have gone.

ADDENDUM.—Fig. 7 *of Paper on "A New Registering Microphotometer." By* DR. W. J. H. MOLL, *University of Utrecht.* (*See Vol.* XXXIII., *Part* IV., 1921, *p.* 207.)

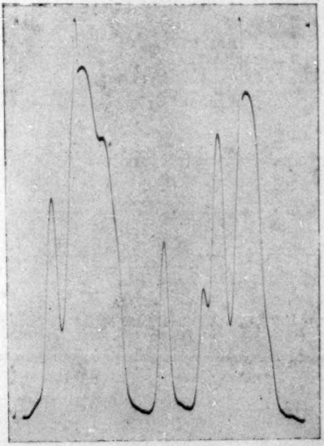

FIG. 7.—RECORD OF THE GREEN MERCURY LINE AS GIVEN BY AN ECHELON.

The three sharp points at the top have been obtained by intercepting the beam of light, by means of a screen in front of the thermopile, for one second.

INDEX.

A.

Page
Absorption of Gases in Discharge Tube 73
Allen, N. A., *on* Carbon Arc 62
Analysis, Micro-, of Gases 287
Appleton, E.V., *on* Retroaction in Triode Oscillator 100
Arc, Carbon, Current Density in 62
Archer, R. M., *see* Owen, Dr. D.
Atoms, Stability of 389

B.

Barr, Dr. Guy, *on* Electric Relay 53
Bond, W. N., *on* Viscosity in Orifice Flows 225
Bragg, Prof., Sir W. H., *on* X-ray Reflection by Diamond 304
Bragg, Prof. Sir W. H., *on* X-ray Spectra of Minute Crystals 222
Butterworth, S., *on* Inductometers 312

C.

Capacity and Eddy Current Effects in Inductometers 312
Chree, Dr. C., *on* an Electro-culture Problem 377
Clack, B. W., *on* Coefficient of Diffusion in Saturated Solutions 259
Crystals, Structure of Minute 222
Current Density in Carbon Arc 62

D.

Detectors, Vacuum Tube 249
Diamond, Intensity of X-ray Reflection by 304
Diffusion, Coefficient of, in Saturated Solutions 259
Discussions :—
    X-ray Spectra 1
    Resistance, Electrical, Absolute Measurement of, and Instruments based on Temperature-Variation of 125

E.

Eddy Current Effects in Inductometers 312
Electrical Resistance, Absolute Measurement of, and Instruments based on Temperature-Variation of 125
Electro-culture Problem, An 377
Encounters between Non-spherical Gas Molecules 362

F.

Page
Fluospar, Reflection of X-rays from 297
Frequencies, Measurement of 243
Fringe System in Uncompensated Interferometers 40

G.

Gases, Micro-analysis of 287
General Electric Co., Micro-analysis of Gases 287
Goucher, F. S., *on* Ionisation, &c., in Helium 13
Gravitating Fluid, Sphere of 116
Griffiths, Dr. A., *and* Mrs. C. H., *on* Viscosity of Water at Low Rates of Shear 231
Griffiths, Dr. E., *on* Specific Heat Apparatus 355
Guild, J., *on* Fringe Systems in Uncompensated Interferometers .. 40
Guild., J., *on* Location of Interference Fringes.......................... 32
Guild, J., *on* Refractometry of Prisms 179
Guthrie Lecture, Sixth 275

H.

Hartshorn, L. (*with* Keeping, E. S.), *on* Vacuum Tube Detectors.. 249
Helium, Ionisation, &c., in 13
Herroun, Prof. E. F. (*with* Wilson, Prof. E.), *on* Magnetic Susceptibility of Oxides 196
Hot-wire Inclinometer 171
Humphry, R. H., *on* Hot-wire Inclinometer 171

I.

Inclinometer, Hot-wire 171
Inductometers, Capacity and Eddy Current Effects in 312
Interference Fringes, Location of 32
Interference Methods, Some Recent Applications of 275
Interferometers, Fringe Systems in 40
Ionisation, &c., by Electron Impact in Helium 13

K.

Keeping, E. S., *see* Hartshorn, L.

L.

Lees, Dr. C. H., *on* Equilibrium of Fluid Sphere 116
Location of Interference Fringes.......................... 32

M.

Page

Magnetic Susceptibility of Oxides 196
Michelson, Prof. A. A., *on* Some Recent Applications of Interference Methods (Sixth Guthrie Lecture) 275
Micro-analysis of Gases.. 287
Microphotometer, Registering 207
Molecules, Gaseous, Non-spherical, Encounters between 362
Moll, Dr. W. J. H., *on* a Registering Microphotometer 207, 395

N.

Nagaoka, H., *on* Neon Lines and Runge's Rule.................. 83
Neon Lines and Runge's Rule 83
Newman, F. H., *on* Absorption of Gases in Discharge Tubes 73
Newman, F. H., *on* Sodium Vapour Discharge Tube.............. 70

O.

Optical System, Tracing Rays through 174
Owen, Dr. D. (*with* Archer, R. M.), *on* Quickness of Response in Thermionic Tube 104
Oxides, Magnetic Susceptibility of 196

P.

Palladium, X-ray Spectrum of, Reflection from Fluorspar........ 297
Parry, H., *on* Measurement of Power by Quadrant Electrometer.. 217
Partridge, G. F., (*see* Smith, B. S.)
Pealing, H., *on* X-ray Reflection from Fluorspar 297
Pletts, J. St. V., *on* Slide Rule Improvements 57
Power, Measurement of, by Quadrant Electrometer.............. 217
Prisms, Refractometry of.. 179

Q.

Quadrant Electrometer, Measurement of Power by 217
Quickness of Response of Current to Voltage in Thermionic Tube .. 104

R.

Radiation, Excitation by Electron Impact in Helium 13
Raman, Prof. C. V., Demonstration of Optical and Acoustical Experiments 388
Refractometry of Prisms 179
Relay for Moderately Heavy Currents.............................. 53
Resistance, Electrical, Absolute Measurement of, and Instruments based on Temperature-Variation of........................ 125
Response in Thermionic Tube, Quickness of 104
Retroaction in Triode Oscillator 100
Runge's Rule and Neon Lines 83
Rutherford, Prof. *Sir E.*, *on* The Stability of Atoms.............. 389

S.

Page

Slide Rule Improvements 57
Smith, B. S. (*with* Partridge, G. F.), *on* Frequency Measurement 243
Smith, T., *on* Tracing Rays through Optical System 174
Sodium Vapour Discharge Tube 71
Specific Heat Apparatus 355
Spectrometer, Ionisation, and Structure of Minute Crystals 222
Spectrum, (*see* X-ray Spectra)
Sphere of Gravitating Fluid, Equilibrium of 116
Stability of Atoms 389
Susceptibility, Magnetic, of Oxides 197

T.

Temperature-Variation of Resistance, Instruments based on 125
Thermal Transpiration Currents 266
Thermionic Tubes or Valves 100, 104
Tracing Rays through an Optical System 174
Transpiration Currents, Thermal 266
Triode (*see* Thermionic)

V.

Vacuum-Tube Detectors 249
Valves (*see* Thermionic Valves)
Viscosity, Effect of, on Orifice Flows 225
Viscosity of Water at Low Rates of Shear 231

W.

Water, Viscosity of, at Low Rates of Shear 231
West, Dr. G. D., *on* Thermal Transpiration Currents 267
Wilson, Prof. E. (*see* Herroun, Prof. E. F.)

X.

X-rays, Intensity of Reflection of, by Diamond 304
X-ray Spectra, Discussion on 1
X-ray Spectra of Minute Crystals 223
X-ray Spectrum of Palladium, Reflection from Fluorspar of 297